Decomposition of quantum field theories

Nankai Symposium, Mathematical Dialogues

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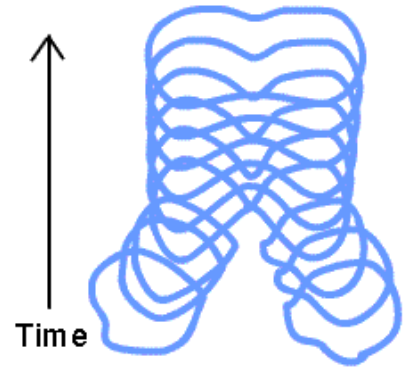
An overview of hep-th/0502027, 0502044, 0502053, 0606034,
0709.3855, 1012.5999, 1307.2269, 1404.3986, ... (many ...),
2101.11619, 2106.00693, 2107.12386, 2107.13552

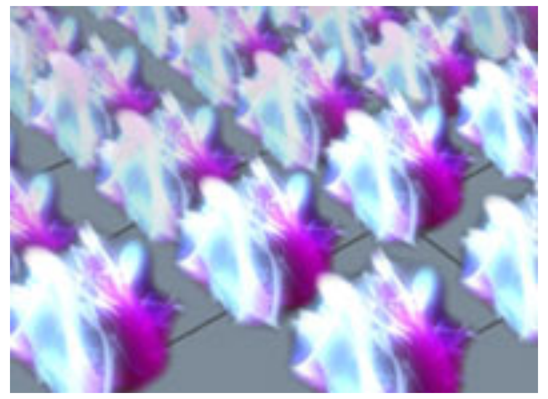
My talk today concerns examples of quantum field theories (QFTs)
which are secretly equivalent to
sums of other quantum field theories.

When this happens, we say the QFT `decomposes.'
Decomposition of the QFT can be applied to give insight
into its properties.

I'm primarily interested in quantum field theories in 1+1 dimensions, because they provide analogues of quantum mechanics for string theory.



To get real-world 4d physics from the 10d physics of string theory, we roll up or `compactify' the 10 dimensions on a compact 6d space.



If I compactify a string on a disjoint union of 6d spaces,
or work with a stringy quantum mechanical system describing a disjoint union,
as arises in decomposition,
then at low energies, one sees multiple four-dimensional universes,
each with its own separate metric and graviton,
which are not mutually interacting.

For this reason, the summands of decomposition are called `universes.'

I'll warm up by describing some general features of sums of QFTs,
and then I'll move on to describe the theories that motivated this work.

What does it mean for one QFT to be a sum of other QFTs?

1) Existence of projection operators

The theory contains topological operators Π_i such that

$$\Pi_i \Pi_j = \delta_{i,j} \Pi_j \qquad \sum_i \Pi_i = 1$$

Correlation functions:

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_m \rangle = \sum_i \langle \Pi_i \mathcal{O}_1 \cdots \mathcal{O}_m \rangle = \sum_i \langle (\Pi_i \mathcal{O}_1) \cdots (\Pi_i \mathcal{O}_m) \rangle = \sum_i \langle \tilde{\mathcal{O}}_1 \cdots \tilde{\mathcal{O}}_m \rangle_i$$

Math analogue:

If a space X has m connected components, then $\dim H^0(X) = m$

— multiple degree-zero elements of cohomology

What does it mean for one QFT to be a sum of other QFTs?

2) Partition functions decompose

$$Z = \sum_{\text{states}} \exp(-\beta H) = \sum_i Z_i = \sum_i \sum \exp(-\beta H_i)$$

(on a connected spacetime)

3) In d spacetime dimensions, has a (possibly noninvertible) (d-1)-form symmetry.

I'll explain what that is in a few minutes....

What sort of QFTs admit a decomposition ? ....

The QFTs I'm interested in, which have a decomposition,
are two-dimensional theories with "global 1-form symmetries,"
and can be described in several ways, such as

(Pantev, ES '05;
Hellerman et al '06)

- Gauge theory w/ trivially-acting subgroup
- Theory w/ restriction on instantons
- Sigma models on gerbes
  = fiber bundles with fibers = `groups' of 1-form symmetries $G^{(1)} = BG$

We'll see in this talk how decomposition (into 'universes') relates these pictures.

Examples:

restriction on instantons = "multiverse interference effect"

1-form symmetry of QFT = translation symmetry along fibers of gerbe

trivial group action b/c $BG = [\text{point}/G]$

The QFTs I'm interested in, which have a decomposition,
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- Gauge theory w/ trivially-acting subgroup
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Decomposition also solves technical issues with such theories.
For example, we don't usually restrict instantons
— technically, this violates the "cluster decomposition" axiom of QFT.
The fact that such theories are equivalent to disjoint unions of others,
solves that problem.

What is a one-form symmetry?....

What is a one-form symmetry?

For this talk, *intuitively*, this will be a `group' that exchanges nonperturbative sectors.

(instantons / bundles)

Example: G gauge theory in which massless matter inv't under $K \subseteq G$

(K assumed finite & abelian)

Then, nonperturbative sectors (G-bundles) are invariant under

$$(G - \text{bundle}) \mapsto (G - \text{bundle}) \otimes (K - \text{bundle})$$

$$A \mapsto A + A'$$

This is the symmetry, involving an action of `group' of K–bundles.

That group is denoted BK or $K^{(1)}$

Let's see how decomposition works in theories with such symmetries....

Decomposition in 2d gauge theories

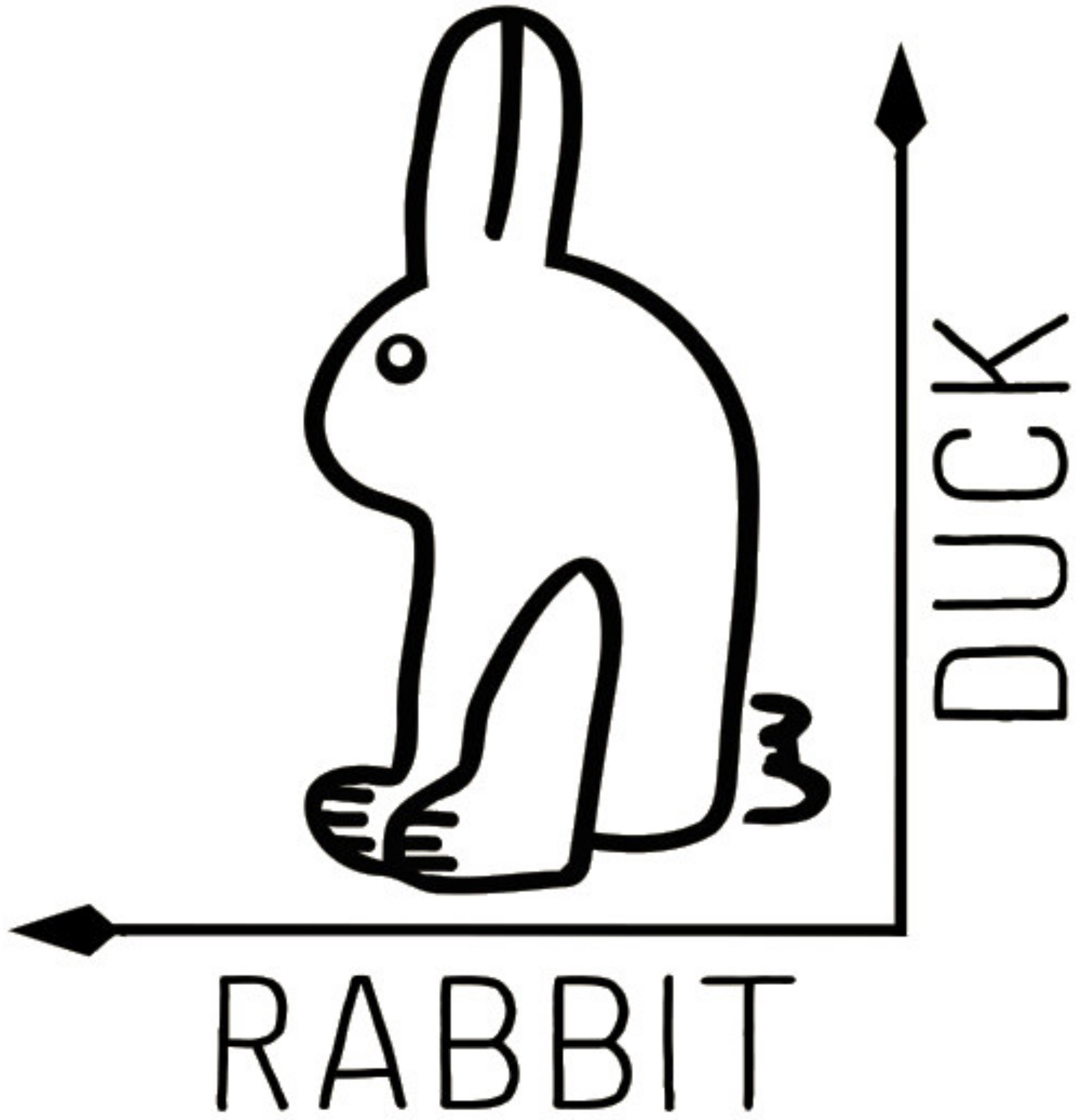
(Hellerman et al '06)

Gauge theory version:

S'pose have $G-$gauge theory, G semisimple, with finite $K \subseteq G$ acting trivially.

For simplicity, assume K is in the center. Has BK 1-form symmetry.

So far, this sounds like just one QFT.



However, I'll outline how, from another perspective, QFTs of this form are also each a disjoint union of other QFTs; they "decompose."

Decomposition in 2d gauge theories

(Hellerman et al '06)

Gauge theory version:

S'pose have $G-$gauge theory, G semisimple, with finite $K \subseteq G$ acting trivially.

For simplicity, assume K is in the center. Has BK 1-form symmetry.

Claim this theory decomposes.
Where are the projection operators?

Math understanding:

Briefly, the projection operators (twist fields, Gukov-Witten) correspond to elements of the center of the group algebra $\mathbb{C}[K]$.

Existence of those projectors (idempotents), forming a basis for the center, is ultimately a consequence of Wedderburn's theorem.

Universes $\longleftrightarrow$ Irreducible representations of K

Partition functions & relation of decomp' to restrictions on instantons....

Decomposition in 2d gauge theories

(Hellerman et al '06)

Gauge theory version:

S'pose have $G-$gauge theory, G semisimple, with finite $K \subseteq G$ acting trivially.

For simplicity, assume K is in the center. Has BK 1-form symmetry.

Statement of decomposition:

$$\text{QFT}(G-\text{gauge theory}) = \coprod_{\text{char's } \hat{K}} \text{QFT}\left(G/K-\text{gauge theory w/ discrete theta angles}\right)$$

Example: pure $SU(2)$ gauge theory = sum $SO(3)_+ + SO(3)_-$ pure gauge theories

where $\pm$ denote discrete theta angles (w_2)

Perturbatively, the $SU(2)$, $SO(3)_\pm$ theories are identical
— differences are all nonperturbative.

Decomposition in 2d gauge theories

(Hellerman et al '06)

Gauge theory version:

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Example: pure $SU(2)$ gauge theory = sum $SO(3)_+ + SO(3)_-$ pure gauge theories

where $\pm$ denote discrete theta angles (w_2)

$SU(2)$ instantons (bundles) $\subset SO(3)$ instantons (bundles)

The discrete theta angles weight the non-$SU(2)$ $SO(3)$ instantons so as to cancel out of the partition function of the disjoint union.

Summing over the $SO(3)$ theories projects out some instantons, giving the $SU(2)$ theory.

Decomposition in 2d gauge theories

(Hellerman et al '06)

Gauge theory version:

S'pose have $G-$gauge theory, G semisimple, with finite $K \subseteq G$ acting trivially.

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Statement of decomposition:

$$\text{QFT}(G-\text{gauge theory}) = \coprod_{\text{char's } \hat{K}} \text{QFT}\left(G/K-\text{gauge theory w/ discrete theta angles}\right)$$

Formally, the partition function of the disjoint union can be written

$$Z = \underbrace{\sum_{\theta \in \hat{K}} \int [DA] \exp(-S) \exp\left[\theta \int \omega_2(A)\right]}_{\text{Disjoint union}} = \int [DA] \exp(-S) \left(\overbrace{\sum_{\theta \in \hat{K}} \exp\left[\theta \int \omega_2(A)\right]}^{\text{projection operator}} \right)$$

where we have moved the summation inside the integral.

("multiverse interference" cancels out some sectors)

Decomposition in 2d gauge theories

(Hellerman et al '06)

projection operator

$$Z = \sum_{\theta \in \hat{K}} \int [DA] \exp(-S) \exp\left[\theta \int \omega_2(A)\right] \quad = \quad \int [DA] \exp(-S) \left(\sum_{\theta \in \hat{K}} \exp\left[\theta \int \omega_2(A)\right] \right)$$

Disjoint union

Decomposition in 2d gauge theories

(Hellerman et al '06)

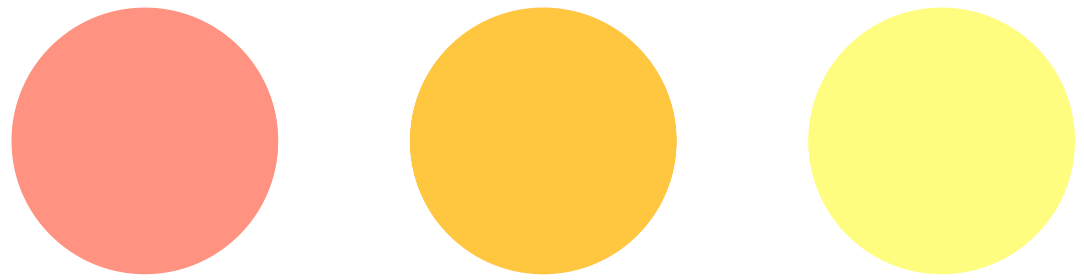
One effect is a projection on nonperturbative sectors:

$$\underbrace{\sum_{\theta \in \hat{K}} \int [DA] \exp(-S) \exp\left[\theta \int \omega_2(A)\right]}_{\text{Disjoint union}} = \int [DA] \exp(-S) \overbrace{\left(\sum_{\theta \in \hat{K}} \exp\left[\theta \int \omega_2(A)\right] \right)}^{\text{projection operator}}$$

Disjoint union of several QFTs / universes = `One' QFT with a restriction on nonperturbative sectors = `multiverse interference'

Schematically,
two theories combine to form a distinct third:

universe ($SO(3)_+$)



universe ($SO(3)_-$)

multiverse interference effect ($SU(2)$)

Before going on, let's quickly check these claims for pure $SU(2)$ Yang-Mills in 2d.

The partition function Z, on a Riemann surface of genus g, is

(Migdal, Rusakov)

$$Z(SU(2)) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps

$$Z(SO(3)_+) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SO(3) reps

(Tachikawa '13)

$$Z(SO(3)_-) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps that are not SO(3) reps

Result: $Z(SU(2)) = Z(SO(3)_+) + Z(SO(3)_-)$ as expected.

Decomposition in 2d gauge theories

Since 2005, decomposition has been checked in many examples in many ways. Examples:

- GLSM's: mirrors, quantum cohomology rings (Coulomb branch) (T Pantev, ES '05; Gu et al '18-'20)
- Orbifolds: partition f'ns, massless spectra, elliptic genera (T Pantev, ES '05)
- Open strings, K theory (Hellerman et al hep-th/0606034)
- Susy gauge theories w/ localization (ES 1404.3986)
- Nonsusy pure Yang-Mills ala Migdal (ES '14; Nguyen, Tanizaki, Unsal '21)
- Adjoint QCD_2 (Komargodski et al '20)
- Plus version for 4d theories w/ 3-form symmetries (Tanizaki, Unsal,'19; Cherman, Jacobson '20)

Applications include:

- Predictions for Gromov-Witten theory (checked by H-H Tseng, Y Jiang, etc starting '08)
- Nonperturbative constructions of geometries in GLSMs (Caldararu et al 0709.3855, Hori '11, ...)
- Elliptic genera (Eager et al '20)
- Anomalies (Robbins et al '21)

We'll discuss applications at the end....

To make this more concrete, let's walk through an example of an orbifold, where everything can be made completely explicit.

Example: Orbifold $[X/D_4]$ in which the $\mathbb{Z}_2$ center acts trivially.

(T Pantev, ES '05)

— has $B\mathbb{Z}_2$ (1-form) symmetry

$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ so this is closely related to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Decomposition predicts

$$\mathrm{QFT}\left([X/D_4]\right) = \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

(consequence of a general formula.)

Let's check this explicitly....

Example, cont'd

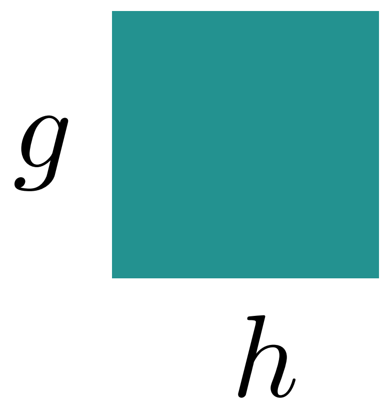
Compute the partition function of $[X/D_4]$ (T Pantev, ES '05)

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

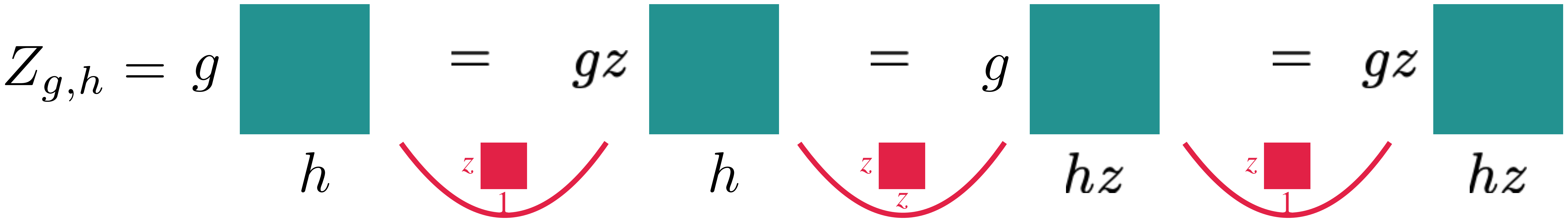
$$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$ where $\overline{a} = \{a, az\}$ etc

$$Z([X/D_4]) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$ where $Z_{g,h} =$



Since z acts trivially,

$Z_{g,h}$ is symmetric under multiplication by z



This is the $B\mathbb{Z}_2$ 1-form symmetry.

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$ where $\overline{a} = \{a, az\}$ etc

$$Z([X/D_4]) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$

where $Z_{g,h} = g$  h

Each D_4 twisted sector that appears is the same as a $\mathbb{Z}_2 \times \mathbb{Z}_2$ twisted sector,

appearing with multiplicity $|\mathbb{Z}_2|^2 = 4$,

except for the sectors $\overline{a}$  $\overline{b}$ $\overline{a}$  $\overline{ab}$ $\overline{b}$  $\overline{ab}$ which do not appear.

Restriction on nonperturbative sectors

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$Z([X/D_4]) = \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \text{(some twisted sectors)} \right)$$

$$= 2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \text{(some twisted sectors)} \right)$$

Different theory than $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Example, cont'd

Compute the partition function of $[X/D_4]$

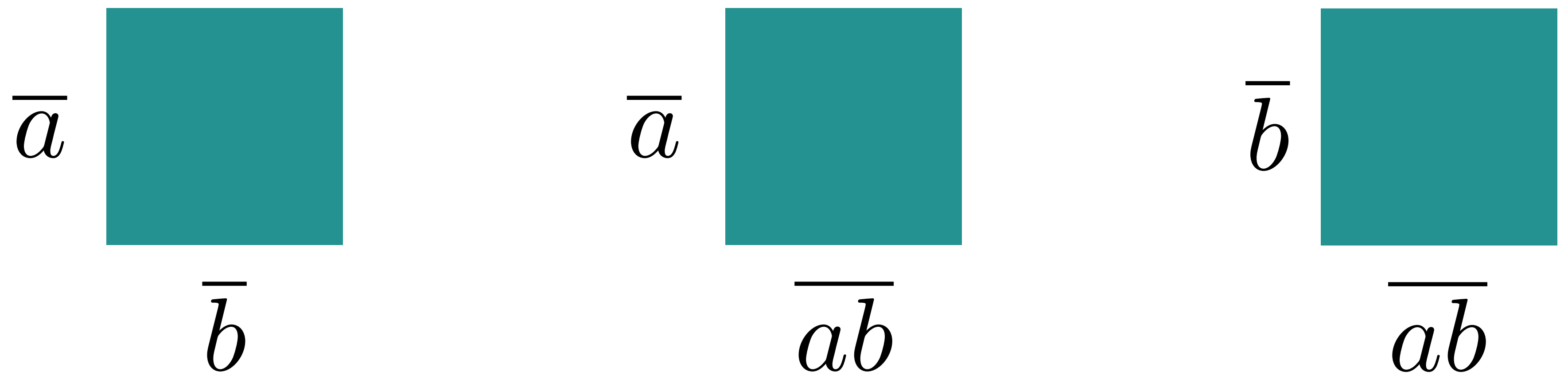
(T Pantev, ES '05)

$$Z([X/D_4]) \;=\; \frac{|\mathbb{Z}_2\times\mathbb{Z}_2|}{|D_4|}|\mathbb{Z}_2|^2\,\big(Z([X/\mathbb{Z}_2\times\mathbb{Z}_2]) \;-\; (\text{some twisted sectors})\big)$$

$$=\; 2\,\big(Z([X/\mathbb{Z}_2\times\mathbb{Z}_2]) \;-\; (\text{some twisted sectors})\big)$$

Discrete torsion is $H^2(\mathbb{Z}_2\times\mathbb{Z}_2, U(1)) = \mathbb{Z}_2$,

and acts as a sign on the twisted sectors



which were omitted above.

$$Z([X/D_4]) \;=\; Z([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{w/o d.t.}}) \;+\; Z([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{d.t.}})$$

Adding the components projects out some sectors — interference effect.

Example, cont'd

Compute the partition function of $[X/D_4]$ (T Pantev, ES '05)

$$Z([X/D_4]) = \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \text{(some twisted sectors)} \right)$$

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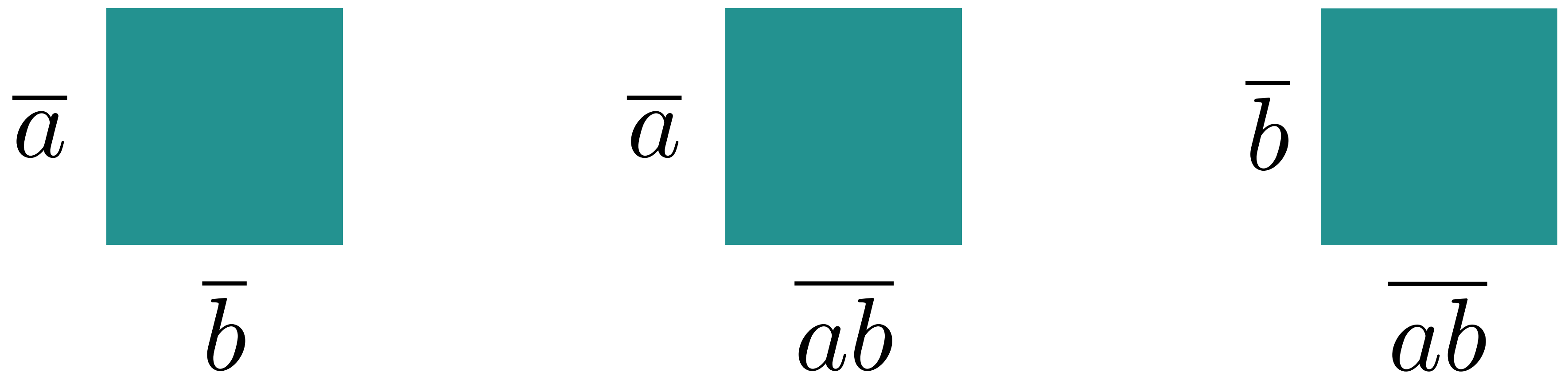


which were omitted above.

$$Z([X/D_4]) = Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}) + Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}})$$

Matches prediction of decomposition

$$\text{QFT}\left([X/D_4]\right) = \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

Example, cont'd

$$\text{QFT}\left([X/D_4]\right) \;=\; \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

At the level of operators, one reason for this is that the theory admits projection operators:

Let $\hat{\imath}$ denote the (dim 0) twist field associated to the trivially-acting $\mathbb{Z}_2$:

$$\Pi_\pm \;=\; \frac{1}{2}\left(1 \pm \hat{\imath}\right)$$

$$\Pi_\pm^2 = \Pi_\pm \qquad\qquad \Pi_\pm \Pi_\mp = 0$$

Massless spectra....

Example, cont'd

Massless states of $[X/D_4]$ for $X = T^6$ (T Pantev, ES '05)

Massless states of $[T^6/D_4]$

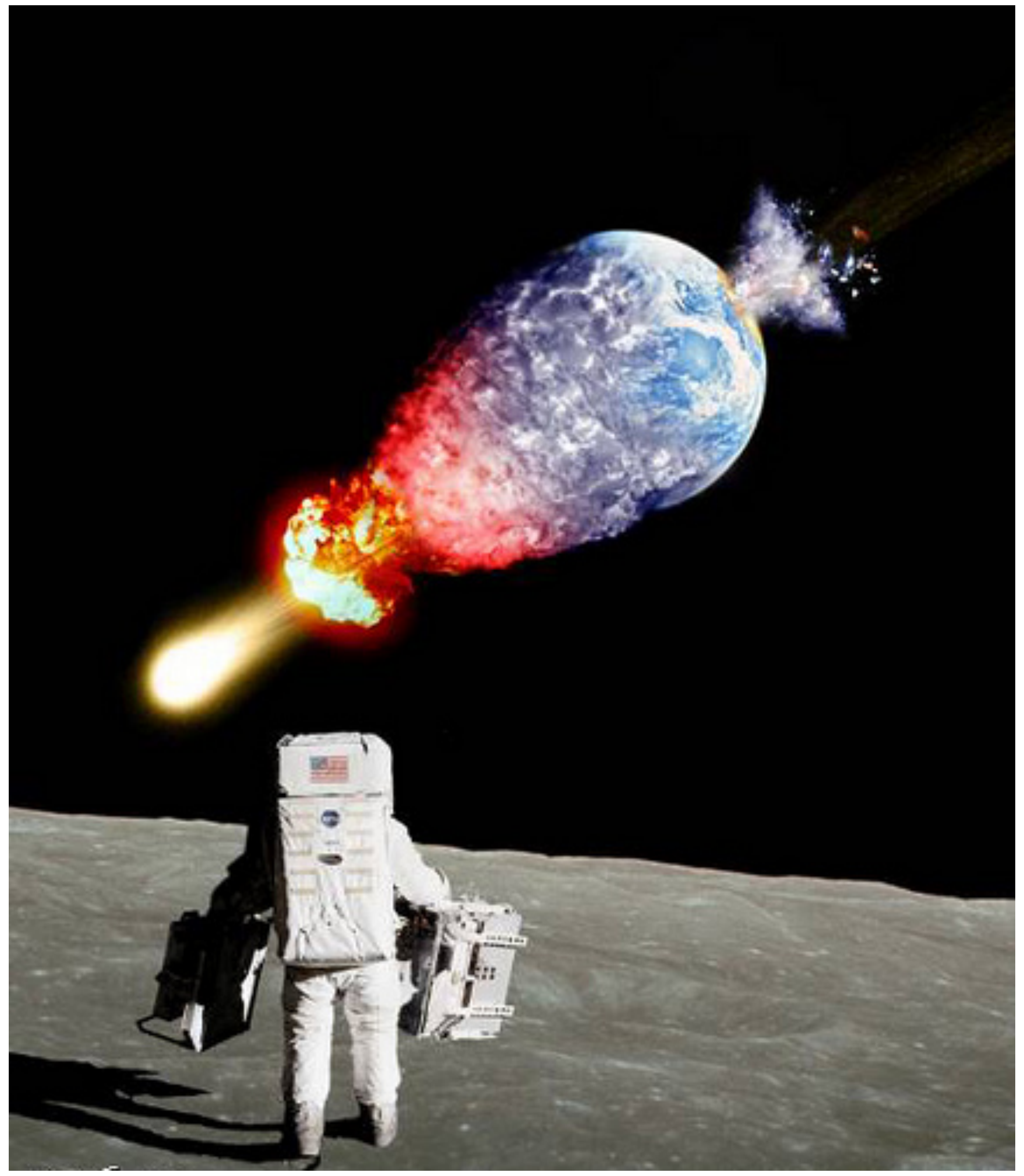
$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 54 & & 0 & \\
2 & & 54 & & 54 & & 2 \\
 & 0 & & 54 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 2 & & &
\end{array}
$$

Signals mult' components / cluster decomp' violation

If we didn't know about decomposition, the 2's in the corners would be a problem...

A big problem!

They signal a violation of an axiom of QFT called "cluster decomposition," the same axiom that's violated by restricting instantons.



Ordinarily, I'd assume that the computation was wrong.

However, decomposition saves the day....

Example, cont'd

Massless states of $[X/D_4]$ for $X = T^6$ (T Pantev, ES '05)

Massless states of $[T^6/D_4]$

$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 54 & & 0 & \\
2 & & 54 & & 54 & & 2 \\
 & 0 & & 54 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 2 & & &
\end{array}
\;=\;
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 51 & & 0 & \\
1 & & 3 & & 3 & & 1 \\
 & 0 & & 51 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
\;+\;
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 3 & & 0 & \\
1 & & 51 & & 51 & & 1 \\
 & 0 & & 3 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Signals mult' components / cluster decomp' violation

states of $[T^6/\mathbb{Z}_2 \times \mathbb{Z}_2]$ w/o d.t.

states of $[T^6/\mathbb{Z}_2 \times \mathbb{Z}_2]$ w/ d.t.

matching the prediction of decomposition

$$\mathrm{QFT}\left([X/D_4]\right) \;=\; \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

In the cases I've described so far,
the universes are all approx'ly the same.

In general, however, they can be different.

Example: $[X/\mathbb{H}]$, $\mathbb{H}$ the 8-element group of unit quaternions,
with $\langle i \rangle \cong \mathbb{Z}_4$ acting trivially on X

In this case, $$[X/\mathbb{H}] \; = \; X \coprod [X/\mathbb{Z}_2] \coprod [X/\mathbb{Z}_2]$$

— 3 universes, not all the same

— has a `noninvertible' symmetry
(beyond the scope of this talk)

So far:

I've reviewed decomposition,

a property of 2d QFTs with finite global 1-form symmetry.

What about QFTs in other dimensions?

- 4d theories w/ finite global 3-form symmetries — Tanizaki, Unsal '19; Cherman, Jacobson '20
- Conjecture same for QFTs in d dims w/ finite global (d-1)-form symmetries, $d > 1$

  Cherman, Jacobson '20

So far:

- Conjecture same for QFTs in d dims w/ finite global (d-1)-form symmetries, d > 1

To that end,

1. Involves a (d-1)-form, which couples to a domain wall

(analogous to Bousso-Polchinski '00, ...)

2. Consistent with reduction on circle:

The (d-1)-dim theory has a (d-2)-form symmetry,

as expected:

if the d-dim'l theory decomposes, its reduction on a circle should decompose too.

Is there any math here?....

Mathematical interpretation:

So far I've just talked abstractly about 2d QFTs & 1-form symmetries.

This has a mathematical interpretation: "gerbes" $\neq$

A G–gerbe is a fiber bundle whose fibers are copies of BG.

A sigma model on a G–gerbe has a global BG symmetry,

just as a sigma model on a G–bundle has a global G symmetry,

from translations on the fibers.

Furthermore, $BG = [\text{point}/G]$

so whenever a group acts trivially,

you should expect a gerbe structure (1-form symmetry) somewhere.

Mathematical interpretation:

Twenty years ago, I was interested in studying
`sigma models on gerbes' as possible sources of new string compactifications.

Potential issues, since solved:

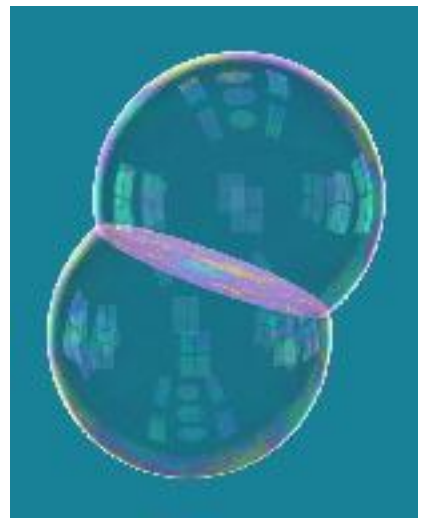
construction of QFT; cluster decomposition; moduli;
mod' invariance & unitarity in orbifolds; potential presentation-dependence.

What we eventually learned was that these theories are well-defined,
but,
are disjoint unions of ordinary theories, at least in (2,2) susy cases,
because of decomposition.

Not really new compactifications, but instead other applications.
I'll list 4 of my favorites next….

Application: GW invariants

The Gromov-Witten (GW) invariants count minimal-area surfaces in a given space.



There exists a def'n of GW invariants of gerbes.

Decomposition predicts,
GW invariants of a gerbe = sum of GW invariants of universes

✓ Checked by (H-H Tseng, Y Jiang, et al '08 on)

Application: GLSMs

(Caldararu et al '07)

Consider the GLSM for e.g. $\mathbb{P}^3[2,2] = T^2$.

This is a $U(1)$ gauge theory, with ϕ_i charge $+1$, p_a charge -2.

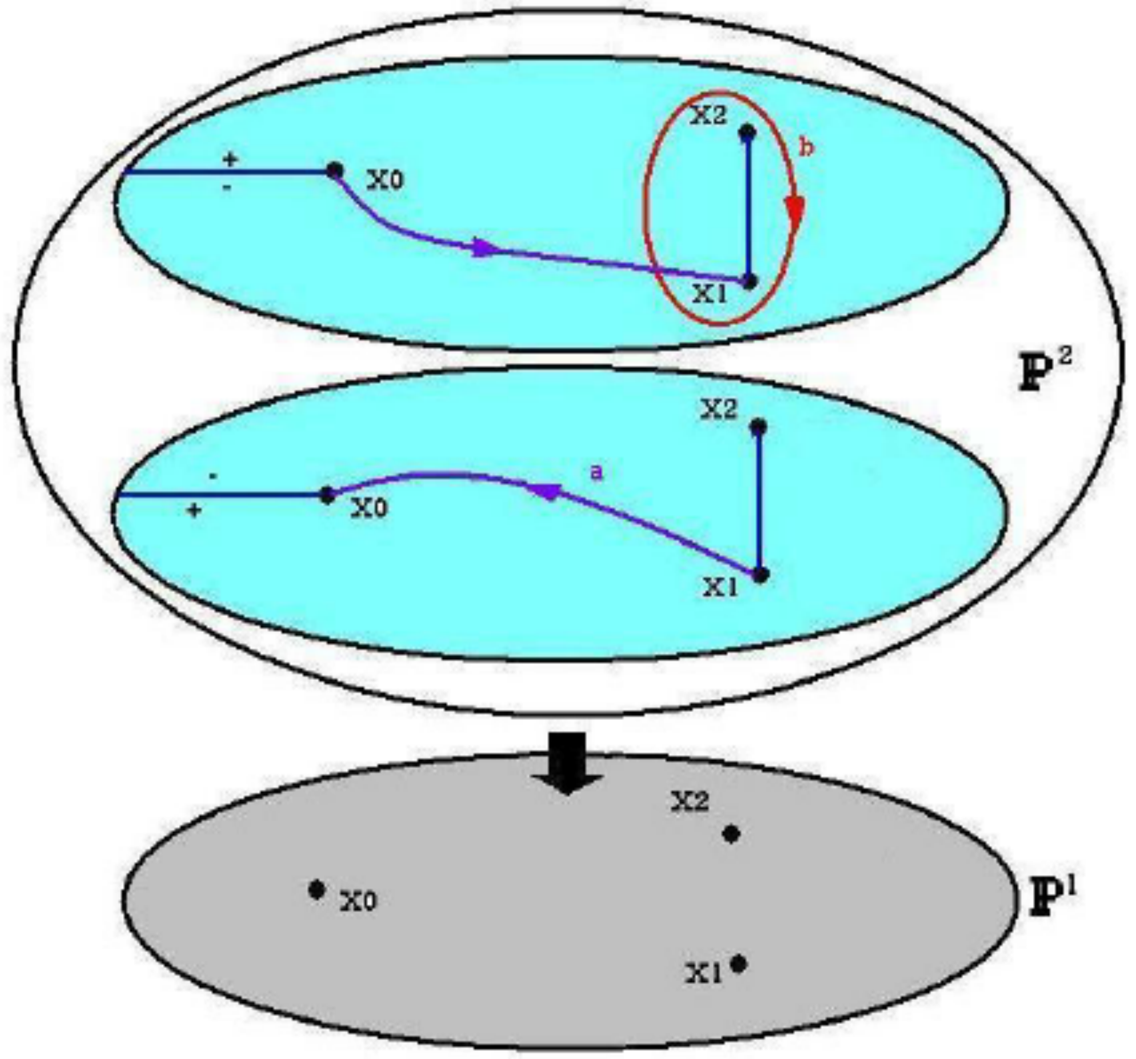
The LG point has superpotential

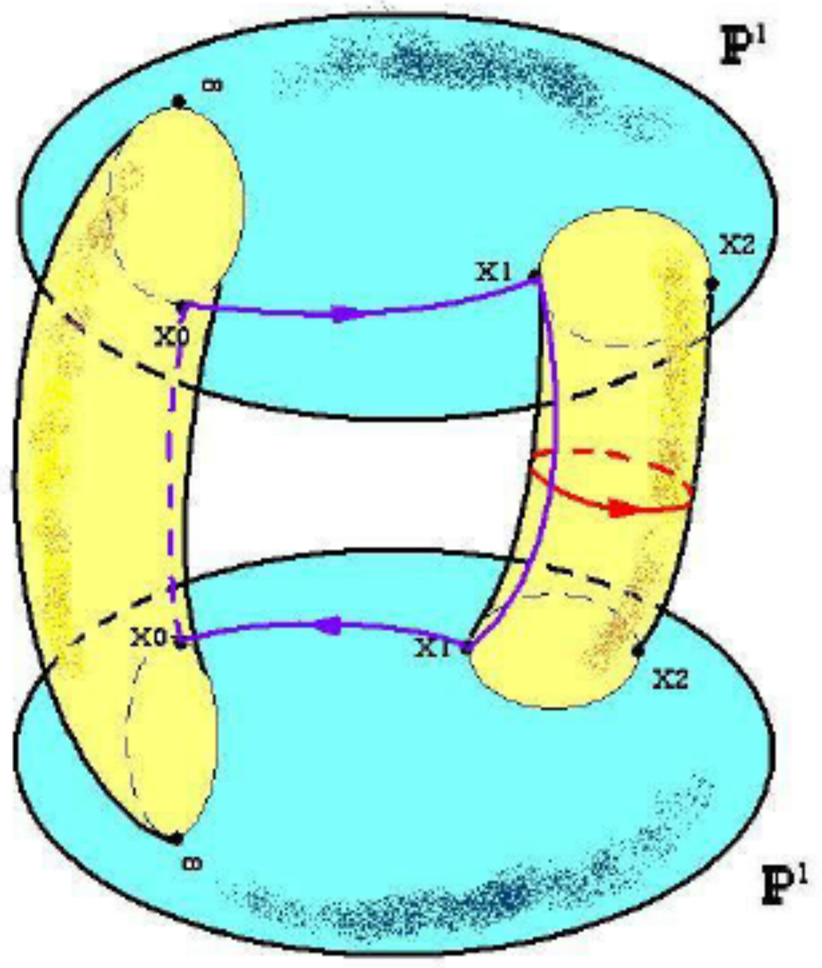
$$W = \sum_{ij} A^{ij}(p)\, \phi_i \phi_j \quad \text{— mass matrix for } \phi \text{ fields.}$$

Away from zeroes of eigenvalues of A^{ij},

looks like sigma model on $\mathbb{P}^1 = \text{Proj}\, \mathbb{C}[p_1, p_2]$, with $B\mathbb{Z}_2$ symmetry.

Decomposition $\Rightarrow$ Double cover of $\mathbb{P}^1$, branched over $\{\det A = 0\} = \{4 \text{ points}\}$





Another T^2 !

geometry

realized

nonperturbatively

via decomposition

Application: elliptic genera of pure susy gauge theories

(R Eager, ES '20)

We can use decomposition to predict elliptic genera of pure (2,2) susy gauge theories, using knowledge of IR susy breaking for various discrete theta angles.

Example: for $SU(k)/\mathbb{Z}_k$, susy unbroken only for discrete theta $\theta = -(1/2)k(k-1) \mod k$

(as derived from 2d nonabelian mirrors)

$$\mathrm{EG}(G/K, \theta) = 0 \quad \text{if susy broken in IR}$$

Decomposition $\Rightarrow$ $$\mathrm{EG}(G) = \sum_{\theta} \mathrm{EG}(G/K, \theta)$$

Can then algebraically recover elliptic genera.

Example: $$\mathrm{EG}(SU(k)/\mathbb{Z}_k, \theta) = (1/k)\mathrm{EG}(SU(k)) \sum_{m=0}^{k-1} (-)^{m(k+1)} \exp(im\theta)$$

For $k = 2$, matches (Kim, Kim, Park '17).

Numerous other low-rank exs checked with susy localization.

Application: anomalies

Suppose the orbifold $[X/G]$ is anomalous, for G finite.

Two methods to resolve the anomaly:

1) Make G bigger. (Wang-Wen-Witten '17, Tachikawa '17)

Replace G by Γ, $1 \longrightarrow K \longrightarrow \Gamma \xrightarrow{\pi} G \longrightarrow 1$

where $\pi^*\alpha$ trivial for $\alpha \in H^3(G, U(1))$ the anomaly,

and replace original orbifold with $[X/\Gamma]_B$ for suitable phases $B \in H^1(G, H^1(K, U(1)))$.

2) Make G smaller.

Replace original orbifold with $[X/\ker f]$ for some hom' $f: G \to H$ s.t. $\alpha|_{\ker f} = 0$

Decomposition: $[X/\Gamma]_B \; = \;$ (copies of) $[X/\ker B]$ (Robbins, ES, Vandermeulen '21)

So the two possibilities are equivalent.

Application: moduli spaces

Gerbe structures are common on moduli spaces of SCFTs.

Moduli stack of susy sigma models = $\mathbb{Z}_2$ gerbe over moduli stack of CYs

Bagger-Witten line bundle = `fractional' bundle over that gerbe

(a bundle on the gerbe that is not a pullback from the underlying moduli space)

(Donagi et al '17, '19)

Example: moduli space of elliptic curves

$$\mathscr{M} = [\mathfrak{h}/SL(2,\mathbb{Z})]$$ for $\mathfrak{h}$ the upper half plane

However, the Bagger-Witten line bundle lives on $\mathscr{N} = [\mathfrak{h}/Mp(2,\mathbb{Z})]$

where $$1 \longrightarrow \mathbb{Z}_2 \longrightarrow Mp(2,\mathbb{Z}) \longrightarrow SL(2,\mathbb{Z}) \longrightarrow 1$$

(Gu, ES '16)

which reflects a subtle $\mathbb{Z}_2$ extending T-duality in susy theories.

(Pantev, ES '16)

Summary

Decomposition: sometimes one QFT secretly $= \sum$ QFTs $= \sum$ universes

Restrictions on instantons arise from such sums as interference effect between universes

Such theories (typically) have one-form symmetries, and arise from sigma models on gerbes

= fiber bundles with fibers $BG = [\text{point}/G]$

Examples include gauge theories w/ trivially-acting subgroups

Applications include Gromov-Witten theory, GLSMs, elliptic genera, anomalies.

Thank you for your time!