Quantization of Fayet-Iliopoulos parameters in supergravity

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joint w/ J Distler arXiv: 1008.0419,

and w/ S Hellerman, arXiv: 1012.5999

Also: N Seiberg, 1005.0002; Banks, Seiberg 1011.5120

A brief partial history of FI in ($\substack{\text{old}\\\text{minimal}}$) supergravity

(Dienes, Thomas, 0911.0677)

Old & complex literature on FI terms; some root issues:

* Any gauge group must be combined with the R symmetry; the FI term contributes to the charges of the gravitino, etc

(Freedman ’77, Stelle-West ’78, Barbieri et al ’82)

which violates electric charge quantization.

(Witten, ``New issues…”, ’86, footnote p 85)

This led to the lore that FI terms couldn’t exist in 4d N=1 sugrav; however, recently….

* Sol’n: quantize the FI term.

(Seiberg, ’10)

Seiberg worked w/ linearly-realized gp actions;
I’ll describe today how to generalize, in classical theory.

Disambiguation:

* In 2009, Komargodski-Seiberg considered sugrav theories obtained by coupling rigid theories to gravity. (Not all sugrav's of this form.)

  Found that Kahler form on sugrav moduli space must be exact, and FI parameter must vanish.

* Seiberg's 2010 paper concerns more general sugrav's, not given by coupling a rigid theory.

  It's in these more general theories (inc. string compactifications) that FI can be nonzero, and this is what I will focus on today.

Outline:

- review Bagger-Witten (basis for FI)
- quantization of FI parameters in sugrav when sugrav moduli space is a **space**

But sometimes one has a gauge theory (equiv'ly, a ``stack'' instead of a space)

- review discrete gauge symmetries in string theory (stacks and special stacks called gerbes)
- exs & prop's of gerby moduli spaces in field and string theory
- Bagger-Witten, FI quantization for gauge theories (ie, when moduli space is a stack)

Review of Bagger-Witten:

(basis for our discussion of FI)

Bagger-Witten's pertinent paper studied
N=1 sugrav in 4d.

Now, as a sugrav theory, it contains a
(low-energy effective) 4d NLSM on a space,
the supergravity moduli space.

They derived a constraint on the metric on that
moduli space
(assuming the moduli space is a smooth manifold).

Review of Bagger-Witten:

Briefly, the supergravity moduli space *M*
(the target space of a 4d NLSM)
comes with a natural line bundle *L*,
whose c_1 = Kahler form.
(hence quantized)

Sketch of derivation:

Across coordinate patches,
Kahler potential $K \mapsto K + f + \overline{f}$

In rigid susy, the action

$$\int d^4\theta\, K(\Phi,\overline{\Phi}) + \int d^2\theta\, W(\Phi) + \text{c.c.}$$ is invariant.

In sugrav, the action

$$\int d^2\Theta\, 2E\left[\frac{3}{8}(\overline{D}\overline{D} - 8R)\exp\left(-K(\Phi,\overline{\Phi})/3\right) + W(\Phi)\right] + \text{c.c.}$$ is not.

Review of Bagger-Witten:

Sketch of derivation:

Across coordinate patches,

Kahler potential $K \mapsto K + f + \overline{f}$

Problem: Sugrav action not invariant.

Fix: demand fermions transform also.

Require

$$\chi^i \mapsto \exp\left(+ \frac{i}{2} \operatorname{Im} f \right) \chi^i \quad \& \quad \psi_\mu \mapsto \exp\left(- \frac{i}{2} \operatorname{Im} f \right) \psi_\mu$$

hence

$$\chi^i \in \Gamma(\phi^*(TM \otimes L)) \qquad \psi_\mu \in \Gamma(TX \otimes \phi^* L^{-1})$$

for some line bdle *L* determined by f's.

L = Bagger-Witten line bundle

Review of Bagger-Witten:

Can show:

- The line bundle L determines metric on the Fermi kinetic terms, & that metric => Kahler form.

- Positivity of the Fermi kinetic energy implies that L^{-2} is a `positive' line bundle (technically, `ample').

- The superpotential W is a meromorphic section of L^2, which is `negative'.

As a result, W has poles.

Review of Bagger-Witten:

B-W also appears in worldsheet physics.

(Distler, Periwal-Strominger, ’90)

Vertex operators for spacetime supercharge, gravitino, scalar superpartners, etc, contain spectral flow operator of the N=2 algebra.

(Recall this is the operator that rotates R $\leftrightarrow$ NS sectors.)

The spectral flow operator is charged under U(1) subset of N=2 algebra.

As walk around loops on SCFT moduli space, the N=2 algebra rotates into itself up to global symmetry transformations — U(1) transformations.

Thus, spectral flow operator (& gravitino etc) are sections of a line bundle over the SCFT moduli space.

Review of Bagger-Witten:

The spectral flow operator is, in a NLSM,
= (holomorphic top-form) f(moduli)

(Lerche-Vafa-Warner '89)

Along cpx moduli, B-W line bundle is the Hodge bundle
= line bundle of hol' top forms over CY moduli space

Picard-Fuchs equ'ns can give holonomies of that line bundle.

(Candelas, de la Ossa, Font, Katz, Morrison, '93)

On (2,2) locus in heterotic compactification,
singularities in spacetime superpotential W
arise from divergent sums of rat'l curves.

Ex: if BW line bundle *L* is trivial,
then no need for W to have poles, hence expect no GW inv'ts.

Review of Bagger-Witten:

Suffice it to say, in string theory,
the Bagger-Witten line bundle is in general nontrivial,
and is related to e.g. Gromov-Witten invariants.

As an aside, when one takes rigid limits of sugrav theories,
one obtains NLSM's with trivial BW line bundles.

(Lapan et al '11, Festuccia-Seiberg '11)

- consequence of units in EFT analysis
- dovetails with fact that in rigid limit, no strings,
  hence no GW inv'ts, so expect BW to be trivial.

Bagger-Witten dates to early ‘80s.

More recently, there has been progress on FI terms in 4d sugrav.

Seiberg in May 2010 argued that, in the special case that the group action on the sugrav moduli space is realized linearly (ie, moduli space = vector space V, group acting as a subgp of GL(V)), the FI term exists & is quantized.

I’ll discuss generalization to nonlinear realizations (ie, gen’l Kahler moduli spaces) today.

Bagger-Witten’s story plays a crucial role.

Quick & dirty argument for FI quantization:

Continuously varying the FI term,
continuously varies the symplectic form on the quotient space.

But that symplectic form = Kahler form,
& Bagger-Witten says is quantized.

Consistency requires FI term be quantized too.

Problem:

— IR limit not nec' same as NLSM, so irrelevant to B-W

Nice intuition, but need to work harder.

To gain a more complete understanding, let's consider gauging the Bagger-Witten story.

Have:

- sugrav moduli space M
- line bundle L over M (Bagger-Witten)
- group action on moduli space M

Need to specify how group acts on L ….

In principle, if we now wish to gauge a group action on the supergravity moduli space *M*, then must specify how the group action lifts to the line bdle *L*.

- **Not** always possible:

  Group actions on spaces do not always lift to bundles.

  Ex: spinors under rotations; rotate 4π instead of 2π.

  — classical constraint on sugrav theories…

- **Not** unique:

  when they do lift, there are multiple lifts, differing by gauge transformations in essence.

  (These different lifts will be the FI parameters.)

We'll see FI as a choice of group action on the Bagger-Witten line bundle directly in sugrav.

First: what is D ?

For linearly realized group action,

If scalars ϕ_i have charges q_i w.r.t. U(1), then

$$D = \sum_i q_i |\phi_i|^2$$

up to additive shift (by Fayet-Iliopoulos parameter).

How to describe D more generally?

Def'n of D more generally:

$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$ describes infinitesimal group action on M

where $X^{(a)} = X^{(a)i} \frac{\partial}{\partial \phi^i}$ ``holomorphic Killing vector''

`Killing' implies

$$\nabla_i X_j^{(a)} + \nabla_j X_i^{(a)} = 0$$

$$\nabla_{\bar{\imath}} X_j^{(a)} + \nabla_j X_{\bar{\imath}}^{(a)} = 0$$

which implies

$$g_{i\bar{\jmath}} X^{(a)\bar{\jmath}} = \mathrm{i} \frac{\partial}{\partial \phi^i} D^{(a)}$$

$$g_{i\bar{\jmath}} X^{(a)i} = -i \frac{\partial}{\partial \phi^{\bar{\jmath}}} D^{(a)}$$

for some $D^{(a)}$ — defines $D^{(a)}$ up to additive shift (FI)

Closer examination of the supergravity:

$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$ describes infinitesimal group action on M

$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc} \epsilon^{(b)} A_\mu^{(c)}$ gauge transformation

$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$

where $F^{(a)} = X^{(a)} K + \mathrm{i}\, \mathrm{D}^{(a)}$

Recall $K \mapsto K + f + \overline{f}$ implies

$$\chi^i \mapsto \exp\left(+ \frac{i}{2} \mathrm{Im}\, f \right) \chi^i \quad \& \quad \psi_\mu \mapsto \exp\left(- \frac{i}{2} \mathrm{Im}\, f \right) \psi_\mu$$

Hence

- gp action on χ^i, ψ_μ includes $\mathrm{Im}\,\mathrm{F}^{(\mathrm{a})}$ terms
- This will define the group action on *L*

Indeed:

$\delta\phi^i = \epsilon^{(a)} X^{(a)i}$ describes infinitesimal group action on M

$\delta A_\mu^{(a)} = \partial_\mu \epsilon^{(a)} + f^{abc} \epsilon^{(b)} A_\mu^{(c)}$ gauge transformation

$$\delta K = \epsilon^{(a)} F^{(a)} + \epsilon^{(a)} \overline{F}^{(a)}$$

where $F^{(a)} = X^{(a)} K + \mathrm{i}\, \mathrm{D}^{(a)}$

$\delta\lambda^{(a)} = f^{abc} \epsilon^{(b)} \lambda^{(c)} - \frac{i}{2} \epsilon^{(a)} (\mathrm{Im}\, F^{(a)}) \lambda^{(a)}$ gaugino

$\delta\chi^i = \epsilon^{(a)} \left[\frac{\partial X^{(a)i}}{\partial \phi^j} \chi^j + \frac{i}{2} \left(\mathrm{Im}\, F^{(a)} \right) \chi^i \right]$ scalar superpartner

$\delta\psi_\mu = - \frac{i}{2} \epsilon^{(a)} \left(\mathrm{Im}\, F^{(a)} \right) \psi_\mu$ gravitino

Encode infinitesimal action on *L*

We need the group to be represented faithfully.

Infinitesimally, the *D*'s can be chosen to obey

$$\left(X^{(a)i}\,\partial_i + X^{(a)\bar{\imath}}\,\partial_{\bar{\imath}}\right)D^{(b)} = -\, f^{abc}D^{(c)}$$

and then

$$\delta^{(b)}\epsilon^{(a)}\mathrm{Im}\,F^{(a)} - \delta^{(a)}\epsilon^{(b)}\mathrm{Im}\,F^{(b)} = -\,\epsilon^{(a)}\epsilon^{(b)}f^{abc}\mathrm{Im}\,F^{(c)}$$

If the group is semisimple,
the constraints above will fix *D*.
If there are U(1) factors, must work harder….

Next: constraints from representing **group** ….

An infinitesimal action is not enough.

Need an action of the **group** on *L*,
not just its Lie algebra.

Lift of $g = \exp\left(i\epsilon^{(a)}T^a\right)$ to line bundle *L*

is $\tilde{g} = \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right)$

Require $\tilde{g}\tilde{h} = \widetilde{gh}$

so that the group is honestly represented.

(This is the part that can't always be done.)

The lifts $\tilde{g}$ might not obey $\tilde{g}\tilde{h} = \widetilde{gh}$ initially,

but we can try to adjust them:

Since $F^{(a)} = X^{(a)}K + iD^{(a)}$,

shifting $D^{(a)} \mapsto D^{(a)} + \alpha^{(a)}$

is equivalent to adding a phase to $\tilde{g}$:

$$\tilde{g} \equiv \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right) \mapsto \exp\left(\frac{i}{2}\epsilon^{(a)}\mathrm{Im}\,F^{(a)}\right)\exp\left(\frac{i}{2}\epsilon^{(a)}\alpha^{(a)}\right)$$

$$= \tilde{g}\exp(i\theta_g)$$

for $\theta_g = \frac{1}{2}\epsilon^{(a)}\alpha^{(a)}$ encoding the shift in $D^{(a)}$.

If the lifts $\tilde{g}$ do not obey $\tilde{g}\tilde{h} = \widetilde{gh}$,

then we can shift $D^{(a)}$ to add phases:

$$\tilde{g} \mapsto \tilde{g}\exp\left(i\theta_g\right)$$

That **might** fix the problem, **maybe**.

Globally, the group $\tilde{G}$ formed by the $\tilde{g}$ is an extension

$$1 \to U(1) \to \tilde{G} \to G \to 1$$

If that extension splits, meaning $\tilde{G} \cong G \oplus U(1)$,
we can fix the problem.

If not, we're stuck — cannot gauge G,
not even classically.

(new consistency condition on classical sugrav)

Let's assume the extension splits, so $\tilde{G} \cong G \oplus U(1)$

and we can fix the problem and gauge G (classically).

In this case, there are multiple consistent $\{\tilde{g}\}$'s, differing by phases.

Those different possibilities correspond to the different possible FI parameters — remember, the phases originate as shifts of $D^{(a)}$.

Let's count them.
We'll see they're quantized.

Count set of possible consistent lifts $\{\tilde{g}\}$:

Start with one set of consistent lifts $\{\tilde{g}\}$,
i.e. lifts obeying $\tilde{g}\tilde{h} = \widetilde{gh}$

Shift *D*: $\tilde{g} \mapsto \tilde{g}' \equiv \tilde{g}\exp\left(i\theta_g\right)$

Demand $\tilde{g}'\tilde{h}' = \widetilde{gh}'$

This implies $\theta_g + \theta_h = \theta_{gh}$

Result: Set of lifts is $\mathrm{Hom}(\mathrm{G},\mathrm{U}(1))$

(= set of possible FI parameters)

So far: set of possible lifts is $\mathrm{Hom}(G, U(1))$

- This is a standard math result
  for lifts of group actions to line bundles.
  (though the sugrav realization is novel)

- Lifts = FI parameters,
  so we see that FI parameters are quantized.

Ex: $G = U(1)$ $\mathrm{Hom}(\mathrm{G}, \mathrm{U}(1)) = \mathbb{Z}$

— integrally many lifts / FI parameters

Ex: G semisimple $\mathrm{Hom}(\mathrm{G}, \mathrm{U}(1)) = 0$

— only one lift / FI parameter

Summary so far:

Although the $D^{(a)}$ were only defined up to const' shift:

$$g_{i\bar{j}} X^{(a)\bar{j}} = \mathrm{i} \frac{\partial}{\partial \phi^i} D^{(a)}$$

the constraint $\tilde{g}\tilde{h} = \widetilde{gh}$

determines their values up to a (quantized) shift by elements of Hom(G,U(1)) = possible values of FI parameter

Supersymmetry breaking

Is sometimes forced upon us.

If the FI parameters could be varied continuously, then we could typically solve $D=0$ just by suitably varying FI.

Since the FI parameters are quantized in sugrav, sometimes one cannot solve $D=0$ for any available FI parameter.

Supersymmetry breaking

Example: $M = \mathbb{P}^1$ $G = SU(2)$ (Bagger, 1983)

$$\mathrm{Hom}(\mathrm{SU}(2), \mathrm{U}(1)) = 0$$

so FI, equivariant lift are unique

For Bagger-Witten $L = O$(n),

$$(D^{(1)})^2 + (D^{(2)})^2 + (D^{(3)})^2 = \left(\frac{n}{2\pi} \right)^2$$

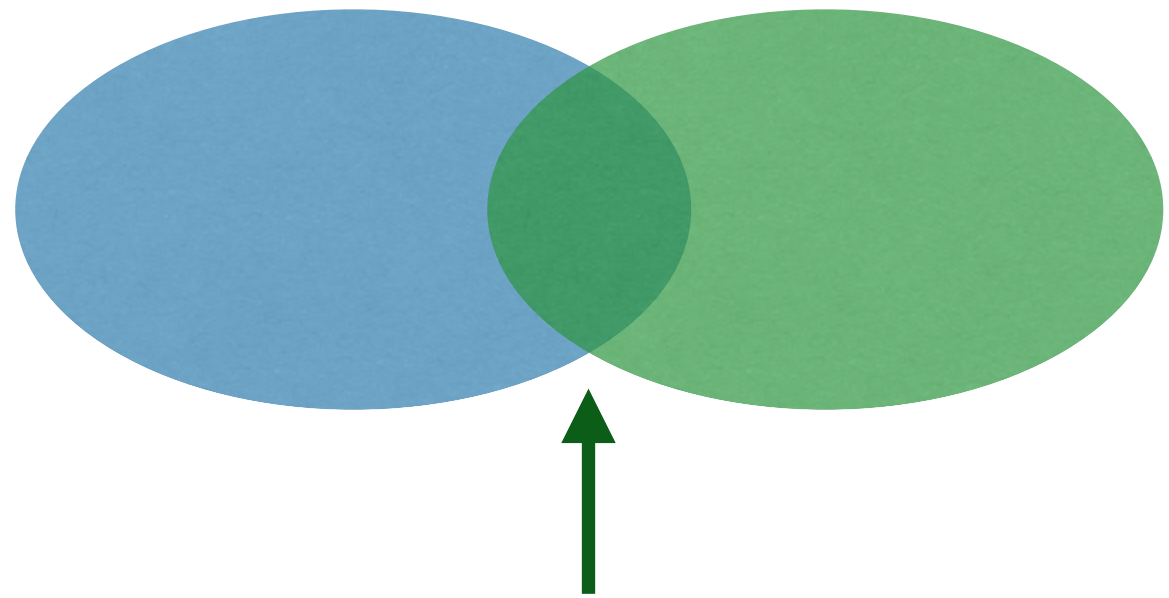
(Use $D^a = \phi T^a \phi$ on $\mathbb{P}^1$, plus fact that *D*'s obey Lie algebra rel'ns to fix the value above.)

susy always broken in this example

Math interpretation

- In rigid susy, gauging ~ symplectic reduction
- Symplectic quotients do **not** have a restriction to integral Kahler classes; this cannot be a symplectic quotient.
- Instead, propose: **GIT quotients**.
- Symplectic / GIT sometimes used interchangeably; however, GIT quotients restrict to integral classes.

Symplectic quotients



GIT quotients

complex Kahler manifolds,
integral Kahler forms

Why should GIT be relevant?

- To specify GIT,
  need to give an ample line bundle on original space,
  that determines a projective embedding.

  (= Bagger-Witten line bundle;
  `ample' from kinetic term positivity)

- Must specify a group action on that line bundle;
  Kahler class ultimately determined by that group action,
  in same fashion as here.

Same structure as here: thus, sugrav = GIT

So far, we've discussed B-W & FI for sugrav's whose rigid limit includes a NLSM on a smooth manifold.

This is inadequate:

- In a heterotic string compactification, for example, get a gauge theory, not a ungauged NLSM.

Even in a weakly-coupled Higgs phase, will still often have a residual finite gauge group.

- Mathematically, moduli spaces of CY's are **never** smooth manifolds.

Instead, they're (Deligne-Mumford) stacks.

These two issues solve one another:
get gauge theory   if & only if   moduli space is a stack.

Briefly,

Working in a finite gauge theory will make the following modifications.

- Still have something like a Bagger-Witten line bundle, but, transition functions no longer close on triple overlaps.

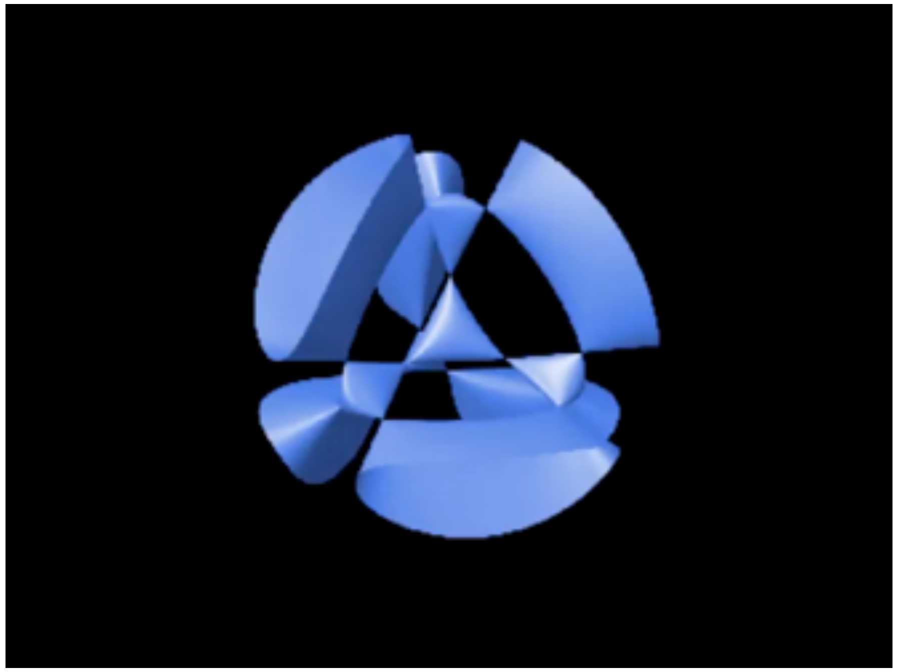
Instead, transition functions only close up to a (finite) gauge transformation.

Result is a generalization of a line bundle, albeit with **fractional** Chern classes.

Get basically same Bagger-Witten story, but now with fractional Kahler classes, and fractional FI parameters.

I’m sure you’re all well-acquainted with gauge theories, so let me focus on explaining the other half: **stacks**.

A stack is a generalization of a space.



Idea: defined by incoming maps.

(and so nicely suited for NLSM’s;
just have path integral sum over what
the def’n gives you)

Example: A space X as a stack

For every other space Y, associate to Y the set of continuous maps Y —> X

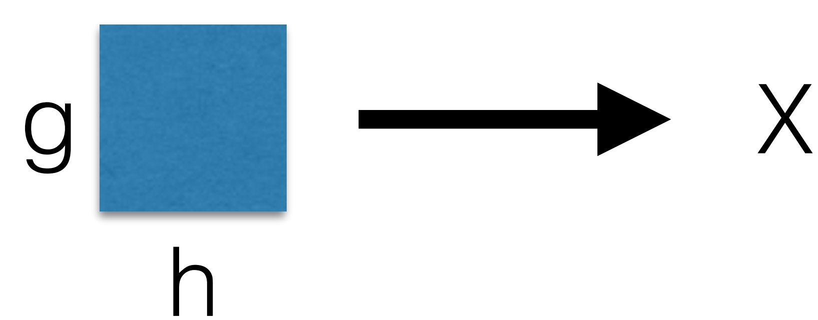
Example: A quotient stack [X/G]

Maps Y —> [X/G] are pairs

(principal G bundle (w/ connection) E on Y, G-equivariant map E —> X)

— the same data appearing in gauged NLSM

If $Y=T^2$ & G finite,



= twisted sector maps in string orbifold

All smooth `Deligne-Mumford' stacks (over $\mathbb{C}$)
can be described as [X/G]
for some X, some G

(G not necessary finite, not necessarily effectively-acting
— these are not all orbifolds)

Program:

A NLSM on a stack
is a G-gauged sigma model on X

Problem: such presentations not unique;
same stack can be described by several X, G

Potential fix: RG flow

2d: extensive checks. 4d: much less work.

Let's consider a particularly interesting kind of stack.

(& then, BW & FI for sugrav's on such)

Consider NLSM's in which the sum over nonperturbative sectors has been restricted;
only sum over maps of degree obeying divisibility property.

(Can also build via coupling to TFT.)

Since stacks describe, in essence, all possible NLSM's, naturally this is a kind of stack.

Specifically, this sort of stack is known as a **gerbe**.

These are equivalent to gauge theories in which a finite subgroup acts trivially.

(ES, Distler, Pantev, Hellerman, '05, GW '01; Seiberg, Banks-Seiberg '10)

Gauge theories w/ finite trivially-acting subgroup

Example: Gerby susy $\mathbb{CP}^N$ model in two dimensions

- 2d U(1) susy gauge theory
- N+1 chiral superfields, charge k
  - — nonminimal charges
  - — global unbroken $\mathbb{Z}_k$ — acts trivially on fields

How can this differ from ordinary susy $\mathbb{CP}^N$ model ?

Answer: nonperturbative effects

The difference lies in nonperturbative effects.
(Perturbatively, having nonminimal charges makes no difference.)

2d: Argument for compact worldsheet:

To specify a field completely,
need to specify what bundle it couples to.

For example, if the gauge field ~ L,
then for Φ to have charge Q means

$$\Phi \in \Gamma(L^{\otimes Q})$$

Different bundles => different zero modes
=> different anomalies => different physics

Argument for noncompact worldsheet:

Utilize the fact that in 2d,
theta angle acts as electric field.

Want Higgs fields to have charge k
at the same same that instanton number is integral.

Latter is correlated to periodicity of theta angle;
can fix to desired value by adding massive charge ± 1
fields — for large enough separation, can excite,
and that sets periodicity.

**(J Distler, R Plesser, Aspen 2004 & hepth/0502027, 0502044, 0502053;
N Seiberg, Banks-Seiberg, 2010)**

So far, only discussed 2d case.

There is a closely analogous argument in related four-dimensional models coupled to gravity.

Instead of theta angle,
use Reissner-Nordstrom black holes.

Idea: if all states in the theory have charge a multiple of k,
then gerbe theory is same as ordinary one,
just rescale charges.

However, if have massive minimally-charged fields,
then a RN BH can Hawking radiate down to charge 1,
and so can sense fields with mass > cutoff.

(J Distler, private communication)

Return to the 2d gerby $\mathbb{CP}^N$ example:

Compare physics of this and ordinary $\mathbb{CP}^N$ model:

Example: Anomalous global U(1)'s

$\mathbb{CP}^{N-1}$: $U(1)_A \mapsto \mathbb{Z}_{2N}$

Here: $U(1)_A \mapsto \mathbb{Z}_{2kN}$

Example: A model correlation functions

$\mathbb{CP}^{N-1}$: $\langle \omega^{N(d+1)-1} \rangle = \mathrm{q}^d$

Here: $\langle \omega^{N(kd+1)-1} \rangle = \mathrm{q}^d$

Example: quantum cohomology

$\mathbb{CP}^{N-1}$: $\mathbb{C}[x]/(x^N - q)$

Here: $\mathbb{C}[x]/(x^{kN} - q)$

Different physics

More generally, for 2d gerbe theories, there are somewhat extensive results.

- quantum cohomology rings
- mirror symmetry, incl. Toda duals
- decomposition conjecture for (2,2) susy theories….

(will describe next)

Decomposition conjecture

(Hellerman, Henriques, Pantev, Sharpe, etc)

In the special case of `banded' gerbes, the decomposition conjecture says

$$\text{CFT(G-gerbe on X)} = \text{CFT}\left(\coprod_{\hat{G}} (X,B) \right)$$

finite gauge theory

disjoint union of spaces

where the B field is determined by the image of

$$H^2(X,Z(G)) \xrightarrow{Z(G)\to U(1)} H^2(X,U(1))$$

More gen'ly, disjoint union of **different** spaces.

Example:

Consider $[X/D_4]$ where the center acts trivially.

$$1 \to \mathbb{Z}_2 \to D_4 \to \mathbb{Z}_2 \times \mathbb{Z}_2 \to 1$$

Can show this orbifold is physically distinct from $[X/\mathbb{Z}_2 \times \mathbb{Z}_2]$; for example,

$$Z([X/D_4]) = Z\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2] \coprod [X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}} \right)$$

Suffice it to say,
there's been considerable work done on the 2d case.

Pertinent here: 4d.

Specifically,
4d NLSM on sugrav moduli stack
= 4d gauged NLSM, finite gauge group

Much less work done;
I'll outline some results and issues.

Four dimensions

Example:

Consider a U(1) susy gauge theory in 4d,
with N (massless) chiral superfields of charge k,
and N of charge -k.

To be physically different from the charge 1 case,
need either:

- topologically nontrivial 4d spacetime
  (so that there are U(1) instantons)

or

- massive fields of charge +1, -1

(parallels 2d case)

Examples of gerby moduli spaces do exist in string theory:

Consider toroidally-compactified $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string.

Low-energy theory has only adjoints,
hence all invariant under $\mathbb{Z}_2$ center of $\mathrm{Spin}(32)/\mathbb{Z}_2$.

But, there are massive states that do see the center.

So: Higgs phase has finite gauge group,
acting trivially on massless matter.

Math'ly, equivalent observation is that the moduli space of flat $\mathrm{Spin}(32)/\mathbb{Z}_2$ connections has $\mathbb{Z}_2$ gerbe structure.

One can get enhanced gerbe structures along various strata.

Ex: toroidally-compactified $E_8 x E_8$ heterotic string

- no center, so no gerbe structure globally

- but, over stratum where $E_8 x E_8$ broken to

$$\mathrm{Spin}(16)/\mathbb{Z}_2 \times \mathrm{Spin}(16)/\mathbb{Z}_2$$

there is a $\mathbb{Z}_2 \times \mathbb{Z}_2$ gerbe structure,
matching the corresponding $\mathrm{Spin}(32)/\mathbb{Z}_2$ compactification.

Related examples in Seiberg duality:

Several years ago, Matt Strassler was very interested in Spin/SO Seiberg duals.

(hepth/9507018, 9510228, 9709081, 9808073)

Prototypical example:

- Spin(8) gauge theory with N_f fields in $\mathbf{8}_V$, and one massive $\mathbf{8}_S$

Seiberg dual to

- $SO(N_f-4)_-$ gauge theory with N_f vectors (from Higgsing $SU(N_f-4)$ theory)

massive $\mathbf{8}_S$ <—> $\mathbf{Z}_2$ monopole

$$\pi_2(SU(N_f-4)/SO(N_f-4)) = \mathbb{Z}_2$$

- Spin(8) gauge theory with N_f fields in $\mathbf{8}_V$,
  and one massive $\mathbf{8}_S$

Seiberg dual to

- $SO(N_f-4)_-$ gauge theory with N_f vectors
  (from Higgsing $SU(N_f-4)$ theory)

massive $\mathbf{8}_S$ <—> $\mathbf{Z}_2$ monopole

Important for his analysis that a $\mathbf{Z}_2$ center of Spin(8)
acted trivially on massless matter,
but nontrivially on the massive $\mathbf{8}_S$

— so $\mathbf{Z}_2$ gerbe structure on moduli space on one side

Apply to BW & quantization of FI parameters:

For a simple example,
consider the (anomalous) 4d gerby $\mathbb{CP}^N$ model:

- U(1) gauge theory
- N+1 chiral superfields charge k

now in supergravity

(The anomaly is irrelevant;
more complicated anomaly-free examples exist.)

- U(1) gauge theory
- N+1 chiral superfields charge k

D terms: $$\sum_i k\,|\phi_i|^2 = r$$

$$\Rightarrow \sum_i |\phi_i|^2 = r/k$$

But r is an integer…

Result looks like ordinary $\mathbb{CP}^N$ model,
but now with fractional Kahler class or FI term.

Interpretation ?

We've argued that FI integrality in sugrav follows b/c FI term is a choice of equivariant structure on the Bagger-Witten line bundle.

Over a gerbe, there are `fractional' line bundles.

Ex: gerbe on $\mathbb{CP}^N$

Has homogeneous coordinates

$$[x_0, \cdots, x_N] \cong [\lambda^k x_0, \cdots, \lambda^k x_N]$$

Can define a line bundle *L* by $y \mapsto \lambda^n y$

Has $c_1 = n/k$ Call it $O(n/k)$

If n/k not integer, then transition functions close on triple overlaps only up to element of $\mathbb{Z}_k$

Call such line bundles **fractional**.

Let's redo Bagger-Witten,
when the sugrav moduli space is a stack or gerbe
(= have NLSM + finite gauge theory)

For same reasons as ordinary case,

$$\chi^i \in \Gamma(\phi^*(TM \otimes L)) \qquad \psi_\mu \in \Gamma(TX \otimes \phi^* L^{-1})$$

Now, however, M is a stack or gerbe.

Stacks & gerbes have more (ie fractional) bundles
than their underlying spaces,
so *L* can be fractional.

That's what's happening in the previous r/k example.

Potential issue:

Fractional line bundles on gerbes have no smooth sections, only multisections with branch cuts.

However, all maps into gerbes
= maps into spaces w/ divisibility constraint,
which turns out to
ensure pullback bundles are honest bundles.

So, even if L is fractional,
$\phi^* L$ is an honest bundle,
and so no branch cuts in χ^i, ψ_μ .

So far:

- outlined how BW, FI can be fractional,
  at least in principle

- Also seen exs of gerby moduli spaces in string theory

- But, do fractional BW, FI ever arise in actual
  string compactifications?

Do fractional BW, FI admit UV completions?

Open question.

Have we missed any subtleties?

Maybe:

The 2d analogue is heterotic string on a gerbe (gauge group acts trivially on space, nontrivially on bundle).

These **sometimes** break modular invariance; details not well understood.

Speculation: possible that (some) 4d examples suffer from (discrete) anomalies.

Summary:

- reviewed Bagger-Witten
- quantization of FI parameters in sugrav,
  for ordinary NLSM's (= moduli space is a space)
- reviewed stacks

  NLSM on stack = gauged NLSM
- exs & prop's of gerby moduli spaces in field & string thy
- Bagger-Witten, FI quantization when moduli space
  is a stack or gerbe — get fractional results