Moduli of heterotic string compactifications

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Introduction

This talk will concern compactifications of heterotic strings.

Recall: in 10D, a heterotic string is specified by
metric + nonabelian gauge field,
so to compactify,
we specify not only a space,
but also a bundle over that space.

Introduction, cont'd

Let X be a space, let $\mathcal{E}$ be a bundle over that space.

Constraints:

$$\mathrm{ch}_2(\mathcal{E}) = \mathrm{ch}_2(TX)$$ (Green-Schwarz anom' canc')

$$F_{ij} = 0 = F_{\overline{ij}}$$ ($\mathcal{E}$ holomorphic)

$$g^{i\bar{j}} F_{i\bar{j}} = 0$$ (Donaldson-Uhlenbeck-Yau equ'n; stability, D-terms)

Introduction, cont'd

Often, we want X to be a Calabi-Yau manifold.

-- complex, Kahler, nowhere-zero hol' top-form

In this case, we often divide possible bundles into 2 classes, corresponding to amount of worldsheet susy.

* (2,2) susy: $\mathcal{E} = TX$ ``standard embedding''

* (0,2) susy: $\mathcal{E} \neq TX$

Introduction, cont'd

Other times, we want X to be non-Kahler:

-- complex, non-Kahler, nowhere-zero hol' top-form

(Strominger, '86)

In this case, there is a nonzero background H flux:

$$\omega = i g_{i\bar{\jmath}} \, dz^i \wedge d\overline{z}^{\bar{\jmath}}$$

$$H = i(\overline{\partial} - \partial)\omega$$

-- vanishes for Kahler metric, nonzero for non-Kahler

Introduction, cont'd

So far I've just reviewed various heterotic compactifications.

In this talk, I'm interested in the possible deformations, the `moduli,' of such heterotic compactifications.

Broadly speaking, the moduli are:

* metric moduli
* bundle moduli

and because of conditions such as $F_{ij} = 0 = F_{\overline{\imath}\overline{\jmath}}$, these can be intertangled.

Introduction, cont'd

In Calabi-Yau compactifications, these moduli have been identified with complex, Kahler, bundle moduli for many years (though updated recently, and worldsheet description missing).

In non-Kahler heterotic compactifications, moduli have been a mystery.

Today, I'll present an expression for moduli of heterotic non-Kahler compactifications (which will also give a worldsheet description of recent updates to Calabi-Yau story).

Introduction, cont'd

Outline of the rest of the talk:

* Describe what is known about moduli.

* Present the result:
infinitesimal moduli,
expressed as cocycles + coboundaries

* Derivation of result.

Moduli

As mentioned earlier, there are 2 sources of moduli:

* metric moduli
* bundle moduli

which are required to obey constraints such as

$$F_{ij} = 0 = F_{\overline{\imath}\overline{\jmath}} \; , \qquad g^{i\overline{\jmath}} F_{i\overline{\jmath}} = 0$$

(so the metric & bundle moduli are linked in general).

Metric moduli:

On Calabi-Yau manifolds, from Yau's theorem, metric moduli decompose into 2 types:

* Deformations of the complex structure, counted by $H^1(X, TX)$

$$N(J_\mu^\nu + \delta J_\mu^\nu) = 0 \implies \overline{\partial}\, \delta J_{\overline{\imath}}^{\jmath} = 0$$

* Deformations of the Kahler structure, counted by $H^1(X, T^*X)$

$$\overline{\partial}\,(\omega + \delta\omega) = 0 \implies \overline{\partial}\, \delta\omega_{i\overline{\jmath}} = 0$$

Metric moduli:

On non-Kahler mflds, no analogue of Yau's thm known, which is part of why moduli of non-Kahler compactifications are mysterious.
We'll present a proposal, later....

Bundle moduli:

Consider an infinitesimal deformation of the gauge field:

$$A_\mu^a + \delta A_\mu^a$$

If we demand $F_{\overline{\imath \jmath}} = 0$ for both the original gauge field and the deformation, then the deformation must satisfy

$$\overline{\partial}\, \delta A = 0$$

Interpretation: $\delta A \in H^1(\mathrm{End}\, \mathcal{E})$

These are the bundle moduli.

So far, I've described the metric & bundle moduli separately. However, b/c of the conditions

$$F_{ij} = 0 = F_{\overline{ij}} \; , \qquad g^{i\bar{j}} F_{i\bar{j}} = 0$$

the metric & bundle moduli are linked, and mix.

For the simplest cases ((2,2) Calabi-Yau compactifications), there's no mixing, but in gen'l (0,2) Calabi-Yau cases these do mix.
(Recent result: Anderson, Gray, Lukas, Ovrut)

We'll see the details as I proceed....

I'll systematically review what's known about moduli in the following three cases:

- (2,2) susy worldsheet (Calabi-Yau)
- (0,2) susy worldsheet (Calabi-Yau)
- Non-Kahler heterotic compactification

and then I'll present our result.

(2,2) susy worldsheet:

This is the `standard embedding,'
in which gauge bundle = tangent bundle.

Here, the allowed moduli are:

* Complex moduli: $Z^i_{\bar{\imath}} \in H^1(X, TX)$
$$\bar{\partial} Z = 0$$

* Kahler moduli: $Y_{i\bar{\imath}} \in H^1(X, T^*X)$
$$\bar{\partial} Y = 0$$

* Bundle moduli: $\Lambda^{\alpha}_{\beta\bar{\imath}} \in H^1(\mathrm{End}\,\mathcal{E})$
$$\bar{\partial} \Lambda = 0$$

(2,2) susy worldsheet:

For later reference, let us expression the moduli in local coordinates, as cocycles mod coboundaries:

Cocycles:

$$Z^i_{\bar{\imath},\bar{k}} - Z^i_{\bar{k},\bar{\imath}} = 0$$

$$Y_{i\bar{\imath},\bar{k}} - Y_{i\bar{k},\bar{\imath}} = 0$$

$$\Lambda^\alpha_{\beta\bar{\imath},\bar{k}} - \Lambda^\alpha_{\beta\bar{k},\bar{\imath}} = 0$$

Coboundaries:

$$Z^i_{\bar{\imath}} \sim Z^i_{\bar{\imath}} + \zeta^i_{,\bar{\imath}}$$

$$Y_{i\bar{\imath}} \sim Y_{i\bar{\imath}} + \mu_{i,\bar{\imath}}$$

$$\Lambda^\alpha_{\beta\bar{\imath}} \sim \Lambda^\alpha_{\beta\bar{\imath}} + \lambda^\alpha_{\beta,\bar{\imath}}$$

(0,2) susy worldsheet (Calabi-Yau):

In this case, the gauge bundle $\neq$ tangent bundle.

It was thought for many years that one still had the same complex, Kahler, bundle moduli in this case.

However, a year ago, L Anderson, J Gray, A Lukas, B Ovrut argued that the complex & bundle moduli **mix**, so that infinitesimally, one has a subset of complex & bundle moduli.

(0,2) susy worldsheet (Calabi-Yau):

One only has a **subset** of complex + bundle moduli, ultimately because of the constraints

$$F_{ij} = 0 = F_{\overline{ij}}\,.$$

These tie together the bundle and complex moduli, so that they are no longer independent.

Correct replacement for complex+bundle is

$$H^1(X, Q)$$

where $$0 \longrightarrow \mathcal{E}^* \otimes \mathcal{E} \longrightarrow Q \xrightarrow{\pi} TX \longrightarrow 0$$

(Atiyah sequence)

(extension determined by F)

(0,2) susy worldsheet (Calabi-Yau):

From Atiyah sequence $0 \longrightarrow \mathcal{E}^* \otimes \mathcal{E} \longrightarrow Q \stackrel{\pi}{\longrightarrow} TX \longrightarrow 0$

we get

$$0 \longrightarrow H^1(X, \mathcal{E}^* \otimes \mathcal{E}) \longrightarrow H^1(X, Q) \stackrel{d\pi}{\longrightarrow} H^1(TX) \stackrel{F}{\longrightarrow} H^2(X, \mathcal{E}^* \otimes \mathcal{E})$$

Interpretation:

If a complex structure modulus in $H^1(X, TX)$ is in the image of $d\pi$, then it came from an element of $H^1(X, Q)$ and survives.

However, a complex structure modulus not in the image of $d\pi$ is lifted by the bundle.

We'll see more details shortly....

Examples are discussed in e.g. Anderson, Gray, Lukas, Ovrut, 1010.0255.

Gen'l form:

Start with a deg (2,2,3) hypersurface in $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^2$.

Define a stable indecomposable rk 2 bundle over an 58-dim'l sublocus of cpx moduli space (not holomorphic elsewhere).

If the complex structure modulus takes one out of that sublocus, then, hol' structure on bundle broken.

Aside: An analogue in D-branes.

If I wrap a D-brane on a cpx submfld S,
and describe the Chan-Paton factors with a
holomorphic vector bundle $\mathcal{E}$,
then naively have moduli

$H^0(S, \mathcal{E}^* \otimes \mathcal{E} \otimes N_{S/X})$ -- brane motions

$\oplus \; H^1(S, \mathcal{E}^* \otimes \mathcal{E})$ -- bundle moduli

However, in general some motions of S can
destroy holomorphic structure on bundle;
only want a subset of brane motions.

Correct moduli: $\mathrm{Ext}^1_X\left(i_* \mathcal{E}, i_* \mathcal{E}\right)$

(Katz, ES, hepth/0208104)

(0,2) susy worldsheet (Calabi-Yau):

For later reference, let us express the moduli in local coordinates, as cocycles mod coboundaries:

Cocycles:

$$Z^i_{\bar{\imath},\bar{k}} - Z^i_{\bar{k},\bar{\imath}} = 0$$

$$Y_{i\bar{\imath},\bar{k}} - Y_{i\bar{k},\bar{\imath}} = 0$$

$$\Lambda^{\alpha}_{\beta\bar{\imath},\bar{k}} - \Lambda^{\alpha}_{\beta\bar{k},\bar{\imath}} = F^{\alpha}_{\beta\bar{k}i} Z^i_{\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i} Z^i_{\bar{k}}$$

(Anderson et al, 1107.5076, equ'n (3.8))

Coboundaries:

$$Z^i_{\bar{\imath}} \sim Z^i_{\bar{\imath}} + \zeta^i_{,\bar{\imath}}$$

$$Y_{i\bar{\imath}} \sim Y_{i\bar{\imath}} + \mu_{i,\bar{\imath}}$$

$$\Lambda^{\alpha}_{\beta\bar{\imath}} \sim \Lambda^{\alpha}_{\beta\bar{\imath}} + \lambda^{\alpha}_{\beta,\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i}\zeta^i$$

Check that this obstructs (some) complex moduli:

$$\Lambda^{\alpha}_{\beta\bar{\imath},\bar{k}} - \Lambda^{\alpha}_{\beta\bar{k},\bar{\imath}} = \underline{F^{\alpha}_{\beta\bar{k}i} Z^{i}_{\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i} Z^{i}_{\bar{k}}}$$

$$\in H^2(X, \mathcal{E}^* \otimes \mathcal{E})$$

$Z^{i}_{\bar{\imath}}$ is a complex structure modulus

The right-hand side above is the image of the map F in

$$0 \longrightarrow H^1(X, \mathcal{E}^* \otimes \mathcal{E}) \longrightarrow H^1(X, Q) \xrightarrow{d\pi} H^1(TX) \xrightarrow{F} H^2(X, \mathcal{E}^* \otimes \mathcal{E})$$

The left-hand side is cohomologically trivial.

Only if the right-hand side is cohomologically trivial can there be a solution.

Check that this reduces correctly on (2,2) locus:

$$\Lambda^{\alpha}_{\beta\bar{\imath},\bar{k}} - \Lambda^{\alpha}_{\beta\bar{k},\bar{\imath}} = F^{\alpha}_{\beta\bar{k}i} Z^{i}_{\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i} Z^{i}_{\bar{k}}$$

On the (2,2) locus, those F's aren't zero,
so I need to explain how this reduces to

$$\Lambda^{\alpha}_{\beta\bar{\imath},\bar{k}} - \Lambda^{\alpha}_{\beta\bar{k},\bar{\imath}} = 0$$

Briefly, on the (2,2) locus,
F = Riemann curvature tensor R,
and in that case, one can redefine $\Lambda^{\alpha}_{\beta\bar{\imath}}$ with a translation to absorb the R terms.

Details next....

Check that this reduces correctly on (2,2) locus:

Cocycle condition:

$$\Lambda^m_{n\bar{\imath},\bar{k}} - \Lambda^m_{n\bar{k},\bar{\imath}} = \underline{R^m{}_{n\bar{k}i} Z^i_{\bar{\imath}} - R^m{}_{n\bar{\imath}i} Z^i_{\bar{k}}}$$

problematic

Define $\tilde{\Lambda}^m_{n\bar{\imath}} = \Lambda^m_{n\bar{\imath}} - \nabla_n Z^m_{\bar{\imath}}$

then

$$\tilde{\Lambda}^m_{n\bar{\imath},\bar{k}} - \tilde{\Lambda}^m_{n\bar{k},\bar{\imath}} = \Lambda^m_{n\bar{\imath},\bar{k}} - \Lambda^m_{n\bar{k},\bar{\imath}} - \nabla_{\bar{k}} \nabla_n Z^m_{\bar{\imath}} + \nabla_{\bar{\imath}} \nabla_n Z^m_{\bar{k}}$$

$$= \underline{R^m{}_{n\bar{k}i} Z^i_{\bar{\imath}} - R^m{}_{n\bar{\imath}i} Z^i_{\bar{k}} - [\nabla_{\bar{k}}, \nabla_n] Z^m_{\bar{\imath}} + [\nabla_{\bar{\imath}}, \nabla_n] Z^m_{\bar{k}}}$$

= 0

$$+\nabla_n(\underline{-Z^m_{\bar{\imath},\bar{k}} + Z^m_{\bar{k},\bar{\imath}}})$$

= 0

$$= 0$$

Check that this reduces correctly on (2,2) locus:

We can treat coboundaries similarly:

Recall $$\tilde{\Lambda}^m_{n\bar{\imath}} = \Lambda^m_{n\bar{\imath}} - \nabla_n Z^m_{\bar{\imath}}$$

where
$$\Lambda^m_{n\bar{\imath}} \sim \Lambda^m_{n\bar{\imath}} + \lambda^m_{n,\bar{\imath}} - R^m{}_{n\bar{\imath}i}\zeta^i, \qquad Z^i_{\bar{\imath}} \sim Z^i_{\bar{\imath}} + \zeta^i_{,\bar{\imath}}$$

hence $$\tilde{\Lambda}^m_{n\bar{\imath}} \sim \tilde{\Lambda}^m_{n\bar{\imath}} + \lambda^m_{n,\bar{\imath}} - \underbrace{R^m{}_{n\bar{\imath}i}\zeta^i - \nabla_n \nabla_{\bar{\imath}}\zeta^i}_{\text{problematic}}$$

Define $$\tilde{\lambda}^m_n = \lambda^m_n - \nabla_n \zeta^m$$

Then $$\tilde{\Lambda}^m_{n\bar{\imath}} \sim \tilde{\Lambda}^m_{n\bar{\imath}} + \tilde{\lambda}^m_{n,\bar{\imath}} + \underbrace{[\nabla_{\bar{\imath}}, \nabla_n]\zeta^m - R^m{}_{n\bar{\imath}i}\zeta^i}_{= 0}$$

Check that this reduces correctly on (2,2) locus:

Started with

$$\Lambda^m_{n\bar{\imath},\bar{k}} - \Lambda^m_{n\bar{k},\bar{\imath}} = R^m{}_{n\bar{k}i} Z^i_{\bar{\imath}} - R^m{}_{n\bar{\imath}i} Z^i_{\bar{k}}$$

$$\Lambda^m_{n\bar{\imath}} \sim \Lambda^m_{n\bar{\imath}} + \lambda^m_{n,\bar{\imath}} - R^m{}_{n\bar{\imath}i}\zeta^i$$

but we can absorb the R terms into redef'ns:

$$\tilde{\Lambda}^m_{n\bar{\imath},\bar{k}} - \tilde{\Lambda}^m_{n\bar{k},\bar{\imath}} = 0$$

$$\tilde{\Lambda}^m_{n\bar{\imath}} \sim \tilde{\Lambda}^m_{n\bar{\imath}} + \tilde{\lambda}^m_{n,\bar{\imath}}$$

and so we recover the cocycles, coboundaries for the (2,2) locus.

Non-Kahler heterotic compactifications:

Here, almost nothing is known about moduli.

Nearly the only thing known is that the `breathing mode', which rescales the entire metric of a CY, is absent.

* Since H appears in multiple places,

$$dH = \alpha' (\mathrm{tr}\, R_H \wedge R_H - \mathrm{tr}\, F \wedge F)$$

is nonlinear in α' so values of α' are isolated

* H is quantized (mod anom') but $H \propto (\bar{\partial} - \partial)\omega$

Non-Kahler heterotic compactifications:

Because that `breathing mode' is absent, one cannot smoothly deform a non-Kahler compactification to a weak coupling, `large radius' limit.

Hence, any claims about rel'ns to geometry, are necessarily somewhat formal.

Non-Kahler heterotic compactifications:

Aside from the absence of the breathing mode, there's no systematic understanding of the moduli of heterotic non-Kahler compactifications.

That said, there are a few special cases where something is known.

Ex: Adams/Lapan compute spectra at LG points in their torsion LSMs, but, interpretation & rel'n to geometry are unclear.

Next: proposal for the answer....

Briefly, our proposal for infinitesimal moduli:

Cocycles:

$$Z^i_{\bar{\imath},\bar{k}} - Z^i_{\bar{k},\bar{\imath}} = 0$$

New

$$Y_{i\bar{\imath},\bar{k}} - Y_{i\bar{k},\bar{\imath}} = \underline{Z^j_{\bar{k}} H_{ji\bar{\imath}} - Z^j_{\bar{\imath}} H_{ji\bar{k}}}$$

$$\Lambda^{\alpha}_{\beta\bar{\imath},\bar{k}} - \Lambda^{\alpha}_{\beta\bar{k},\bar{\imath}} = F^{\alpha}_{\beta\bar{k}i} Z^i_{\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i} Z^i_{\bar{k}}$$

Coboundaries:

$$Z^i_{\bar{\imath}} \sim Z^i_{\bar{\imath}} + \left(\zeta^i + g^{i\bar{\jmath}}\xi_{\bar{\jmath}}\right)_{,\bar{\imath}} + g^{i\bar{k}}\left(\xi_{\bar{\imath},\bar{k}} - \xi_{\bar{k},\bar{\imath}}\right)$$

$$Y_{i\bar{\imath}} \sim Y_{i\bar{\imath}} + \mu_{i,\bar{\imath}} + \xi_{\bar{\imath},i} + \underline{H_{i\bar{\imath}j}\left(\zeta^j + g^{j\bar{\jmath}}\xi_{\bar{\jmath}}\right)}$$

$$\Lambda^{\alpha}_{\beta\bar{\imath}} \sim \Lambda^{\alpha}_{\beta\bar{\imath}} + \lambda^{\alpha}_{\beta,\bar{\imath}} - F^{\alpha}_{\beta\bar{\imath}i}\left(\zeta^i + g^{i\bar{\jmath}}\xi_{\bar{\jmath}}\right)$$

Check:

For (0,2) Calabi-Yau compactifications, H=0, so cocycles reduce to

$$Z^i_{\bar\imath,\bar k} - Z^i_{\bar k,\bar\imath} = 0$$

$$Y_{i\bar\imath,\bar k} - Y_{i\bar k,\bar\imath} = 0$$

$$\Lambda^\alpha_{\beta\bar\imath,\bar k} - \Lambda^\alpha_{\beta\bar k,\bar\imath} = F^\alpha_{\beta\bar k i} Z^i_{\bar\imath} - F^\alpha_{\beta\bar\imath i} Z^i_{\bar k}$$

which is what we described earlier.

Check:

For heterotic non-Kahler compactifications, where H is not zero, about the only thing we know is that the Kahler `breathing mode' is obstructed.

In $Y_{i\bar{\imath},\bar{k}} - Y_{i\bar{k},\bar{\imath}} = Z^j_{\bar{k}} H_{ji\bar{\imath}} - Z^j_{\bar{\imath}} H_{ji\bar{k}}$

if we take Z=0, and take $Y_{i\bar{\imath}} \propto g_{i\bar{\imath}}$

(so as to describe the breathing mode), then since the space is non-Kahler, $\overline{\partial} Y \neq 0$, and so we see the breathing mode is obstructed.

Mathematical interpretation

How to interpret the structure mathematically as some sort of cohomology theory?

Begin with the pure metric part:

$$Z^i_{\bar{\imath},\bar{k}} - Z^i_{\bar{k},\bar{\imath}} = 0$$

$$Y_{i\bar{\imath},\bar{k}} - Y_{i\bar{k},\bar{\imath}} = Z^j_{\bar{k}} H_{ji\bar{\imath}} - Z^j_{\bar{\imath}} H_{ji\bar{k}}$$

At tree level, we can interpret this using an analogue of the Atiyah sequence from earlier....

Mathematical interpretation

Tree level: Since $dH = 0 = \overline{\partial}(H_{(2,1)})$

the (2,1) part of H, at tree level, defines an element of

$$H^1(\wedge^2 T^* X) \subseteq H^1(T^* X \otimes T^* X)$$

and hence an extension

$$0 \longrightarrow T^* X \longrightarrow Q \longrightarrow TX \longrightarrow 0$$

The metric moduli are then elements of $H^1(Q)$.

When H=0, then $Q = T^* X \oplus TX$

and $H^1(Q) = H^1(T^* X) + H^1(TX)$

-- standard Calabi-Yau result

Mathematical interpretation

Similarly, for the full heterotic moduli,
F+H defines an extension

$$0 \longrightarrow T^*X \oplus \mathrm{End}\,\mathcal{E} \longrightarrow Q \longrightarrow TX \longrightarrow 0$$

and the heterotic moduli are then elements of

$$H^1(Q)$$

Mathematical interpretation

Examples? In progress.

Interpretation for nonzero α'? Unknown.

Derivation directly from study of metric moduli?
Desired, not known.

Any rel'n to Hitchin's generalized complex geometry?
Unknown at present.

Physics: why has this been missed in spectrum computations?

After all, people have computed heterotic massless spectra for a quarter century now....

Answer: We usually assume that at large radius, we can reduce to free fields and compute zero energy spectrum.

However,

$$T_L \propto g_{i\bar{\jmath}} \partial \phi^i \partial \overline{\phi}^{\bar{\jmath}} + \overline{\gamma}_\beta \partial \gamma^\beta + \underline{A^\alpha_{\beta j} \partial \phi^j \overline{\gamma}_\alpha \gamma^\beta}$$

not quadratic

hence difficult to pick out massless part of spectrum.

Physical intuition:

Why is the worldsheet BRST cohomology changing?

-- perturbative corrections to OPE's.

On the worldsheet, BRST operator $Q \propto g_{i\overline{\jmath}} \overline{\partial} \phi^i \psi^{\overline{\jmath}}$

Bdle modulus $\Lambda^{\alpha}_{\beta \overline{\imath}} \gamma_{\alpha} \gamma^{\beta} \psi^{\overline{\imath}}$, Cpx modulus $Z^{i}_{\overline{\imath}} \overline{\partial} \phi_i \psi^{\overline{\imath}}$

$$Q \cdot (\mathrm{moduli}) = \left(g_{i\overline{\jmath}} \overline{\partial} \phi^i \psi^{\overline{\jmath}} \right) \cdot \left(\Lambda^{\alpha}_{\beta \overline{\imath}} \gamma_{\alpha} \gamma^{\beta} \psi^{\overline{\imath}} \right)$$

$$+ \left(g_{i\overline{\jmath}} \overline{\partial} \phi^i \psi^{\overline{\jmath}} \right) \left(Z^{i}_{\overline{\imath}} \overline{\partial} \phi_i \psi^{\overline{\imath}} \right) \left(\int d^2 z F^{\alpha}_{\beta \overline{k} m} \gamma_{\alpha} \gamma^{\beta} \psi^{\overline{k}} \psi^{m} \right)$$

Compare

$$\Lambda^{\alpha}_{\beta \overline{\imath}, \overline{k}} - \Lambda^{\alpha}_{\beta \overline{k}, \overline{\imath}} = F^{\alpha}_{\beta \overline{k} i} Z^{i}_{\overline{\imath}} - F^{\alpha}_{\beta \overline{\imath} i} Z^{i}_{\overline{k}}$$

Physical intuition:

Why is the worldsheet BRST cohomology changing?

Similarly, for complex moduli,

Cpx modulus $Z^i_{\bar\imath}\,\overline{\partial}\phi_i\psi^{\bar\imath}$, Kahler modulus $Y_{i\bar\imath}\,\overline{\partial}\phi^i\psi^{\bar\imath}$

$$Q\cdot(\text{moduli}) = \left(g_{i\bar\jmath}\overline{\partial}\phi^i\psi^{\bar\jmath}\right)\cdot\left(Y_{k\overline{m}}\,\overline{\partial}\phi^k\psi^{\overline{m}}\right) + \left(g_{i\bar\jmath}\overline{\partial}\phi^i\psi^{\bar\jmath}\right)\left(Z^k_{\bar k}\,\overline{\partial}\phi_k\psi^{\bar k}\right)\left(\int d^2z H_{m\bar n j}\partial\phi^m\psi^{\bar n}\psi^j\right)$$

Compare

$$Y_{i\bar\imath,\bar k} - Y_{i\bar k,\bar\imath} = Z^j_{\bar k}H_{ji\bar\imath} - Z^j_{\bar\imath}H_{ji\bar k}$$

So far I have described the result,
checked that the result is consistent,
and given some intuition for why it arises.

In principle, I could push the OPE computations I outlined further to give a derivation,
but instead I'll use a different approach.

I'll classify marginal operators we could add to a 2d UV theory,
in the spirit of Beasley-Witten's analysis of 4d SQCD.

(an idea I must attribute to my collaborator, I Melnikov)

To that end, a brief review of (0,2) superspace....

(0,2) superspace:

Coordinates $(z, \overline{z}, \theta, \overline{\theta})$

$$D = \frac{\partial}{\partial\theta} + \overline{\theta\partial} \qquad \overline{D} = \frac{\partial}{\partial\overline{\theta}} + \theta\overline{\partial}$$

$$Q = -\frac{\partial}{\partial\theta} + \overline{\theta\partial} \qquad \overline{Q} = -\frac{\partial}{\partial\overline{\theta}} + \theta\overline{\partial}$$

$$\{D, \overline{D}\} = +2\overline{\partial} \qquad \{Q, \overline{Q}\} = -2\overline{\partial}$$

D, Q have $U(1)_R$ charge -1

$\overline{D}$, $\overline{Q}$ have $U(1)_R$ charge +1

(0,2) superfields:

Chiral superfields:

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta\overline{\theta}\overline{\partial}\phi$$

boson

right-moving fermion

Fermi superfields:

$$\Gamma = \gamma + \sqrt{2}\theta G + \theta\overline{\theta}\overline{\partial}\gamma$$

left-moving fermion

auxiliary field

$$\overline{D}\Phi = 0 = \overline{D}\Gamma$$

(Dine-Seiberg PLB 180 '86)

(0,2) Lagrangian:

$$= D\overline{D}\left[\frac{1}{2}\left(K_i(\Phi,\overline{\Phi})\partial\Phi^i - \overline{K}_{\overline{\imath}}(\Phi,\overline{\Phi})\partial\overline{\Phi}^{\overline{\imath}}\right) - \underline{H_{\beta\overline{\alpha}}(\Phi,\overline{\Phi})}\overline{\Gamma}^{\overline{\alpha}}\Gamma^{\beta}\right]$$

fiber metric

The K_i is a potential for the metric, just as, in (2,2) cases, the Kahler potential determines the metric.

Since the metric is not Kahler in gen'l here,

$g_{i\overline{\jmath}} \neq \partial_i \partial_{\overline{\jmath}} K$ for any K

Instead, we'll see next $g_{i\overline{\jmath}} = \frac{1}{2}\left(K_{i,\overline{\jmath}} + \overline{K}_{\overline{\jmath},i}\right)$

(0,2) Lagrangian:

$$\omega = i g_{i\bar{\jmath}} dz^i \wedge d\bar{z}^{\bar{\jmath}} \qquad H = dB = i(\bar{\partial} - \partial)\omega$$

At leading order, $dH = 0$ so $\partial\bar{\partial}\omega = 0$

Implies $g_{i[\bar{\jmath},\bar{k}]m} = g_{m[\bar{\jmath},\bar{k}]i}$

Implies $g_{i[\bar{\jmath},\bar{k}]} = \partial_i \overline{W}_{\bar{\jmath}\bar{k}}$ for some $\overline{W}$ s.t. $\overline{\partial W} = 0$

$\overline{\partial W} = 0$ implies locally $\overline{W}_{\bar{\jmath}\bar{k}} = (1/2)(\overline{K}_{\bar{\jmath},\bar{k}} - \overline{K}_{\bar{k},\bar{\jmath}})$

for some $\overline{K}_{\bar{\imath}}$

Combining with complex conjugate, we see

$$g_{i\bar{\jmath}} = \frac{1}{2}\left(K_{i,\bar{\jmath}} + \overline{K}_{\bar{\jmath},i}\right)$$

(Dine-Seiberg PLB 180 `86)

(0,2) Lagrangian:

$$= D\overline{D}\left[\frac{1}{2}\left(K_i(\Phi,\overline{\Phi})\partial\Phi^i - \overline{K}_{\bar{\imath}}(\Phi,\overline{\Phi})\partial\overline{\Phi}^{\bar{\imath}}\right) - \underline{H_{\beta\overline{\alpha}}(\Phi,\overline{\Phi})}\overline{\Gamma}^{\overline{\alpha}}\Gamma^{\beta}\right]$$

fiber metric

$$= g_{i\bar{\jmath}}\left(\partial\phi^i\overline{\partial\phi}^{\bar{\jmath}} + \partial\overline{\phi}^{\bar{\jmath}}\overline{\partial}\phi^i\right) + B_{i\bar{\jmath}}\left(\partial\phi^i\overline{\partial\phi}^{\bar{\jmath}} - \partial\overline{\phi}^{\bar{\jmath}}\overline{\partial}\phi^i\right)$$

$$+2g_{i\bar{\jmath}}\overline{\psi}^{\bar{\jmath}}\partial\psi^i + 2\overline{\psi}^{\bar{\imath}}\left(\partial\phi^k\Omega^{-}_{\bar{\imath}kj} + \partial\overline{\phi}^{\bar{k}}\Omega^{-}_{\bar{\imath}\bar{k}j}\right)\psi^j$$

$$+\overline{\gamma}_{\alpha}\left(\overline{\partial}\gamma^{\alpha} + \overline{\partial}\phi^j A^{\alpha}_{\beta j}\gamma^{\beta}\right) + \overline{\gamma}_{\alpha}F^{\alpha}_{\beta\bar{\jmath}k}\gamma^{\beta}\psi^k\overline{\psi}^{\bar{\jmath}}$$

where

$$g_{i\bar{\jmath}} = \frac{1}{2}\left(K_{i,\bar{\jmath}} + \overline{K}_{\bar{\jmath},i}\right) \qquad B_{i\bar{\jmath}} = \frac{1}{2}\left(K_{i,\bar{\jmath}} - \overline{K}_{\bar{\jmath},i}\right)$$

$$\Omega^{-}_{\bar{\imath}kj} = \Gamma_{\bar{\imath}kj} - \frac{1}{2}H_{\bar{\imath}kj} \qquad H = dB = i(\overline{\partial} - \partial)\omega$$

In this language, a susy marginal operator should be of the form DX

where X is a (0,2) chiral superfield with classical dim 1, $U(1)_R$ charge +1.

((2,2): Seiberg et al, 1005.3546; (0,2): Adam, M, Plesser, unpub)

Check:

Under a susy transformation,

$$\int d^2z DX \mapsto \int d^2z D(-\xi Q - \overline{\xi Q})X$$

Up to total derivatives, $Q = -D$, $\overline{Q} = -\overline{D}$

$$\int d^2z DX \mapsto \int d^2z D(\xi D + \overline{\xi D})X = 0$$

In this language, a susy marginal operator should be of the form DX

where X is a (0,2) chiral superfield with classical dim 1, $U(1)_R$ charge +1.

Most general possibility:

$$X = \left[\overline{\Gamma}_\alpha \Gamma^\beta \Lambda^\alpha_{\beta\bar{\imath}}(\Phi, \overline{\Phi}) + \partial \Phi^i Y_{i\bar{\imath}}(\Phi, \overline{\Phi}) + \partial \overline{\Phi}^{\bar{\jmath}} g_{i\bar{\jmath}} Z^i_{\bar{\imath}}(\Phi, \overline{\Phi}) \right] \overline{D\Phi}^{\bar{\imath}}$$

This defines $Z^i_{\bar{\imath}}, Y_{i\bar{\imath}}, \Lambda^\alpha_{\beta\bar{\imath}}$

Susy marginal operators on worldsheet:

$$X = \left[\overline{\Gamma}_\alpha \Gamma^\beta \Lambda^\alpha_{\beta \bar{\imath}}(\Phi, \overline{\Phi}) + \partial \Phi^i Y_{i\bar{\imath}}(\Phi, \overline{\Phi}) + \partial \overline{\Phi}^{\bar{\jmath}} g_{i\bar{\jmath}} Z^i_{\bar{\imath}}(\Phi, \overline{\Phi}) \right] \overline{D \Phi}^{\bar{\imath}}$$

Demand X be chiral on-shell, ie, $\overline{D} X = 0$.

This gives the cocycle conditions:

$$Z^i_{\bar{\imath}, \bar{k}} - Z^i_{\bar{k}, \bar{\imath}} = 0$$

$$Y_{i\bar{\imath}, \bar{k}} - Y_{i\bar{k}, \bar{\imath}} = Z^j_{\bar{k}} H_{ji\bar{\imath}} - Z^j_{\bar{\imath}} H_{ji\bar{k}}$$

$$\Lambda^\alpha_{\beta \bar{\imath}, \bar{k}} - \Lambda^\alpha_{\beta \bar{k}, \bar{\imath}} = F^\alpha_{\beta \bar{k} i} Z^i_{\bar{\imath}} - F^\alpha_{\beta \bar{\imath} i} Z^i_{\bar{k}}$$

Not all solutions/cocycles correspond to distinct infinitesimal moduli.

For example, in SCFT, two marginal operators that differ by a superspace derivative,
define the same deformation
(though the action itself can change).

Ex: in (2,2) theory,
changing the Kahler form by an exact 2-form
does change the action, but
does **not** deform the SCFT.

In the present case,
if two marginal operators X, X' differ by $\overline{D}Y$,
for some superfield Y,
then they define the same SCFT deformation.

If X is chiral, then $\overline{D}X = 0$

Note $\overline{D}X' = \overline{D}X + \overline{D}^2 Y = 0$

so $X' = X + \overline{D}Y$ is also chiral

Lagrangian changes by

$$DX' = DX + \underline{D\overline{D}Y}$$

nonzero

but SCFT unchanged.

Coboundaries resulting from $X \mapsto X + \overline{D}Y$:

Dimensions and symmetries require

$$Y = \overline{\Gamma}_\alpha \Gamma^\beta \lambda^\alpha_\beta + \partial\Phi^i \mu_i + \partial\overline{\Phi}^{\bar{\imath}} g_{i\bar{\imath}} \zeta^i$$

for some $\lambda^\alpha_\beta, \mu_i, \zeta^i$

Resulting shifts:

$$Z^i_{\bar{\imath}} \mapsto Z^i_{\bar{\imath}} + \zeta^i_{,\bar{\imath}}$$

$$Y_{i\bar{\imath}} \mapsto Y_{i\bar{\imath}} + \mu_{i,\bar{\imath}} + H_{i\bar{\imath}j}\zeta^j$$

$$\Lambda^\alpha_{\beta\bar{\imath}} \mapsto \Lambda^\alpha_{\beta\bar{\imath}} + \lambda^\alpha_{\beta,\bar{\imath}} - F^\alpha{}_{\beta\bar{\imath}i}\zeta^i$$

Another source of coboundaries:

$$X \mapsto X + \partial Y' \qquad \text{for chiral } Y'$$

Lagrangian changes by total derivative:

$$DX \mapsto DX + \partial D Y'$$

Dimensional analysis, symmetries imply $Y' = \overline{D\Phi}^{\bar{\imath}} \xi_{\bar{\imath}}$

Combine with previous action to get full coboundaries:

$$Z^i_{\bar{\imath}} \sim Z^i_{\bar{\imath}} + \left(\zeta^i + g^{i\bar{\jmath}} \xi_{\bar{\jmath}}\right)_{,\bar{\imath}} + g^{i\bar{k}} \left(\xi_{\bar{\imath},\bar{k}} - \xi_{\bar{k},\bar{\imath}}\right)$$

$$Y_{i\bar{\imath}} \sim Y_{i\bar{\imath}} + \mu_{i,\bar{\imath}} + \xi_{\bar{\imath},i} + H_{i\bar{\imath}j} \left(\zeta^j + g^{j\bar{\jmath}} \xi_{\bar{\jmath}}\right)$$

$$\Lambda^\alpha_{\beta\bar{\imath}} \sim \Lambda^\alpha_{\beta\bar{\imath}} + \lambda^\alpha_{\beta,\bar{\imath}} - F^\alpha_{\beta\bar{\imath}i} \left(\zeta^i + g^{i\bar{\jmath}} \xi_{\bar{\jmath}}\right)$$

So far, I've discussed moduli.

What about charged matter?

In a heterotic CY compactification, charged matter believed to be counted by

$$H^*(X, \Lambda^* \mathcal{E})$$

Is this modified?

Is the spectrum of charged matter modified?

On the one hand, there is a PDE one could write down which would mix states:

$$h^{\alpha_1 \cdots \alpha_m}_{[\bar{\imath}_1 \cdots \bar{\imath}_n, \bar{\imath}_{n+1}]} + h'^{[\alpha_1 \cdots \alpha_{m-1}|\beta|}_{\bar{\imath}_1 \cdots \bar{\imath}_n \bar{\jmath}} F^{\alpha_m] \quad \bar{\jmath}}_{\beta \bar{\imath}_{n+1}} = 0$$

However, the different elements of $H^*(X, \Lambda^* \mathcal{E})$ typically correspond to different representations of the low-energy gauge group.

Thus, we currently believe no modification to charged matter spectrum, only to singlet matter.

Summary:

* Discussed moduli in heterotic string compactifications.

Issues:

-- what are moduli in non-Kahler cases?

-- worldsheet understanding of recent results of Anderson, Ovrut, et al ?

* Presented solutions:
a proposal for infinitesimal moduli of all compactifications, including non-Kahler cases.

Thank you for your time!