GLSM's, gerbes, and Kuznetsov's homological projective duality

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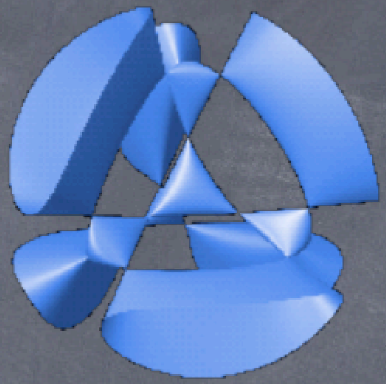
T Pantev, ES, hepth/0502027, 0502044, 0502053
S Hellerman, A Henriques, T Pantev, ES, M Ando, hepth/0606034
R Donagi, ES, arxiv: 0704.1761
A Caldararu, J Distler, S Hellerman, T Pantev, ES, arXiv: 0709.3855
J. Guffin, ES, arXiv: 0801.3836, 0803.3955

Outline

- Basics of string compactifications on stacks
- Cluster decomposition conjecture for strings on gerbes:
  CFT(gerbe) = CFT(disjoint union of spaces)
- Application to GLSM's; realization of Kuznetsov's homological projective duality
- LG models; h.p.d. via matrix factorizations

Stacks



Stacks are a mild generalization of spaces.

One would like to understand strings on stacks:

-- new string compactifications

-- better understand certain existing string compactifications

Next: how to construct QFT's for strings propagating on stacks?

Stacks

How to make sense of strings on stacks concretely?

Most (smooth, Deligne-Mumford) stacks can be presented as a global quotient

$$[X/G]$$

for X a space and G a group.

(G need not be finite; need not act effectively.)

To such a presentation, associate a ``G-gauged sigma model on X.''

Problem: such presentations not unique

Stacks

If to [X/G] we associate "G-gauged sigma model,"
then:

$$[\mathbf{C}^2/\mathbf{Z}_2]$$

defines a 2d theory with a symmetry
called conformal invariance

$$=$$

$$\neq$$

$$[X/\mathbf{C}^\times]$$

$$\left(X = \frac{\mathbf{C}^2 \times \mathbf{C}^\times}{\mathbf{Z}_2} \right)$$

defines a 2d theory
w/o conformal invariance

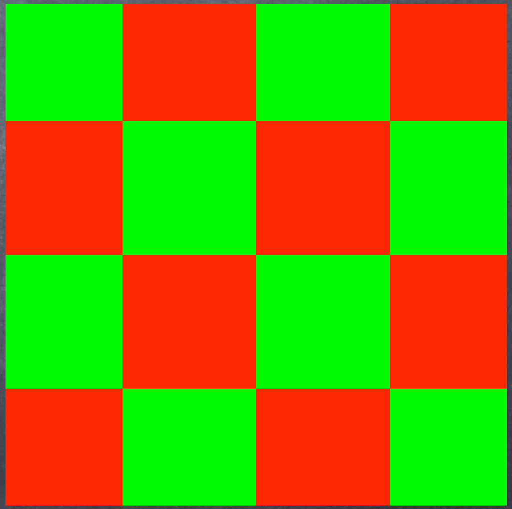
Same stack, different physics!

Potential presentation-dependence problem:
fix with renormalization group flow
(Can't be checked explicitly, though.)

Renormalization (semi)group flow

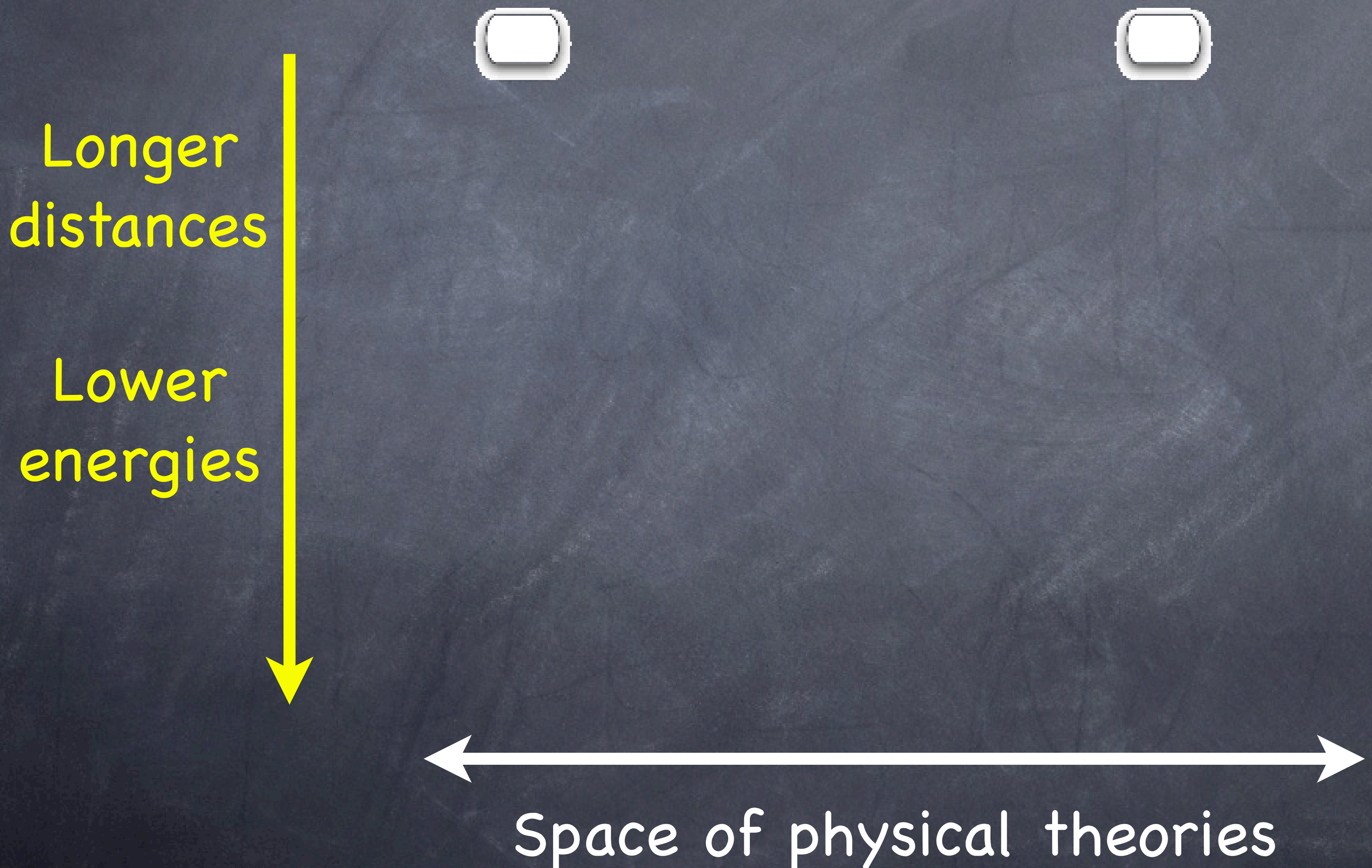
Constructs a series of theories that are approximations to the previous ones, valid at longer and longer distance scales.

The effect is much like starting with a picture and then standing further and further away from it, to get successive approximations; final result might look very different from start.



Problem: cannot follow it explicitly.

Renormalization group

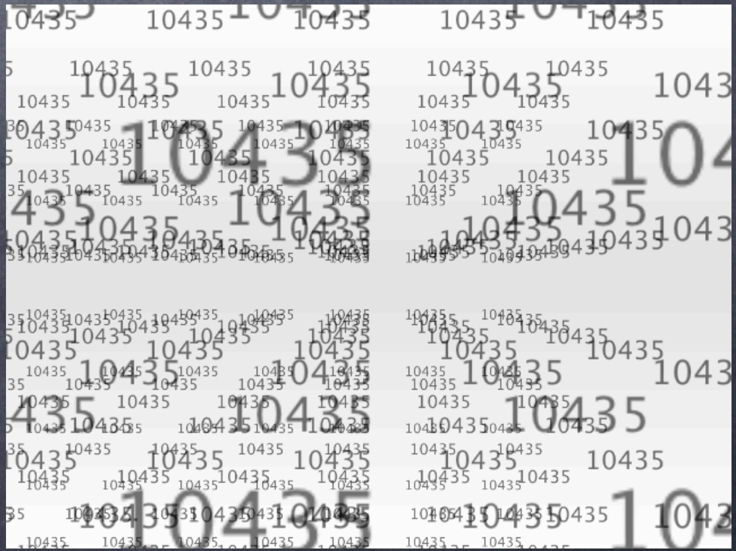


Renormalization group

-- is a powerful tool, but unfortunately we really can't follow it completely explicitly in general.

-- can't really prove in any sense that two theories will flow under renormalization group to same point.

Instead, we do lots of calculations, perform lots of consistency tests, and if all works out, then we believe it.



The problems here are analogous to the derived-categories-in-physics program.

There, to a given object in a derived category, one picks a representative with a physical description (as branes/antibranes/tachyons).

Alas, such representatives are not unique.

It is conjectured that different representatives give rise to the same low-energy physics, via boundary renormalization group flow.

Only indirect tests possible, though.

Stacks

Other issues: deformation theory
massless spectra

To justify application of stacks to physics,
need to conduct tests of presentation-dependence,
understand issues above.

This was the subject of several papers.

For the rest of today's talk,
I want to focus on special kinds of stacks, namely,
gerbes.

(= quotient by noneffectively-acting group)

Gerbes

Gerbes have add'l problems when viewed from this physical perspective.

Example: The naive massless spectrum calculation contains multiple dimension zero operators, which manifestly violates cluster decomposition, one of the foundational axioms of quantum field theory.

There is a single known loophole: if the target space is disconnected. We **think** that's what's going on....

General decomposition conjecture

Consider $[X/H]$ where

$$1 \longrightarrow G \longrightarrow H \longrightarrow K \longrightarrow 1$$

and G acts trivially.

We now believe, for (2,2) CFT's,

$$\mathrm{CFT}([X/H]) = \mathrm{CFT}\left(\left[(X \times \hat{G})/K \right] \right)$$

(together with some B field), where $\hat{G}$ is the set of irreps of G

Decomposition conjecture

For banded gerbes, K acts trivially upon $\hat{G}$ so the decomposition conjecture reduces to

$$\mathrm{CFT}(G - \mathrm{gerbe\ on\ } X) \, = \, \mathrm{CFT}\left(\coprod_{\hat{G}} (X, B) \right)$$

where the B field is determined by the image of

$$H^2(X, Z(G)) \stackrel{Z(G) \rightarrow U(1)}{\longrightarrow} H^2(X, U(1))$$

Banded Example:

Consider $[X/D_4]$ where the center acts trivially.

$$1 \longrightarrow \mathbf{Z}_2 \longrightarrow D_4 \longrightarrow \mathbf{Z}_2 \times \mathbf{Z}_2 \longrightarrow 1$$

The decomposition conjecture predicts

$$\mathrm{CFT}\left([X/D_4]\right) = \mathrm{CFT}\left([X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2] \right)$$

One of the effective orbifolds has vanishing discrete torsion, the other has nonvanishing discrete torsion.

Checks: can show partition functions match:

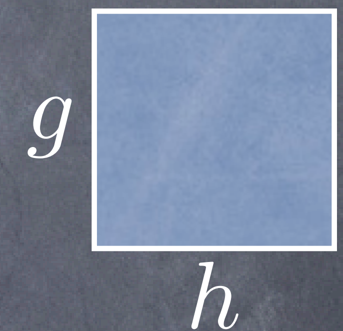
$$Z([X/D_4]) = Z\left([X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2] \right)$$

Check genus one partition functions:

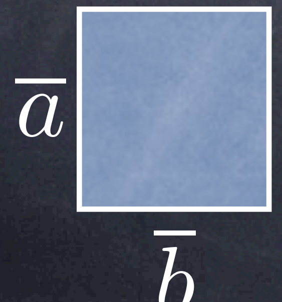
$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

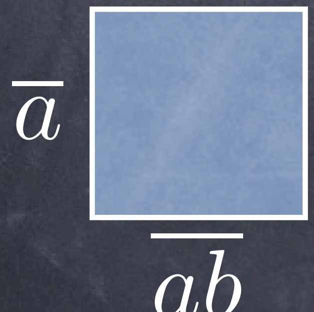
$$\mathbf{Z}_2 \times \mathbf{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$

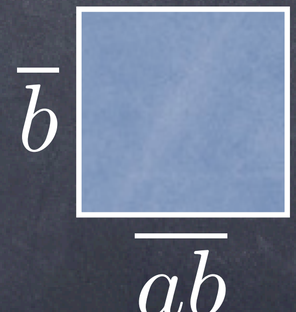
$$Z(D_4) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$



Each of the $Z_{g,h}$ twisted sectors that appears, is the same as a $\mathbf{Z}_2 \times \mathbf{Z}_2$ sector, appearing with multiplicity $|\mathbf{Z}_2|^2 = 4$ except for the







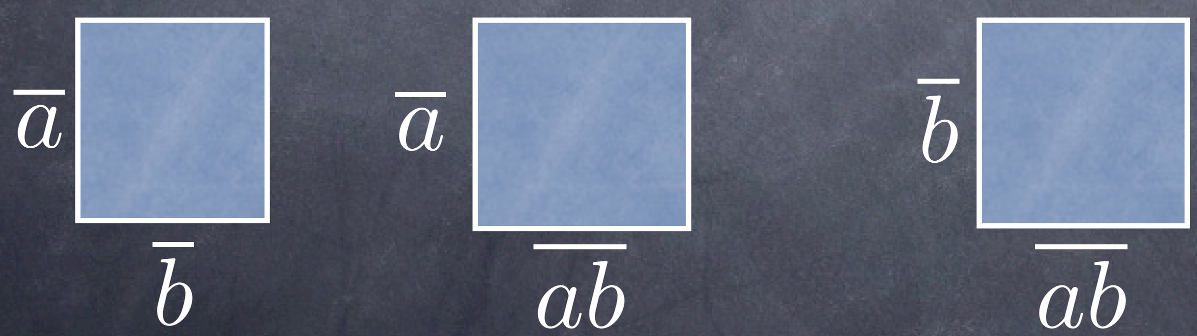
sectors.

Partition functions, cont'd

$$
\begin{aligned}
Z(D_4) &= \frac{|\mathbf{Z}_2 \times \mathbf{Z}_2|}{|D_4|} |\mathbf{Z}_2|^2 \left(Z(\mathbf{Z}_2 \times \mathbf{Z}_2) \; - \; (\text{some twisted sectors}) \right) \\
&= 2 \left(Z(\mathbf{Z}_2 \times \mathbf{Z}_2) \; - \; (\text{some twisted sectors}) \right)
\end{aligned}
$$

(In ordinary QFT, ignore multiplicative factors,
but string theory is a 2d QFT coupled to gravity,
and so numerical factors are important.)

Discrete torsion acts as a sign on the



twisted sectors

so we see that $Z([X/D_4]) = Z\left([X/\mathbf{Z}_2 \times \mathbf{Z}_2] \coprod [X/\mathbf{Z}_2 \times \mathbf{Z}_2] \right)$
with discrete torsion in one component.

A quick check of this example comes from comparing massless spectra:

Spectrum for $[T^6/D_4]$:

$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 54 & & 0 & \\
2 & & 54 & & 54 & & 2 \\
 & 0 & & 54 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 2 & & &
\end{array}
$$

and for each $[T^6/\mathbf{Z}_2 \times \mathbf{Z}_2]$:

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 3 & & 0 & \\
1 & & 51 & & 51 & & 1 \\
 & 0 & & 3 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
\qquad
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 51 & & 0 & \\
1 & & 3 & & 3 & & 1 \\
 & 0 & & 51 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Sum matches. $\checkmark$

Nonbanded example:

Consider $[X/\mathbf{H}]$ where $\mathbf{H}$ is the eight-element group of quaternions, and a $\mathbf{Z}_4$ acts trivially.

$$1 \longrightarrow < i > (\cong \mathbf{Z}_4) \longrightarrow \mathbf{H} \longrightarrow \mathbf{Z}_2 \longrightarrow 1$$

The decomposition conjecture predicts

$$\mathrm{CFT}([X/\mathbf{H}]) = \mathrm{CFT}\left([X/\mathbf{Z}_2] \coprod [X/\mathbf{Z}_2] \coprod X \right)$$

Straightforward to show that this is true at the level of partition functions, etc.

Another class of examples:
global quotients by nonfinite groups

The banded $\mathbf{Z}_k$ gerbe over $\mathbf{P}^N$
with characteristic class $-1 \bmod k$
can be described mathematically as the quotient

$$\left[\frac{ \mathbf{C}^{N+1} - \{0\} }{ \mathbf{C}^{\times} } \right]$$

where the $\mathbf{C}^{\times}$ acts as rotations by k times

which physically can be described by a U(1) susy gauge theory with N+1 chiral fields, of charge k

How can this be different from ordinary $\mathbf{P}^N$ model?

The difference lies in nonperturbative effects. (Perturbatively, having nonminimal charges makes no difference.)

To specify Higgs fields completely, need to specify what bundle they couple to.

If the gauge field $\sim L$
then Φ charge Q implies

$$\Phi \in \Gamma(L^{\otimes Q})$$

Different bundles => different zero modes
=> different anomalies => different physics

(Noncompact worldsheet - theta angle)

(J Distler, R Plesser)

Return to the example $\left[\frac{\mathbf{C}^{N+1} - \{0\}}{\mathbf{C}^\times}\right]$

Example: Anomalous global U(1)'s

$$\mathbf{P}^{N-1}: \quad U(1)_A \mapsto \mathbf{Z}_{2N}$$

$$\text{Here}: \quad U(1)_A \mapsto \mathbf{Z}_{2kN}$$

Example: A model correlation functions

$$\mathbf{P}^{N-1}: \quad < X^{N(d+1)-1} > = q^d$$

$$\text{Here}: \quad < X^{N(kd+1)-1} > = q^d$$

Example: quantum cohomology

$$\mathbf{P}^{N-1}: \quad \mathbf{C}[x]/(x^N - q)$$

$$\text{Here}: \quad \mathbf{C}[x]/(x^{kN} - q)$$

Different physics

Other tests:

* **K theory:**

H-equivariant K theory of X

=

twisted K-equivariant K theory of $X \times \hat{G}$

* **Sheaf theory**

A sheaf on a banded G-gerbe

=

a twisted sheaf on the underlying space,
twisted by image of an element of $H^2(X,Z(G))$

& Ext groups follow the decomposition.

Gromov-Witten prediction

Notice that there is a prediction here for Gromov-Witten theory of gerbes:

GW of $[X/H]$

should match

GW of $\left[(X \times \hat{G})/K \right]$

Banded $\mathbf{Z}_k$ gerbes:
E Andreini, Y Jiang, H-H Tseng, 0812.4477

Quantum cohomology

Some old results of Morrison-Plesser (q.c. from gauge theory) generalize from toric varieties to toric stacks.

Let the toric stack be described in the form

$$\left[\frac{\mathbf{C}^N \; - \; E}{(\mathbf{C}^\times)^n} \right]$$

E some exceptional set

Q_i^a the weight of the i[th] vector under a[th] $\mathbf{C}^\times$

then Batyrev's conjecture becomes

$\mathbf{C}[\sigma_1, \cdots, \sigma_n]$ modulo the relations

$$\prod_{i=1}^{N} \left(\sum_{b=1}^{n} Q_i^b \sigma_b \right)^{Q_i^a} = q_a$$

(ES, T Pantev, '05)

Quantum cohomology

Ex: Quantum cohomology ring of $\mathbf{P}^N$ is

$$\mathbf{C}[x]/(x^{N+1} - q)$$

Quantum cohomology ring of $\mathbf{Z}_k$ gerbe over $\mathbf{P}^N$ with characteristic class -n mod k is

$$\mathbf{C}[x,y]/(y^k - q_2, x^{N+1} - y^n q_1)$$

Aside: these calculations give us a check of the massless spectrum -- in physics, can derive q.c. ring w/o knowing massless spectrum.

Mirrors to stacks

There exist mirror constructions for any model realizable as a 2d abelian gauge theory.

For toric stacks (BCS '04), there is such a description.

Standard mirror constructions now produce character-valued fields, a new effect, which ties into the stacky fan description of (BCS '04).

(ES, T Pantev, '05)

Toda duals

Ex: The LG mirror of $\mathbf{P}^N$ is described by the holomorphic function

$$W = \exp(-Y_1) + \cdots + \exp(-Y_N) + \exp(Y_1 + \cdots + Y_N)$$

The analogous duals to $\mathbf{Z}_k$ gerbes over $\mathbf{P}^N$ are described by

$$W = \exp(-Y_1) + \cdots + \exp(-Y_N) + \Upsilon^n \exp(Y_1 + \cdots + Y_N)$$

where Υ is a character-valued field
(discrete Fourier transform of components in decomp' conjecture)

(ES, T Pantev, '05;
E Mann, '06)

Summary so far:

string compactifications on stacks exist

CFT(string on gerbe)
= CFT(string on disjoint union of spaces)

GLSM's

This result can be applied to understand GLSM's.

GLSM's are families of 2d gauge theories that RG flow to families of CFT's.

Example: $\mathbf{P}^7[2,2,2,2]$

one-parameter
Kahler moduli space

NLSM on $\mathbf{P}^7[2,2,2,2]$ ⟷ LG point

GLSM's

Example, cont'd: $\mathbf{P}^7[2,2,2,2]$

Have 8 fields ϕ_i of charge 1 (homog' coords on $\mathbf{P}^7$), plus another 4 fields p_a of charge -2.

Superpotential $W = \sum_a p_a G_a(\phi)$

D-terms $D = \sum_i |\phi_i|^2 - 2 \sum_a |p_a|^2 - r$

$r \gg 0$ ==> NLSM on $\mathbf{P}^7[2,2,2,2]$

GLSM's

Example, cont'd: $\mathbf{P}^7[2,2,2,2]$

At the Landau-Ginzburg point, have superpotential

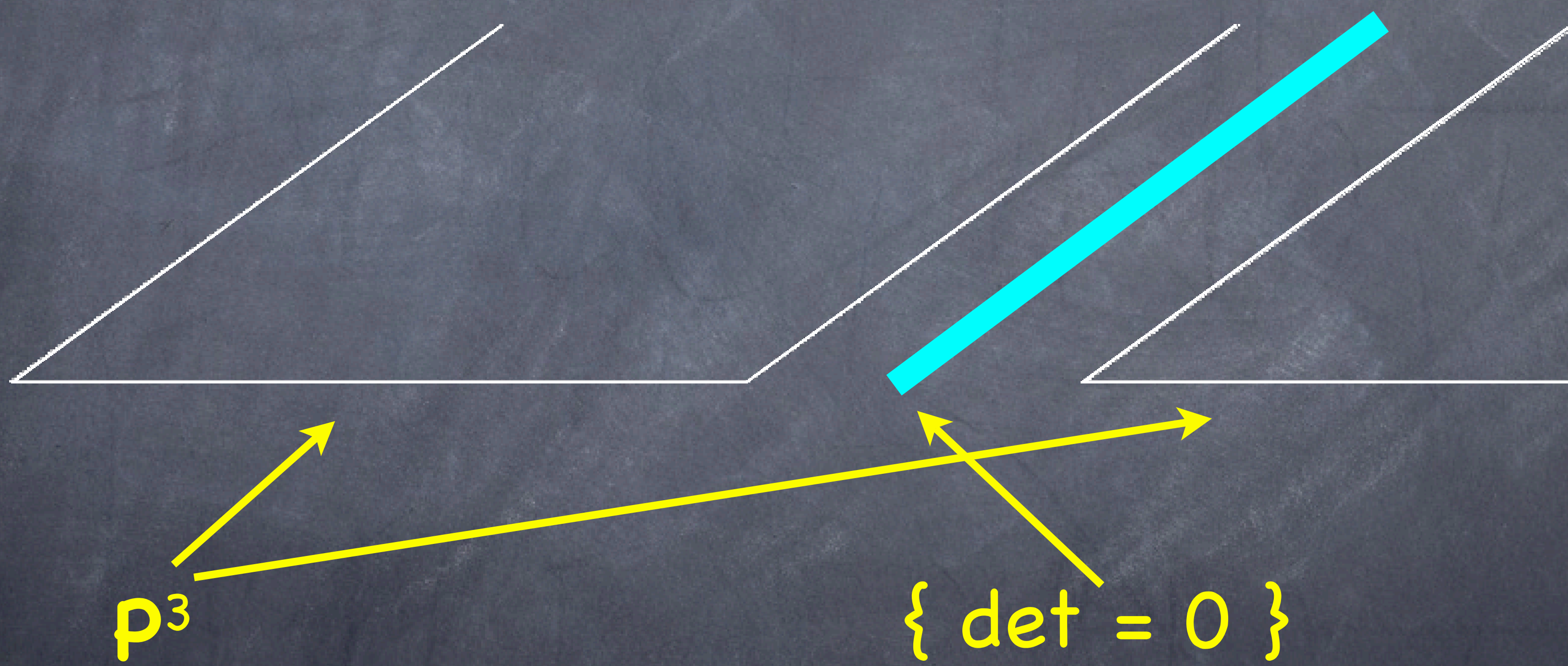
$$\sum_a p_a G_a(\phi) = \sum_{ij} \phi_i A^{ij}(p) \phi_j$$

* mass terms for the ϕ_i, away from locus $\{\det A = 0\}$.

* leaves just the p fields, of charge -2

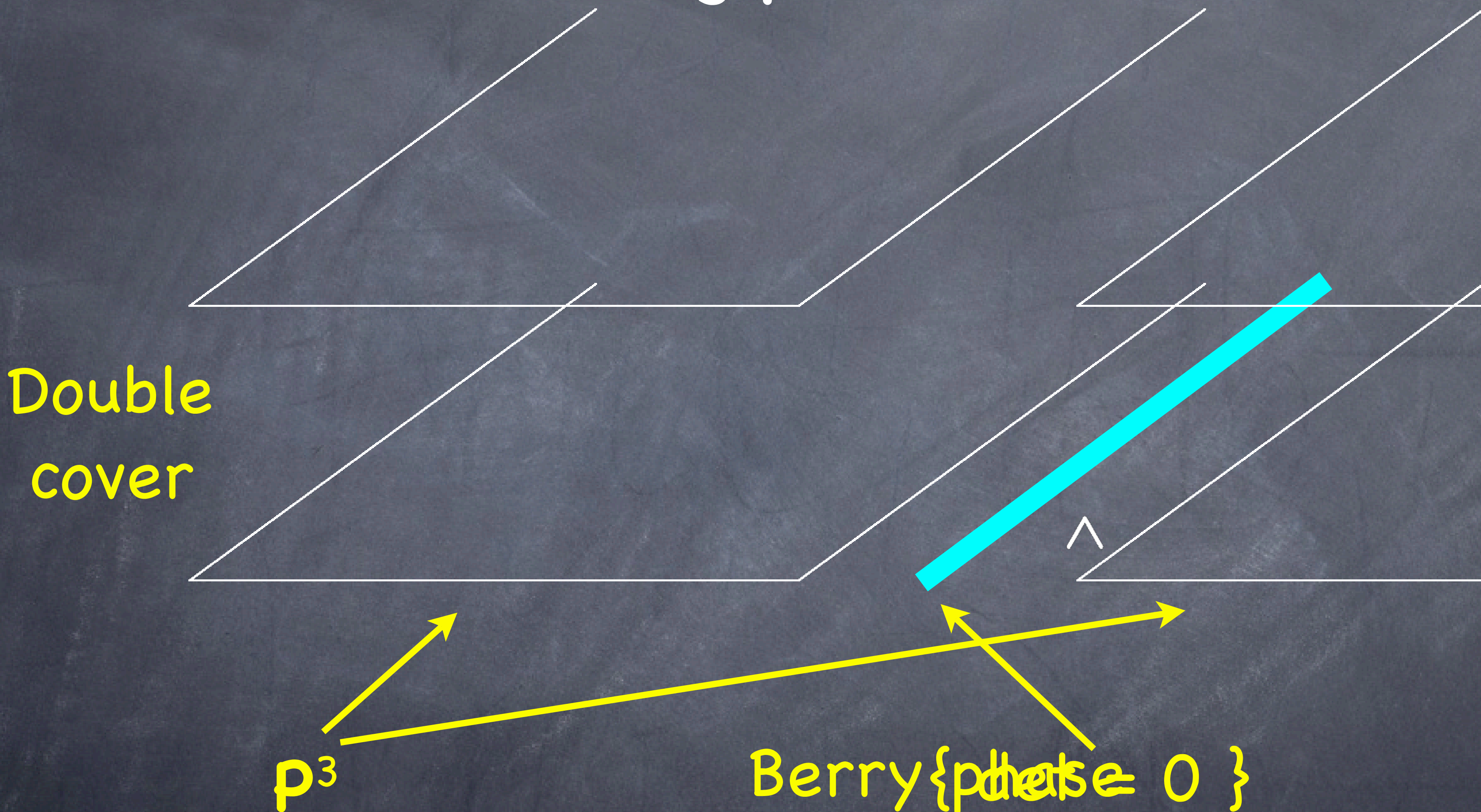
* $\mathbf{Z}_2$ gerbe, hence double cover

The Landau-Ginzburg point:



Because we have a $\mathbf{Z}_2$ gerbe over $\mathbf{P}^3$ - det....

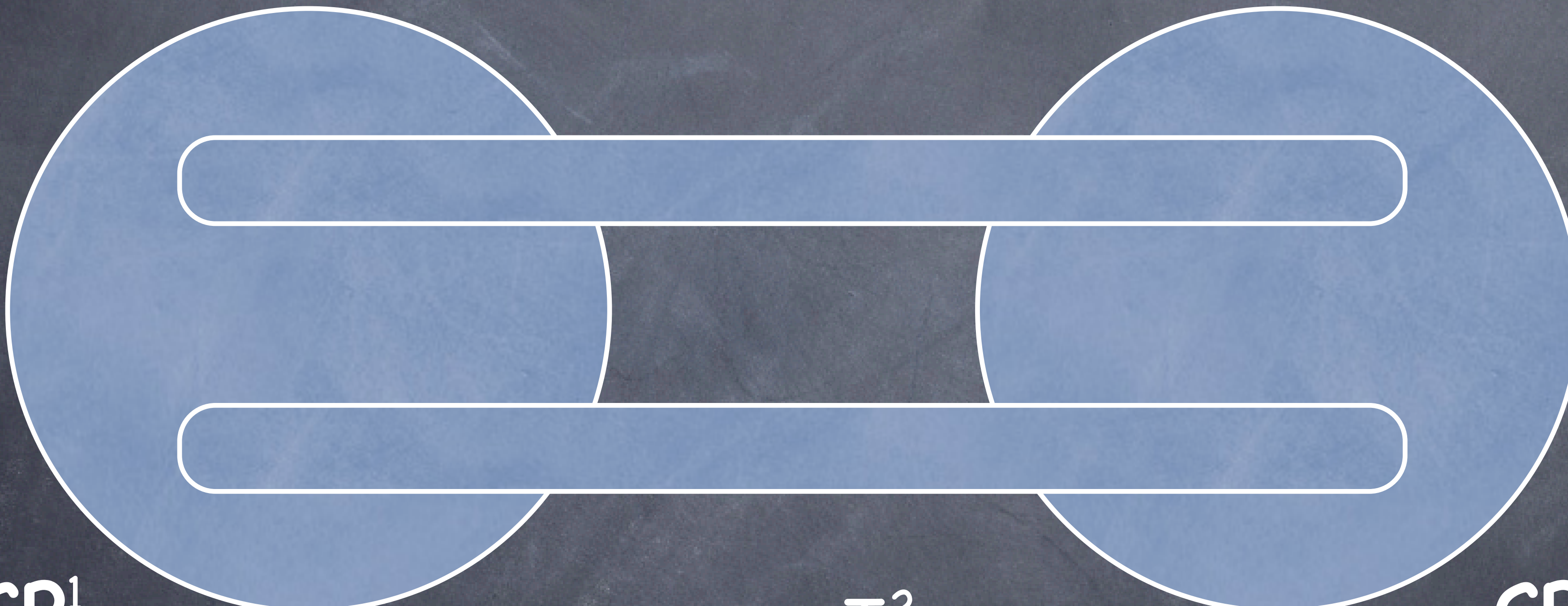
The Landau-Ginzburg point:



Result: branched double cover of $\mathbf{P}^3$

Aside: analogue for GLSM for $\mathbf{P}^3[2,2]$:

Branched double cover of $\mathbf{P}^1$ over deg 4 locus



$= T^2$

So a GLSM for $\mathbf{P}^3[2,2]$ relates

$T^2 \xleftrightarrow{\text{Kahler}} T^2$ (no surprise)

Back to $\mathbf{P}^7[2,2,2,2]$. Summary so far:

The GLSM realizes:

$\mathbf{P}^7[2,2,2,2]$ $\xleftrightarrow{\text{Kahler}}$ branched double cover of $\mathbf{P}^3$

(Clemens' octic double solid)

where RHS realized at LG point via local $\mathbf{Z}_2$ gerbe structure + Berry phase.

(S. Hellerman, A. Henriques, T. Pantev, ES, M Ando, '06; R Donagi, ES, '07; A. Caldararu, J. Distler, S. Hellerman, T. Pantev, E.S., arXiv: 0709.3855)

Non-birational twisted derived equivalence

Novel physical realization of geometry

Rewrite with Landau-Ginzburg models:

large radius ← GLSM for $\mathbf{P}^7[2,2,2,2]$ → LG

RG

RG

LG model on Tot($O(-2)^4$ --> $\mathbf{P}^7$)

GLSM

Kahler

LG model on Tot($O(-1)^8$ --> $\mathbf{P}^3_{[2,2,2,2]}$)

RG

RG

NLSM on $\mathbf{P}^7[2,2,2,2]$

NLSM on branched double cover of $\mathbf{P}^3$, branched over deg 8 locus

Puzzle:

the branched double cover will be singular,
but the GLSM is smooth at those singularities.

Solution?....

We believe the GLSM is actually describing
a `noncommutative resolution' of the branched double cover worked out by Kuznetsov.

Kuznetsov has defined
`homological projective duality'
that relates $\mathbf{P}^7[2,2,2,2]$ to the noncommutative resolution above.

Check that we are seeing K's noncomm' resolution:

K defines a `noncommutative space' via its sheaves -- so for example, a Landau-Ginzburg model can be a noncommutative space via matrix factorizations.

Here, K's noncomm' res'n = $(\mathbf{P}^3, B)$
where B is the sheaf of even parts of Clifford algebras associated with the universal quadric over $\mathbf{P}^3$ defined by the GLSM superpotential.

B ~ structure sheaf; other sheaves ~ B-modules.

Physics?......

What are the B-branes at the LG point of GLSM?

To answer this, we back up the RG flow to an intermediate point, a Landau-Ginzburg model (ie, integrate out gauge field of GLSM).

============= GLSM for $\mathbf{P}^n[2,\ldots,2]$ ================

↓ RG		↓ RG
LG model on Tot($O(-2)^k$ --> $\mathbf{P}^n$)	←GLSM / Kahler→	LG model on Tot($O(-1)^{n+1}$ --> $\mathbf{P}^{k-1}{}_{[2,\ldots,2]}$)
↓ RG		↓ RG
NLSM on $\mathbf{P}^n[2,\ldots,2]$	←GLSM / Kahler→	branched double cover of $\mathbf{P}^{k-1}$

Then, compute B-branes in LG (= matrix factorizations)

Physics:

B-branes in the RG limit theory
= B-branes in the intermediate LG theory.

Claim: matrix factorizations in intermediate LG
= Kuznetsov's B-modules

K has a rigorous proof of this;
B-branes = Kuznetsov's nc res'n sheaves.

Intuition....

Local picture:

Matrix factorization for a quadratic superpotential: even though the bulk theory is massive, one still has D0-branes with a Clifford algebra structure.

(Kapustin, Li)

Here: a `hybrid LG model' fibered over $\mathbf{P}^3$, gives sheaves of Clifford algebras (determined by the universal quadric / GLSM superpotential) and modules thereof.

So: open string sector duplicates Kuznetsov's def'n.

Note we have a physical realization of nontrivial examples of Kontsevich's \`noncommutative spaces' realized in gauged linear sigma models.

Furthermore, after \`backing up' RG flow to Landau-Ginzburg models, h.p.d. (on linear sections) becomes an Orlov/Walcher/Hori-type equivalence of matrix factorizations in LG models on birational spaces.

(? Kuznetsov = Orlov ?)

Other notes:

* It is now possible in principle to compute GW invariants of a noncommutative resolution -- compute them in the LG model upstairs, use the fact that A model is invariant under RG.

(Guffin, ES, 0801.3836, 0801.3955)

* We applied Born-Oppenheimer very briefly here; it also implies a more general statement, that matrix factorizations `behave nicely' in families

Summary so far:

The GLSM realizes:

$P^7[2,2,2,2]$ ⟵ Kahler ⟶ branched double cover of $\mathbf{P}^3$

where RHS realized at LG point via local $\mathbf{Z}_2$ gerbe structure + Berry phase.

(A. Caldararu, J. Distler, S. Hellerman, T. Pantev, E.S., arXiv: 0709.3855)

Novel physical realization of geometry

Non-birational twisted derived equivalence

Physical realization of Kuznetsov's homological projective duality

More examples:

CI of n quadrics in $\mathbf{P}^{2n-1}$ ⟷ (Kahler) branched double cover of $\mathbf{P}^{n-1}$, branched over deg 2n locus

Both sides CY

Homologically projective dual

Rewrite with Landau-Ginzburg models:

LG model on
Tot($O(-2)^k$ --> $\mathbf{P}^n$)

GLSM
Kahler

LG model on
Tot($O(-1)^{n+1}$ --> $\mathbf{P}^{k-1}{}_{[2,\ldots,2]}$)

RG

RG

NLSM on
$\mathbf{P}^n[2,\ldots,2]$

Kuznetsov's
h.p.d.

NLSM on n.c. res'n of
branched double cover
of $\mathbf{P}^{k-1}$,
branched over deg n+1 locus

A math conjecture:

Kuznetsov defines his h.p.d. in terms of coherent sheaves. In the physics language

LG model on	GLSM ⟷ Kahler	LG model on
Tot($O(-2)^k$ --> $\mathbf{P}^n$)		Tot($O(-1)^{n+1}$ --> $\mathbf{P}^{k-1}_{[2,\ldots,2]}$)

Kuznetsov's h.p.d. becomes a statement about matrix factorizations, analogous to those in Orlov's work.

Math conjecture: Kuznetsov's h.p.d. has an alternative (& hopefully easier) description in terms of matrix factorizations between LG models on birational spaces.

More examples:

CI of 2 quadrics in the total space of

$$\mathbf{P}\left(\mathcal{O}(-1,0)^{\oplus 2} \oplus \mathcal{O}(0,-1)^{\oplus 2}\right) \longrightarrow \mathbf{P}^1 \times \mathbf{P}^1$$

$$\overset{\text{Kahler}}{\longleftrightarrow}$$

branched double cover of $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^1$,
branched over deg (4,4,4) locus

* In fact, the GLSM has 8 Kahler phases,
4 of each of the above.

* Related to an example of Vafa-Witten involving discrete torsion

(Caldararu, Borisov)

* Believed to be homologically projective dual

A non-CY example:

CI 2 quadrics in $\mathbf{P}^{2g+1}$ ⟷ (Kahler) branched double cover of $\mathbf{P}^1$, over deg 2g+2 (= genus g curve)

Homologically projective dual.

Here, r flows under RG -- not a const parameter.

Semiclassically, Kahler moduli space falls apart into 2 chunks.

Positively curved

Negatively curved

r flows: ⇢ ⇢ ⇢

Aside:

One of the lessons of this analysis is that gerbe structures are commonplace, even generic, in the hybrid LG models arising in GLSM's.

To understand the LG points of typical GLSM's, requires understanding gerbes in physics.

So far we have discussed several GLSM's s.t.:

* the LG point realizes geometry in an unusual way

* the geometric phases are not birational

* instead, related by Kuznetsov's homological projective duality

We conjecture that Kuznetsov's homological projective duality applies much more generally to GLSM's.....

More Kuznetsov duals:

Another class of examples, also realizing Kuznetsov's h.p.d., were realized in GLSM's by Hori-Tong.

$G(2,7)[1^7]$ ←Kahler→ Pfaffian CY

(Rodland, Kuznetsov, Borisov-Caldararu, Hori-Tong)

* unusual geometric realization (via strong coupling effects in nonabelian GLSM)

* non-birational

More Kuznetsov duals:

$G(2,N)[1^m]$ (N odd) $\xleftrightarrow{\text{Kahler}}$ vanishing locus in $\mathbf{P}^{m-1}$ of Pfaffians

Check r flow:

$K = O(m-N)$ $\qquad\qquad$ $K = O(N-m)$

Opp sign, so r flows in same direction,
consistent with GLSM.

r flows: $\dashrightarrow$ $\quad$ $\dashrightarrow$ $\quad$ $\dashrightarrow$

More Kuznetsov duals:

So far we have discussed how Kuznetsov's h.p.d. realizes Kahler phases of several GLSM's with exotic physics.

We conjecture it also applies to ordinary GLSM's.

Ex: flops

Some flops are already known to be related by h.p.d.; K is working on the general case.

So far we have discussed several GLSM's s.t.:

* the LG point realizes geometry in an unusual way

* the geometric phases are not birational

* instead, related by Kuznetsov's homological projective duality

Conjecture: **all** phases of GLSM's are related by Kuznetsov's h.p.d.

Summary

- Basics of string compactifications on stacks
- Cluster decomposition conjecture for strings on gerbes:
  CFT(gerbe) = CFT(disjoint union of spaces)
- Application to GLSM's; realization of Kuznetsov's homological projective duality
- LG models; h.p.d. & matrix factorizations

Mathematics	Physics
Geometry: Gromov-Witten, Donaldson-Thomas, quantum cohomology, etc	Supersymmetric field theories
Homotopy, categories: derived categories, stacks, etc.	Renormalization group