Some developments in heterotic compactifications

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As you all know,
there's been a lot of interest in the last few years in the landscape program.

One of the issues in the landscape program,
is that those string vacua are counted by low-energy effective field theories,
and it is not clear that all of those have consistent UV completions -- not all of them may come from an underlying quantum gravity.

(Banks, Vafa)

One potential such problem arises in heterotic $E_8 x E_8$ strings.

Let's briefly review heterotic strings.

10d: metric, B field, nonabelian gauge field

($E_8 x E_8$ or $Spin(32)/\mathbf{Z}_2$)

If compactify on X, then need gauge bundle E -> X such that $tr(F^2) = tr(R^2)$ in cohomology (anomaly cancellation / Green-Schwarz)

and $g^{ij^*} F_{ij^*} = 0$ (Donaldson-Uhlenbeck-Yau)

Heterotic nonlinear sigma model:

$$L = g_{\mu\nu}\partial\phi^\mu\partial\phi^\nu + i g_{i\bar{\jmath}}\overline{\psi}_+^{\bar{\jmath}} D_- \psi_+^i + h_{a\bar{b}}\overline{\lambda}_-^{\bar{b}} D_+ \lambda_-^a + \cdots$$

* 2d QFT, in fact a CFT

ψ_+ couple to TX

λ_- couple to gauge bundle

* Possesses (N=2) supersymmetry on right-movers, ie: ϕ, ψ_+

Call this chiral supersymmetry ``(0,2) supersymmetry"

Heterotic nonlinear sigma model:

$$L = g_{\mu\nu} \partial \phi^\mu \partial \phi^\nu + i g_{i\bar{\jmath}} \overline{\psi}_+^{\bar{\jmath}} D_- \psi_+^i + h_{a\bar{b}} \overline{\lambda}_-^{\bar{b}} D_+ \lambda_-^a + \cdots$$

In a critical string, there are:

* 10 real bosons ϕ
* 10 real fermions ψ_+
* 32 real fermions (or, 2 groups of 16) λ_-

so as to get central charge (26,10)

How to describe E_8 with 16 λ_-?

How to describe E_8?

The conventional worldsheet construction builds each E_8 using a ($\mathbf{Z}_2$ orbifold of) the fermions λ_-

$$L = g_{\mu\nu}\partial\phi^\mu\partial\phi^\nu + ig_{i\bar{j}}\overline{\psi}^{\bar{j}}_+ D_- \psi^i_+ + h_{a\bar{b}}\overline{\lambda}^{\bar{b}}_- D_+ \lambda^a_- + \cdots$$

The fermions realize a Spin(16) current algebra at level 1, and the $\mathbf{Z}_2$ orbifold gives Spin(16)/$\mathbf{Z}_2$.

Spin(16)/$\mathbf{Z}_2$ is a subgroup of E_8,
and we use it to realize the E_8.

How to realize E_8 with $Spin(16)/\mathbf{Z}_2$?

Adjoint rep of E_8 decomposes into adjoint of $Spin(16)/\mathbf{Z}_2$ + spinor:

$$\mathbf{248} = \mathbf{120} + \mathbf{128}$$

left R sector

left NS sector

So we take currents transforming in adjoint, spinor of $Spin(16)/\mathbf{Z}_2$, and form E_8 via commutation relations.

More, in fact: all E_8 d.o.f. are realized via $Spin(16)/\mathbf{Z}_2$

This construction has served us well for many years,
but,
in order to describe an E_8 bundle w/ connection,
that bundle and connection must be reducible to
$Spin(16)/\mathbf{Z}_2$.

After all, all info in kinetic term $h_{\alpha\beta}\lambda_-^\alpha D_+ \lambda_-^\beta$

Can this always be done?

Briefly: Bundles -- yes (in dim 9 or less)

Connections/gauge fields -- no.

Heterotic swampland?

Summary of this talk:

Part 1: Reducibility of E_8 bundles w/ connection to $Spin(16)/\mathbf{Z}_2$.
Worldsheet descriptions?

Part 2: Alternative constructions of 10d heterotic strings using other subgroups of E_8.
-- gen'l Kac-Moody algebras,
typically no free field representations

Part 3: Realize in compactifications with `fibered WZW models';
physical realization of elliptic genera of Ando, Liu

No swampland; new worldsheet constructions instead.

(A Henriques)

Reducibility of bundles

If H is a subgroup of G,
then obstructions to reducing a p-pal G bdle on M to a p-pal H bundle live in $H^k(M, \pi_{k-1}(G/H))$

Use the fiber sequence

$$E_8/\mathrm{Spin}(16)/\mathbf{Z}_2 \longrightarrow B\mathrm{Spin}(16)/\mathbf{Z}_2 \longrightarrow BE_8$$

π_i :	1	2	3	4	5	6	7	8	9	10	11
$E_8/\mathrm{Spin}(16)/\mathbf{Z}_2$	0	Z_2	0	0	0	0	0	Z	Z_2	Z_2	0
$B\mathrm{Spin}(16)/\mathbf{Z}_2$	0	Z_2	0	Z	0	0	0	Z	Z_2	Z_2	0
BE_8	0	0	0	Z	0	0	0	0	0	0	0

Obs': $H^3(M,\mathbf{Z}_2)$, $H^9(M,\mathbf{Z})$, $H^{10}(M,\mathbf{Z}_2)$

(A Henriques)

Reducibility of bundles

The obstruction in $H^3(M,\mathbf{Z}_2)$ vanishes because it is a pullback from $H^3(BE_8,\mathbf{Z}_2) = 0$.

It can be shown, via a cobordism invariance argument, that on an oriented manifold, the obstruction in $H^9(M,\mathbf{Z})$ will vanish.

The obstruction in $H^{10}(M,\mathbf{Z}_2)$ need not vanish. It counts the number of pos'-chirality zero modes of the ten-dim'l Dirac operator, mod 2, and has appeared in physics in work of Diaconescu-Moore-Witten on K theory.

So far:

In dim 9 or less,
all principal E_8 bundles can be reduced to
principal $Spin(16)/\mathbf{Z}_2$ bundles.

Next:

Reducibility of connections (gauge fields)

(R. Thomas)

Reducibility of connections

On a p-pal G bundle,
even a trivial p-pal G bundle,
one can find connections with holonomy that fill out all of G,
and so cannot be understood as connections on a p-pal H bundle for H a subgroup of G:
just take a conn' whose curvature generates the Lie algebra of G.

Thus, just b/c the bundles can be reduced, doesn't mean we're out of the woods yet.

(R. Thomas)

Reducibility of connections

We'll build an example of an anomaly-free gauge field satisfying DUY condition

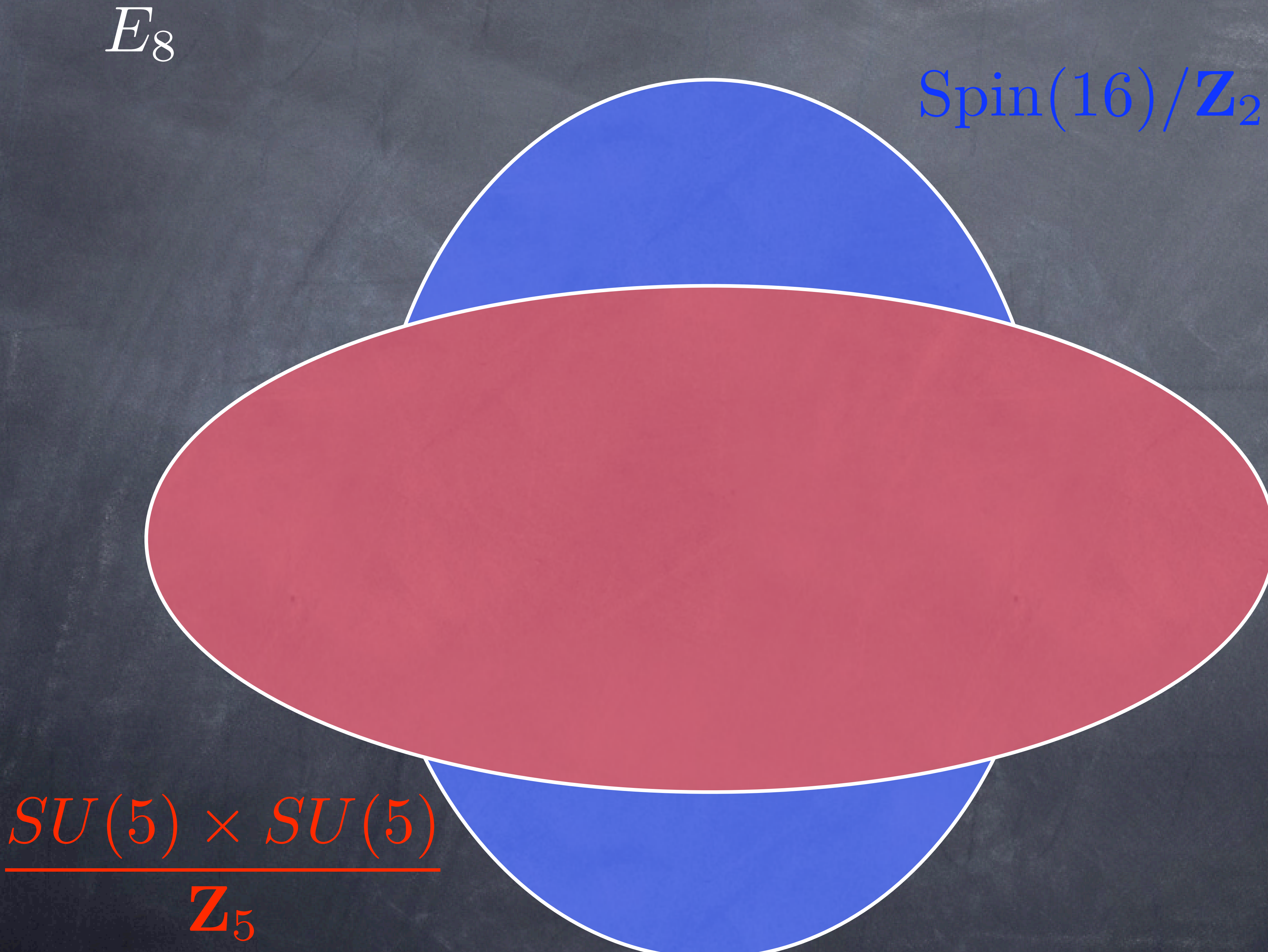
that does not sit inside Spin(16)/$\mathbf{Z}_2$.

The basic trick is to use the fact that E_8 has an

(SU(5)xSU(5))/$\mathbf{Z}_5$ subgroup

that does not sit inside Spin(16)/$\mathbf{Z}_2$.

We'll build an (SU(5)xSU(5))/$\mathbf{Z}_5$ connection.



E_8
$\mathrm{Spin}(16)/\mathbf{Z}_2$
$\frac{SU(5)\times SU(5)}{\mathbf{Z}_5}$

Reducibility of connections

Build a stable SU(5) bundle on an elliptically-fibered K3 using Friedman-Morgan-Witten technology.

Rk 5 bundle with $c_1=0$, $c_2=12$ has spectral cover in linear system $|5\sigma + 12f|$, describing a curve of genus

$$g = 5c_2 - 5^2 + 1 = 36$$

together with a line bundle of degree

$$-(5 + g - 1) = -40$$

(R. Thomas)

Reducibility of connections

Result is a (family of) stable SU(5) bundles with $c_2=12$ on K3.

Holonomy generically fills out all of SU(5).

Put two together,

and project to $\mathbf{Z}_5$ quotient,

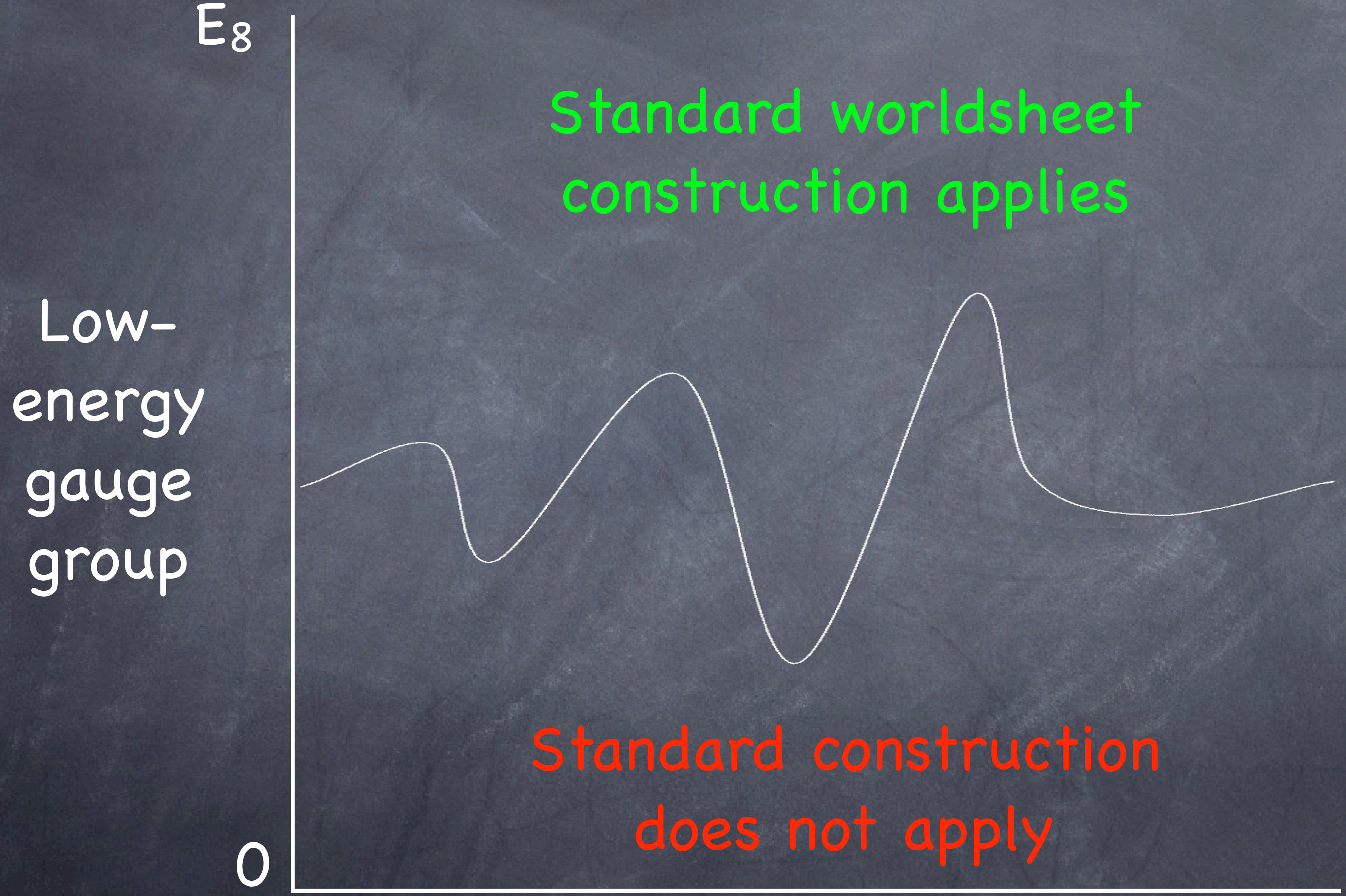
to get $(SU(5)\times SU(5))/\mathbf{Z}_5$ bundle w/ connection that satisfies anomaly cancellation + DUY.

(R. Thomas)

Reducibility of connections

Thus, we have an example of a consistent heterotic sugrav background,
in which the E_8 bundle cannot be reduced to
$Spin(16)/\mathbf{Z}_2$,
and so cannot be described with ordinary heterotic worldsheet theory.

Lessons for the Landscape



Statistics on trad'l CFT's artificially favors large gps

So far:

* E8 bundles in dim < 10 can be reduced to Spin(16)/$\mathbf{Z}_2$ bundles
* but connections (gauge fields) cannot

Heterotic swampland?

Next:

Alternative constructions of 10d heterotic strings using subgroups of E_8 other than Spin(16)/$\mathbf{Z}_2$

Alternative E_8 constructions

Example:

E_8 has an (SU(5) x SU(5))/$\mathbf{Z}_5$ subgroup.

Can it be used instead of Spin(16)/$\mathbf{Z}_2$?

One issue:

There are free field representations of U(n), Spin(n) at level 1, but not SU(n)...

... so we'll need to work with the current algebras more abstractly.

Alternative E_8 constructions

So, we'll take current algebras for two copies of SU(5) at level 1, and orbifold by a $\mathbf{Z}_5$

Check: central charge of each SU(5) = 4,
so adds up to 8
= central charge of E_8 ✓

Next, more convincing tests....

Alternative E_8 constructions

Check: fusion rules

Conformal familes of SU(5) current algebra obey

[**5**]x[**5**] = [**10**], [**10**]x[**5**] = [**5**], etc

so the combination

[**1**] + [**5**,**10***] + [**5***,**10**] + [**10**,**5**] + [**10***,**5***]

squares into itself; identify with [**1**] of E8 level 1.

Contains the E_8 adjoint decomposition

248 = (1,24) + (24,1) + (5,10*) + (5*,10) + (10,5) + (10*,5*)

Alternative E_8 constructions

Best check: characters

For Spin(16)/**Z**$_2$, corresponding to the decomposition

248 = 120 + 128

there is a decomposition of characters/left-moving partition f'ns:

$$\chi_{E_8}(\mathbf{1}, q) \,=\, \chi_{Spin(16)}(\mathbf{1}, q) \,+\, \chi_{Spin(16)}(\mathbf{128}, q)$$

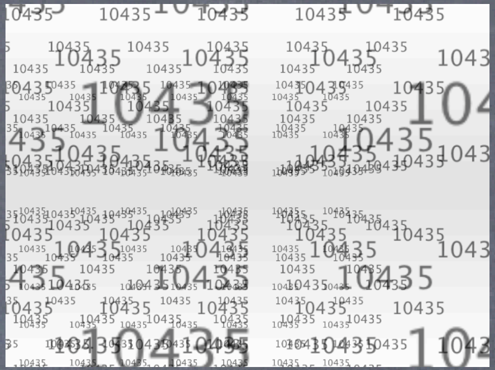
For SU(5)^2/**Z**$_5$, from the decomp' of adjoint

248 = (1,24) + (24,1) + (5,10*) + (5*,10) + (10,5) + (10*,5*)

get a prediction for characters:

$$\chi_{E_8}(\mathbf{1}, q) \,=\, \chi_{SU(5)}(\mathbf{1}, q)^2 \,+\, 4\,\chi_{SU(5)}(\mathbf{5}, q)\,\chi_{SU(5)}(\mathbf{10}, q)$$

Alternative E_8 constructions



Check: characters

$$\chi_{SU(5)}(\mathbf{1},q) = \frac{1}{\eta(\tau)^4} \sum_{\vec{m}\in\mathbf{Z}^4} q^{\left(\sum m_i^2 + (\sum m_i)^2\right)/2}$$

$$\chi_{SU(5)}(\mathbf{5},q) = \frac{1}{\eta(\tau)^4} \sum_{\vec{m}\in\mathbf{Z}^4, \sum m_i \equiv 1 \, mod \, 5} q^{\left(\sum m_i^2 - \frac{1}{5}(\sum m_i)^2\right)/2}$$

$$\chi_{SU(5)}(\mathbf{10},q) = \frac{1}{\eta(\tau)^4} \sum_{\vec{m}\in\mathbf{Z}^4, \sum m_i \equiv 2 \, mod \, 5} q^{\left(\sum m_i^2 - \frac{1}{5}(\sum m_i)^2\right)/2}$$

Can show (E. Scheidegger) (Kac, Sanielevici)

$$\chi_{E_8}(\mathbf{1},q) = \chi_{SU(5)}(\mathbf{1},q)^2 + 4\,\chi_{SU(5)}(\mathbf{5},q)\,\chi_{SU(5)}(\mathbf{10},q)$$

so E_8 worldsheet d.o.f. can be replaced by $SU(5)^2$

Alternative E_8 constructions

Analogous statement for Spin(16)/$\mathbf{Z}_2$ is

$$\chi_{E_8}(\mathbf{1},q) = \chi_{Spin(16)}(\mathbf{1},q) + \chi_{Spin(16)}(\mathbf{128},q)$$

There is a $\mathbf{Z}_2$ orbifold implicit here -- the **1** character is from untwisted sector, the **128** character is from twisted sector.

Sim'ly, in the expression

$$\chi_{E_8}(\mathbf{1},q) = \chi_{SU(5)}(\mathbf{1},q)^2 + 4\,\chi_{SU(5)}(\mathbf{5},q)\,\chi_{SU(5)}(\mathbf{10},q)$$

there is a $\mathbf{Z}_5$ orbifold implicit. Good!: $SU(5)^2/\mathbf{Z}_5$

Alternative E_8 constructions

Another max-rank subgroup: $SU(9)/\mathbf{Z}_3$.

Check: central charge = 8 = that of E_8.

E_8 conformal family decomposes as

[1] = [1] + [84] + [84*]

Can show

$$\chi_{E_8}(\mathbf{1}, q) = \chi_{SU(9)}(\mathbf{1}, q) + 2\,\chi_{SU(9)}(\mathbf{84}, q)$$

(Note $\mathbf{Z}_3$ orbifold implicit.)

So, can describe E_8 w.s. d.o.f. with $SU(9)/\mathbf{Z}_3$.

Alternative E_8 constructions

A non-max-rank possibility: $G_2 \times F_4$

Central charge of G_2 at level 1 = 14/5
Central charge of F_4 at level 1 = 52/10
Sum = 8
= central charge of E_8 at level 1 ✓

Even better:

$$\chi_{E_8}(\mathbf{1},q) = \chi_{G_2}(\mathbf{1},q)\,\chi_{F_4}(\mathbf{1},q) + \chi_{G_2}(\mathbf{7},q)\,\chi_{F_4}(\mathbf{26},q)$$ ✓

Alternative E_8 constructions

$$\chi_{E_8}(\mathbf{1},q) = \chi_{G_2}(\mathbf{1},q)\,\chi_{F_4}(\mathbf{1},q) + \chi_{G_2}(\mathbf{7},q)\,\chi_{F_4}(\mathbf{26},q)$$

Problem: This has structure of $\mathbf{Z}_2$ orbifold twisted sectors,

but, G_2 & F_4 both centerless.

Conclusion: $G_2 \times F_4$ can't be realized.

(Though it does come close.)

So far:

* not all p-pal E_8 bundles w/ connection can be described using trad'l heterotic worldsheet construction

* in 10d there exist alternative constructions of the E_8's, using gen'l Kac-Moody algebras

Next:

Fiber Kac-Moody algebras over gen'l mflds using `fibered WZW models'

(J Distler, ES; J Gates, W Siegel, etc)

Fibered WZW models

First, recall ordinary WZW models.

$$S = -\frac{k}{2\pi}\int_{\Sigma} \mathrm{Tr}\ [g^{-1}\partial g g^{-1}\overline{\partial} g] - \frac{ik}{2\pi}\int_{B} d^3y \epsilon^{ijk} \mathrm{Tr}\ [g^{-1}\partial_i g g^{-1}\partial_j g g^{-1}\partial_k g]$$

Looks like sigma model on mfld G w/ H flux.

Has a global $G_L \times G_R$ symmetry, with currents

$$J(z) = g^{-1}\partial g \qquad \overline{J}(\overline{z}) = \overline{\partial} g\, g^{-1}$$

obeying $$\overline{\partial} J(z) = \partial \overline{J}(\overline{z}) = 0$$

-- realizes G Kac-Moody algebra at level k

Fibered WZW models

Let P be a principal G bundle over X, with connection A.

Replace the left-movers of ordinary heterotic with WZW model with left-multiplication gauged with A.

$$\frac{1}{\alpha'}\int_\Sigma \left(g_{i\bar{\jmath}} \partial_\alpha \phi^i \partial^\alpha \phi^{\bar{\jmath}} + \cdots \right)$$

$$- \frac{k}{4\pi}\int_\Sigma \mathrm{Tr}\, \left(g^{-1}\partial g g^{-1} \overline{\partial} g \right) - \frac{ik}{12\pi}\int_B d^3 y \epsilon^{ijk} \mathrm{Tr}\, \left(g^{-1}\partial_i g g^{-1}\partial_j g g^{-1}\partial_k g \right)$$

$$- \frac{k}{2\pi}\mathrm{Tr}\, \left((\partial\phi^\mu) A_\mu \overline{\partial} g g^{-1} + \frac{1}{2}(\partial\phi^\mu \overline{\partial}\phi^\nu) A_\mu A_\nu \right)$$

NLSM on X

WZW

Gauge left-multiplication

Fibered WZW models

A WZW model action is invariant under gauging symmetric group multiplications,
but not under the chiral group multiplications that we have here.

Under $g \mapsto hg$

$$A_\mu \mapsto hA_\mu h^{-1} + h\partial_\mu h^{-1}$$

the classical action is not invariant.

As expected -- this is bosonization of chiral anomaly.

... but this does create a potential well-definedness issue in our fibered WZW construction ....

Fibered WZW models

In add'n to the classical contribution, the classical action also picks up a quantum correction across coord' patches, due to right-moving chiral fermi anomaly.

To make the action gauge-invariant, we proceed in the usual form for heterotic strings:

assign a transformation law to the B field.

Turns out this implies

$$k \, \mathrm{ch}_2(\mathcal{E}) \, = \, \mathrm{ch}_2(TX)$$

Anom' canc' at level k

If that is obeyed, then action well-defined globally.

Fibered WZW models

The right-moving fermion kinetic terms on the worldsheet couple to H flux:

$$\frac{i}{2} g_{\mu\nu} \psi_+^\mu D_{\overline{z}} \psi_+^\nu$$

where

$$D_{\overline{z}} \psi_+^\mu \; = \; \overline{\partial} \psi_+^\mu \; + \; \overline{\partial} \phi^\mu \left(\Gamma^\nu{}_{\sigma\mu} \; - \; H^\nu{}_{\sigma\mu} \right) \psi_+^\sigma$$

To make fermion kinetic terms gauge-invariant, set

$$H \; = \; dB \; + \; (\alpha') \left(k CS(A) \; - \; CS(\omega) \right)$$

➔ $k \, \mathrm{ch}_2(\mathcal{E}) \; = \; \mathrm{ch}_2(TX)$ Anomaly-cancellation

Fibered WZW models

Demand (0,2) supersymmetry on base.
Discover an old faux-susy-anomaly in subleading terms in α' (Sen)

Susy trans' in ordinary heterotic string:

$$\delta\lambda_- = -i\epsilon\psi_+^\mu A_\mu \lambda_-$$

-- same as a chiral gauge transformation, with parameter $-i\epsilon\psi_+^\mu A_\mu$

-- b/c of chiral anomaly, there is a quantum contribution to susy trans' at order α'

-- appears classically in bosonized description

Fibered WZW models

(0,2) supersymmetry:

One fermi-terms in susy transformations of:

NLSM Base: $$\frac{1}{\alpha'}\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu\,(H\,-\,dB)_{\bar{\imath}\mu\nu}$$

WZW fiber: $$-k\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu CS(A)_{\bar{\imath}\mu\nu}$$

Quantum: $$\int_\Sigma (i\alpha\psi^{\bar{\imath}})\bar{\partial}\phi^\mu\partial\phi^\nu CS(\omega)_{\bar{\imath}\mu\nu}$$

→ $$H\,=\,dB\,+\,\alpha'\,(kCS(A)\,-\,CS(\omega))$$ for susy to close

Fibered WZW models

Yet another check of $k\,\mathrm{ch}_2(\mathcal{E}) \;=\; \mathrm{ch}_2(TX)$

Take an ordinary heterotic string on S^1, and orbifold by a $\mathbf{Z}_2$ that translates on the S^1 and simultaneously exchanges the E_8's.

Result is 9d theory with level 2 E_8 algebra.

Covering space: $\mathrm{ch}_2(\mathcal{E}) \,+\, \mathrm{ch}_2(\mathcal{E}) \;=\; \mathrm{ch}_2(TX)$

Quotient: $2\,\mathrm{ch}_2(\mathcal{E}) \;=\; \mathrm{ch}_2(TX)$

Level = 2: Exactly consistent.

Fibered WZW models

Massless spectrum:

In an ordinary WZW model, the massless spectrum is counted by WZW primaries, which are associated to integrable rep's of G.

Here, for each integrable rep R of the principal G bundle P,
we get an associated vector bundle E_R.

Massless spectrum = $H^*(X, E_R)$ for each R

Fibered WZW models

Massless spectrum:

Example: G = SU(n), level 1

Here the integrable reps are the fundamental **n** and its exterior powers.

Massless spectrum: $H^*(X, \mathrm{Alt}^* E)$ ✓

(Distler-Greene, '88)

Fibered WZW models

Massless spectra:

Check that Serre duality closes these states back into themselves:

When X has trivial canonical bundle,

$$H^{\cdot}(X, \mathcal{E}_R) \cong H^{n-\cdot}(X, \mathcal{E}_R^{\vee})^* \cong H^{n-\cdot}(X, \mathcal{E}_{R*})^*$$

R is an integrable rep iff R* is integrable, so all is OK

Fibered WZW models

Elliptic genera:

= 1-loop partition function

In std case, has the form (Witten)

$$q^{-d/24-r/48} \left| \hat{A}(TM)\mathrm{ch}\left(\bigotimes_{k=1/2,3/2,\cdots} \Lambda_{q^k}\mathcal{E} \bigotimes_{\ell=1,2,3,\cdots} S_{q^\ell}TM \right)[M] \right.$$

where

$$\Lambda_q \mathcal{E} = 1 + q\mathcal{E} + q^2\Lambda^2\mathcal{E} + \cdots$$

$$S_q T = 1 + qT + \mathrm{Sym}^2 T + \cdots$$

Fibered WZW models

Elliptic genera:

Anom' cancellation shows up as a condition for the elliptic genus to have good modular properties.

$$\hat{A}(TM)\mathrm{ch}(S_{q^\ell}T) = \eta(q^2)^{-8m}\exp\left\{\sum_{k=1}^{\infty} G_{2k}(q^2)\frac{1}{(2k)!}\mathrm{Tr}\ \left(\frac{iR}{2\pi}\right)^{2k}\right.$$

$G_{2k}(q^2)$ have good mod' prop's for k>1 but not for k=1

For this by itself, to insure good prop's, need Tr R^2 exact

Fibered WZW models

Elliptic genera:

These fibered WZW constructions realize the `new' elliptic genera of Ando, Liu.

Ordinary elliptic genera describe left-movers coupled to a level 1 current algebra;
these, have left-moving level k current algebra.

Black hole applications?

Conclusions

* standard heterotic worldsheet constructions do **not** suffice to describe all heterotic sugrav vacua

* but more general constructions exist which describe the others

-- build E_8 from other subgroups

-- fibered WZW models