Gauging 1-form symmetries in two-dimensional theories

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Eric Sharpe
Virginia Tech

Based on 1911.05080 & also building on hep-th/0502027, 0502044, 0502053, 0606034, 0709.3855, 1012.5999, 1307.2269, 1404.3986, 1508.04770, …

Main theme:

- Two-dimensional theories with finite global 1-form symmetries
  = disjoint union of theories with no 1-form symmetry (`universes’).

  This is *Decomposition* (Hellerman et al ’06)

- Gauging the 1-form symmetry = projection onto components.

  (ES 1911.05080)

Secondary theme:

Two-dimensional theories with global 1-form symmetries have several descriptions:

- Gauge theory w/ trivially-acting subgroup
- Restriction on instanton sectors
- Sigma models on gerbes
  = fiber bundles with fibers = $G^{(1)} = BG$
- Coupling a QFT to a TQFT

We'll see in this talk how decomposition (into 'universes')

implements a projection on nonperturbative sectors (``multiverse interference effect''),

relating some of these pictures.

Outline:

- Brief overview of 1-form symmetries in 2d theories
- Brief review of decomposition of 2d theories w/ (finite, global) 1-form symmetries

For the rest of the talk, I want to focus on one (or if time, two) simple concrete examples:

1. An orbifold with a 1-form symmetry
    - Explicit description of decomposition
    - Explicitly gauge the 1-form symmetry
2. Pure nonsusy SU(2) Yang-Mills
    - Explicit description of decomposition
    - Explicitly gauge the 1-form symmetry

This only scratches the surface —
there are more ex's, more kinds of ex's,
and fun applications,
but only time in this talk for a few basics.

What is a one-form symmetry?

Often described in terms of actions on defects,

but in this talk, we'll focus on them in local QFTs.

For this talk, *intuitively*, this will be a \`group' that exchanges nonperturbative sectors.

Example: G gauge theory in which massless matter inv't under $K \subseteq G$

(K assumed finite & abelian)

Then, nonperturbative sectors are invariant under

$$(G - \text{bundle}) \mapsto (G - \text{bundle}) \otimes (K - \text{bundle})$$

$$A \mapsto A + A'$$

This is the symmetry, involving an action of \`group' of K–bundles.

That group is denoted BK or $K^{(1)}$

Suppose you have a 2d QFT w/ a finite global 1-form symmetry.

An old result: (Hellerman et al '06)

such theories decompose into disjoint unions of theories w/o 1-form symmetry.

Let's make that concrete....

Decomposition in 2d gauge theories

(Hellerman et al '06)

This is an old story, but sometimes not appreciated, so I'll review....

Gauge theory version:

S'pose have $G-$gauge theory, G semisimple, with finite $K \subseteq G$ acting trivially.

For simplicity, assume K is in the center. Has BK symmetry.

$$\mathrm{QFT}(G-\text{gauge theory}) = \coprod_{\text{char's } \hat{K}} \mathrm{QFT}\left(G/K-\text{gauge theory w/ discrete theta angles}\right)$$

Example: pure $SU(2) = SO(3)_+ + SO(3)_-$

where ± denote discrete theta angles (w_2)

(Another version exists for NLSMs.)

One effect is a projection on nonperturbative sectors:

$$\underbrace{\sum_{\theta \in \hat{K}} \int [DA] \exp(-S) \exp\left[\theta \int \omega_2(A)\right]}_{\text{Disjoint sum}} = \int [DA] \exp(-S) \overbrace{\left(\sum_{\theta \in \hat{K}} \exp\left[\theta \int \omega_2(A)\right]\right)}^{\text{projection operator}}$$

Decomposition in 2d gauge theories

Since 2006, decomposition has been checked in many examples in many ways. Examples:

- GLSM's: mirrors, quantum cohomology rings (Coulomb branch) (T Pantev, ES, hep-th/0502053)
- Orbifolds: partition f'ns, massless spectra, elliptic genera (T Pantev, ES '05)
- Open strings, K theory (Hellerman et al hep-th/0606034)
- Susy gauge theories w/ localization (ES 1404.3986)
- Nonsusy pure Yang-Mills ala Migdal
- Plus version for 4d theories w/ 3-form symmetries (Tanizaki, Unsal, 1912.01033)

Applications:

- Predictions for Gromov-Witten theory (checked by H-H Tseng, Y Jiang, etc starting '08)
- Approximate 1-form symmetries used to understand phases of some GLSMs (Caldararu et al 0709.3855, Hori '11, ...)
- Moduli spaces (Donagi et al, '17...)

Decomposition in 2d gauge theories

A recent computation: vacua of pure susy gauge theories (Gu, ES, Zou 2005.10845)

For pure G/K gauge theory, get susy vacua for one discrete theta angle; susy broken for others.

Examples:

Group	Theta	IR tw' chiral R charges
$SU(k)/\mathbb{Z}_k$	$-(1/2)k(k-1) \mod k$	2, 3, 4, ..., k
$SO(4k)/\mathbb{Z}_2$	$k(2k-1) \mod 2,\ 0 \mod 2$	2k; 2, 4, 6, ..., 4k - 2
$SO(4k+2)/\mathbb{Z}_2$	$2k(2k-1) \mod 4$	2k+1; 2, 4, 6, ..., 4k
$Sp(2k)/\mathbb{Z}_2$	$(1/2)k(k+1) \mod 2$	2, 4, 6, ..., 2k
$E_6/\mathbb{Z}_3$	$0 \mod 3$	2, 5, 6, 8, 9, 12
$E_7/\mathbb{Z}_2$	$1 \mod 2$	2, 6, 8, 10, 12, 14, 18

For that one value, IR same as pure G gauge theory

Consistent w/ decomposition: $$G = \coprod_{\theta \in \hat{K}} (G/K)_\theta$$

Suffice to say, decomposition is well-established.

Next, I will walk through a simple example,

first to demonstrate decomposition explicitly,

then to describe gauging of the one-form symmetry.

Example: Orbifold $[X/D_4]$ in which the $\mathbb{Z}_2$ center acts trivially.

— has $B\mathbb{Z}_2$ (1-form) symmetry

$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ so this is closely related to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Decomposition predicts

$$\mathrm{CFT}\left([X/D_4]\right) = \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)$$

(consequence of a general formula.)

Let's check this explicitly....

Example, cont'd

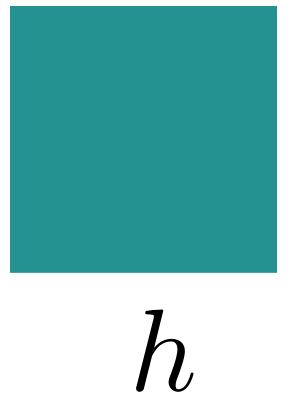
Compute the partition function of $[X/D_4]$

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

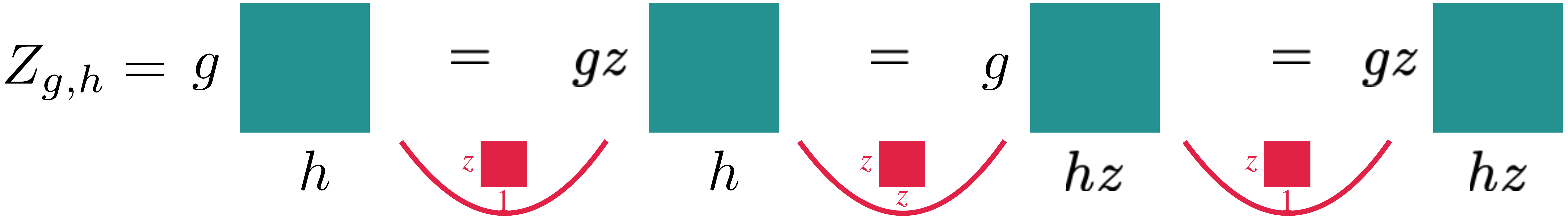
$$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$ where $\overline{a} = \{a, az\}$ etc

$$Z([X/D_4]) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$ where $Z_{g,h} = g$



Since z acts trivially,

$Z_{g,h}$ is symmetric under multiplication by z



This is the $B\mathbb{Z}_2$ 1-form symmetry.

Example, cont'd

Compute the partition function of $[X/D_4]$

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

$$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$ where $\overline{a} = \{a, az\}$ etc

$$Z([X/D_4]) = \frac{1}{|D_4|} \sum_{g,h \in D_4, gh=hg} Z_{g,h}$$ where $Z_{g,h} = g$  h

Each D_4 twisted sector that appears is the same as a $\mathbb{Z}_2 \times \mathbb{Z}_2$ twisted sector,

appearing with multiplicity $|\mathbb{Z}_2|^2 = 4$,

except for the sectors $\overline{a}$  $\overline{b}$, $\overline{a}$  $\overline{ab}$, $\overline{b}$  $\overline{ab}$ which do not appear.

Restriction on nonperturbative sectors

Example, cont'd

Compute the partition function of $[X/D_4]$

$$Z([X/D_4]) = \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \text{(some twisted sectors)} \right)$$

$$= 2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \text{(some twisted sectors)} \right)$$

Different theory than $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Example, cont'd

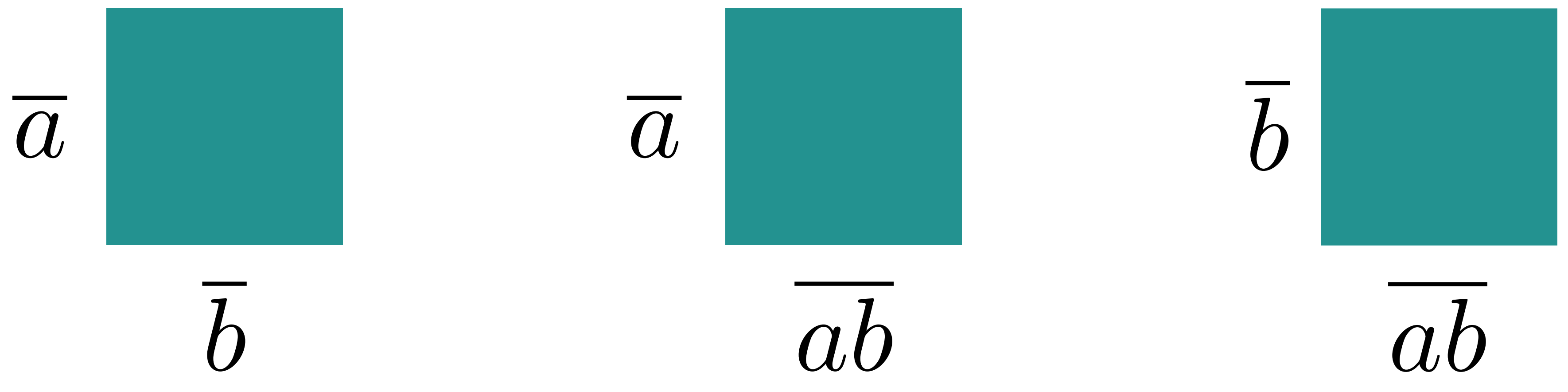
Compute the partition function of $[X/D_4]$

$$Z([X/D_4]) \;=\; \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|}|\mathbb{Z}_2|^2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) \;-\; (\text{some twisted sectors}) \right)$$

$$= \; 2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) \;-\; (\text{some twisted sectors}) \right)$$

Discrete torsion is $H^2(\mathbb{Z}_2 \times \mathbb{Z}_2, U(1)) = \mathbb{Z}_2$,

and acts as a sign on the twisted sectors



which were omitted above.

$$Z([X/D_4]) \;=\; Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}) \;+\; Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}})$$

Adding the components projects out some sectors — interference effect.

Example, cont'd

Compute the partition function of $[X/D_4]$

$$Z([X/D_4]) \;=\; \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) \;-\; (\text{some twisted sectors}) \right)$$

$$= \; 2 \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) \;-\; (\text{some twisted sectors}) \right)$$

Discrete torsion is $H^2(\mathbb{Z}_2 \times \mathbb{Z}_2, U(1)) = \mathbb{Z}_2$,

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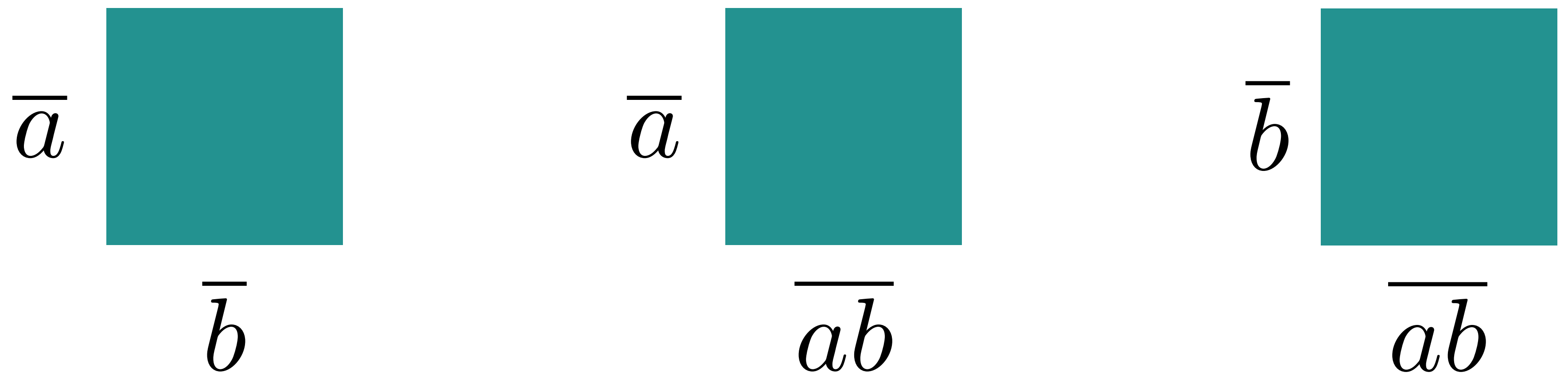


which were omitted above.

$$Z([X/D_4]) \;=\; Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}) \;+\; Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}})$$

Matches prediction of decomposition

$$\mathrm{CFT}\left([X/D_4]\right) \;=\; \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)$$

Example, cont'd

$$\mathrm{CFT}\left([X/D_4]\right) = \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)$$

At the level of operators, one reason for this is that the theory admits projection operators:

Let $\hat{i}$ denote the (dim 0) twist field associated to the trivially-acting $\mathbb{Z}_2$:

$$\Pi_{\pm} = \frac{1}{2}\left(1 \pm \hat{i}\right)$$

$$\Pi_{\pm}^2 = \Pi_{\pm} \qquad \Pi_{\pm}\Pi_{\mp} = 0$$

Massless spectra....

Example, cont'd

Massless spectra for $X = T^6$

(T Pantev, ES '05)

Massless spectrum of D_4 orbifold

$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 54 & & 0 & \\
2 & & 54 & & 54 & & 2 \\
 & 0 & & 54 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 2 & & &
\end{array}
\quad = \quad
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 51 & & 0 & \\
1 & & 3 & & 3 & & 1 \\
 & 0 & & 51 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
\quad + \quad
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 3 & & 0 & \\
1 & & 51 & & 51 & & 1 \\
 & 0 & & 3 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Signals mult' components / cluster decomp' violation

spectrum of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orb' w/o d.t.

spectrum of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orb' w/ d.t.

matching the prediction of decomposition

$$
\mathrm{CFT}\left([X/D_4]\right) = \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)
$$

Example, cont'd

Next: gauge $B\mathbb{Z}_2$

In broad brushstrokes,

$$Z\left([[X/D_4]/BG]\right) = \frac{1}{|G|}\sum_{G-\text{gerbes}} (\text{sectors twisted by gerbe})$$

Here,

$$Z\left([[X/D_4]/B\mathbb{Z}_2]\right) = \frac{1}{|\mathbb{Z}_2|}\sum_{z\in H^2(\Sigma,\mathbb{Z}_2)} \epsilon(z)\left(\frac{1}{|D_4|}\sum_{gh=hgz} g\,\underset{h}{\square}^{z}\right)$$

sum over (banded) gerbes

phase (analogue of d.t.)

sector twisted by gerbe

Example, cont'd

$$Z\left([[X/D_4]/B\mathbb{Z}_2]\right) = \frac{1}{|\mathbb{Z}_2|} \sum_{\substack{z \in H^2(\Sigma,\mathbb{Z}_2) \\ = \mathbb{Z}_2}} \epsilon(z) \left(\frac{1}{|D_4|} \sum_{gh=hgz} g \; \right)$$

The g

are gerbe-twisted orbifold twisted sectors $gh = hgz$

For z in the center of the orbifold group,

$SL(2,\mathbb{Z})$: g

$\mapsto$ $g^a h^b$

$g^c h^d$

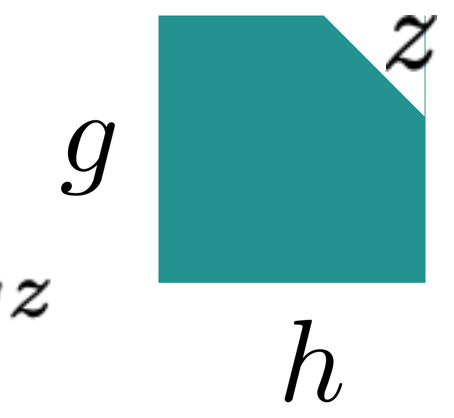
so z preserved.

(More generally, modular transformations map z to a conjugate.)

The phases $\epsilon(z)$ form a group homomorphism: $\epsilon : \mathbb{Z}_2 \to U(1)$, $\epsilon(gh) = \epsilon(g)\epsilon(h)$

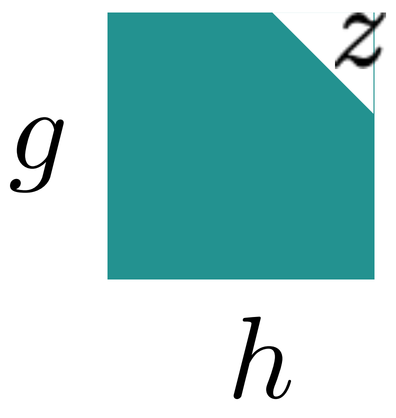
(consistent with multiloop factorization)

Example, cont'd

$$Z\left([[X/D_4]/B\mathbb{Z}_2]\right) = \frac{1}{|\mathbb{Z}_2|} \sum_{\substack{z \in H^2(\Sigma, \mathbb{Z}_2) \\ = \mathbb{Z}_2}} \epsilon(z) \left(\frac{1}{|D_4|} \sum_{gh = hgz} \right.$$ 

$$\left. \right)$$

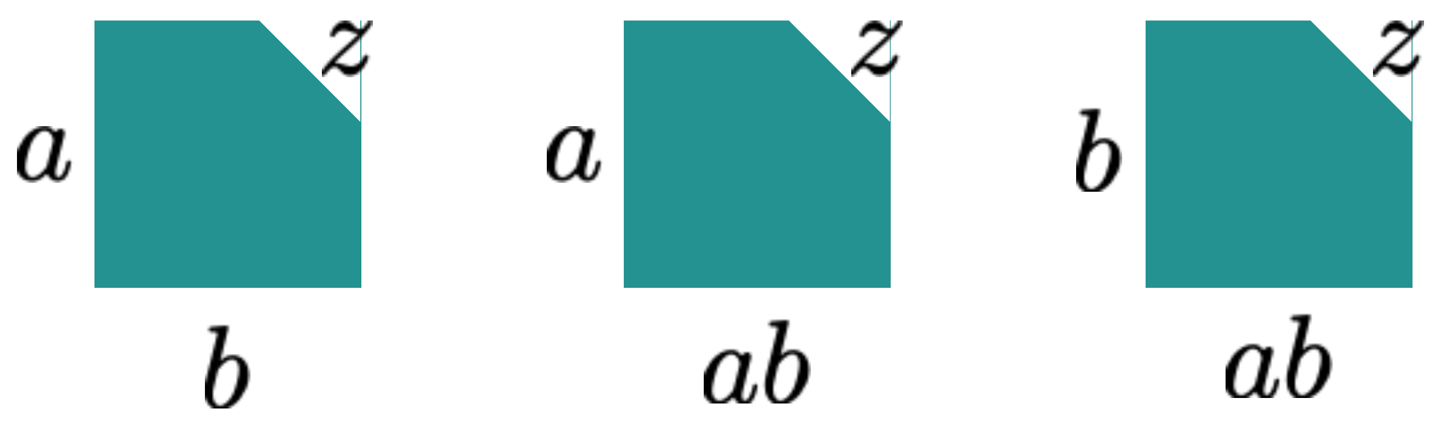
$z = 1:$

$$\frac{1}{|D_4|} \sum_{gh=hgz}$$ 

$=$ ordinary partition function $Z([X/D_4])$

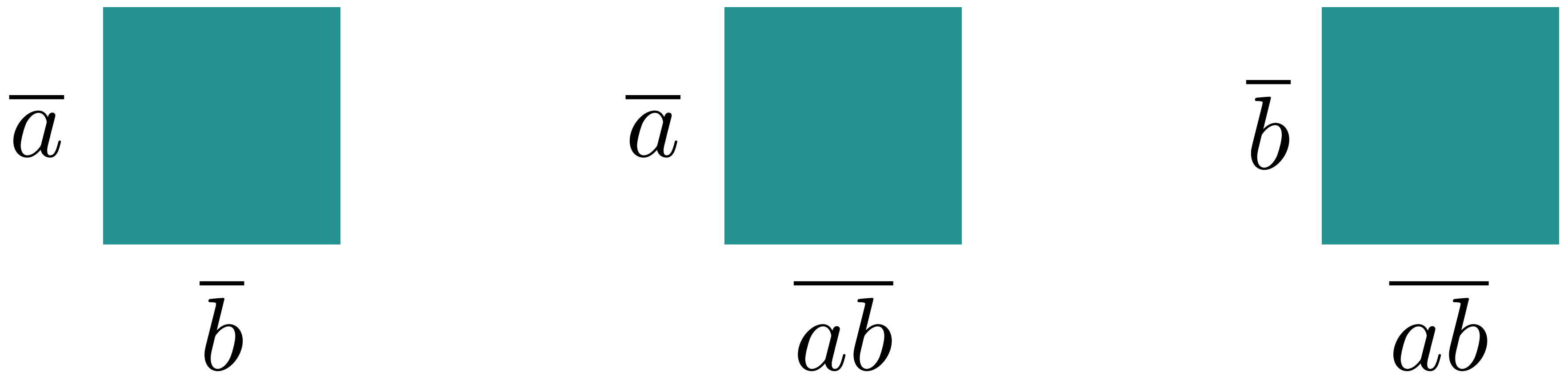
$= 2\left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) \;-\; (\text{some twisted sectors})\right)$

$z \neq 1:$ Only contributing sectors are



(plus perm's from mult' by z's)

This reproduces the sectors excluded from the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold:



Example, cont'd

$$Z\left([[X/D_4]/B\mathbb{Z}_2]\right) = \frac{1}{|\mathbb{Z}_2|} \sum_{z \in H^2(\Sigma, \mathbb{Z}_2)} \epsilon(z) \left(\frac{1}{|D_4|} \sum_{gh = hgz} g \right.$$  $$\left. {}^{z}_{h} \right)$$

$$= \frac{1}{|\mathbb{Z}_2|} \left[2\epsilon(+1) \left(Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) - \frac{\text{(excluded)}}{|\mathbb{Z}_2 \times \mathbb{Z}_2|} \right) + 2\epsilon(-1) \frac{\text{(excluded)}}{|\mathbb{Z}_2 \times \mathbb{Z}_2|} \right]$$

where $\epsilon(+1) = +1$ in all cases

Put this together:

$\epsilon(-1) = +1$: $$Z\left([[X/D_4]/B\mathbb{Z}_2]\right) = \frac{2}{|\mathbb{Z}_2|} Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) = Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2])$$

the partition function of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion.

$\epsilon(-1) = -1$: $Z\left([[X/D_4]/B\mathbb{Z}_2]\right) =$ partition function of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

with discrete torsion

Example, cont'd

$$\epsilon(-1) = +1: \quad Z([[X/D_4]/B\mathbb{Z}_2]) = \frac{2}{|\mathbb{Z}_2|} Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2]) = Z([X/\mathbb{Z}_2 \times \mathbb{Z}_2])$$

the partition function of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion.

$\epsilon(-1) = -1: \quad Z([[X/D_4]/B\mathbb{Z}_2]) =$ partition function of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold **with** discrete torsion

Recall decomposition in this case:

$$\mathrm{CFT}([X/D_4]) = \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}})$$

Result: gauging the 1-form symmetry has projected onto the components

Furthermore, this is a general story — we'll see another example next.

Another example of decomposition:

Pure nonsusy 2d SU(2) Yang-Mills

Decomposition: $SU(2) \;=\; SO(3)_+ + SO(3)_-$ due to global $B\mathbb{Z}_2$ center symmetry

(Migdal, Rusakov)

$$Z(SU(2)) \;=\; \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps

$$Z(SO(3)_+) \;=\; \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SO(3) reps

(Tachikawa '13)

$$Z(SO(3)_-) \;=\; \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps

that are not SO(3) reps

Result: $Z(SU(2)) \;=\; Z(SO(3)_+) + Z(SO(3)_-)$

In fact, this is easy to generalize…

Another example of decomposition:

Pure nonsusy 2d G Yang-Mills

More generally, if G has center K,

a pure 2d nonsusy $G-$gauge theory has BK symmetry,

and decomposes as

$$G \;=\; \coprod_{\theta \in \hat{K}} (G/K)_\theta$$

where the θ are discrete theta angles,

coupling to analogues of Stiefel-Whitney classes.

Hilbert spaces...

Another example of decomposition:

Pure nonsusy 2d G Yang-Mills

Hilbert spaces:

The Hilbert space of a pure G YM theory is $\mathscr{H}(G)$ = L² class f'ns on G

These decompose under action of center: $f(gz) = \theta(z) f(g)$

$\mathscr{H}\left((G/K)_\theta\right)$ = L² class f'ns on G such that $f(gz) = \theta(z) f(g)$

As a result, $$\mathscr{H}(G) = \sum_{\theta \in \hat{K}} \mathscr{H}\left((G/K)_\theta\right)$$

which is consistent with decomposition: $$G = \coprod_{\theta \in \hat{K}} (G/K)_\theta$$

Next, I'll outline how to gauge BK to project onto decomposition components.

Another example of decomposition:

Pure nonsusy 2d G Yang-Mills

Broadly speaking, the partition function of a BK-gauged theory has the form

$$\frac{1}{|K|} \sum_{z \in H^2(K) = K} \epsilon(z) Z(z)$$

where $Z(z)$ is the partition f'n in sector twisted by K-gerbe z,

and $\epsilon(z)$ is a phase

We'll write $\epsilon(z) = \exp(-i\lambda z)$ for $\lambda \in \hat{K}$

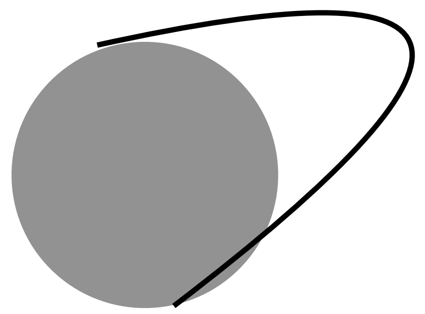
We'll define $Z(z)$ next....

Another example of decomposition:

Pure nonsusy 2d G Yang-Mills

To define the gerbe-twisted gauge theory partition f'n, we'll need `twisted caps.'

Ordinary cap:



$$Z_{\rm cap}(U) \;=\; \sum_R (\dim R) \chi_R(U) \exp(-A C_2(R))$$

$$Z_{S^2} \;=\; \int dU \overline{Z}_{\rm cap}(U) Z_{\rm cap}(U) \;=\; \sum_R (\dim R)^2 \exp(-A C_2(R))$$

Twisted cap:

$$Z_{\rm cap,tw}(U,z) \;=\; \sum_R (\dim R) \chi_R(U) \exp(i w(R)(z)) \exp(-A C_2(R))$$

$$Z(z) \;=\; \sum_R (\dim R)^{2-2g} \exp(i w(R)(z)) \exp(-A C_2(R))$$

where $\chi_R(zU) \;=\; \exp(iw(R)(z))\chi_R(U)$ $\quad$ $w =$ `n-ality' of representation R

Another example of decomposition:

Pure nonsusy 2d G Yang-Mills

Putting this together,

$$Z(G/BK,\lambda) = \frac{1}{|K|}\sum_{z\in H^2(K)=K}\epsilon(z)Z(z)$$

$$= \sum_R \underbrace{\left(\frac{1}{|K|}\sum_{z\in K}\exp(i(w(R)-\lambda)(z))\right)}_{\text{projects onto rep's of n-ality } \lambda}(\dim R)^{2-2g}\exp(-AC_2(R))$$

$$= \sum_{R,w(R)=\lambda}(\dim R)^{2-2g}\exp(-AC_2(R))$$

$$= Z\left((G/K)_\lambda\right)$$

Thus, gauging BK w/ phase determined by λ selects one component of the decomposition

$$G = \coprod_{\theta\in\hat{K}}(G/K)_\theta$$

So far:

I've reviewed decomposition,

a property of 2d QFTs with finite global 1-form symmetry,

and the gauging of that 1-form symmetry.

What about QFTs in other dimensions?

- 4d theories w/ finite global 3-form symmetries — Tanizaki, Unsal, 1912.01033
- Conjecture same for QFTs in d dims w/ finite global (d-1)-form symmetries, d > 1

So far:

- Conjecture same for QFTs in d dims w/ finite global (d-1)-form symmetries, d > 1

To that end,

1. Involves a (d-1)-form, which couples to a domain wall

   (analogous to Bousso-Polchinski '00, ...)

2. Consistent with reduction on circle:

   The (d-1)-dim theory has a (d-2)-form symmetry,

   as expected:

   if the d-dim'l theory decomposes, its reduction on a circle should decompose too.

Is there any math here?....

Mathematical interpretation:

So far I've just talked abstractly about 2d theories & 1-form symmetries.

This has a mathematical interpretation: "gerbes"

A G–gerbe is a fiber bundle whose fibers are copies of BG.

A sigma model on a G–gerbe has a global BG symmetry,

just as a sigma model on a G–bundle has a global G symmetry,

from translations on the fibers.

Furthermore, $BG = [\mathrm{point}/G]$

so whenever a group acts trivially,

you should expect a gerbe structure (1-form symmetry) somewhere.

Mathematical interpretation:

Twenty years ago, I was interested in studying

`sigma models on gerbes' as possible sources of new string compactifications.

Potential issues, since solved:

construction of QFT; cluster decomposition; moduli;

mod' invariance & unitarity in orbifolds; potential presentation-dependence.

What we eventually learned was that these theories are well-defined,

but,

are disjoint unions of ordinary theories, at least in (2,2) susy cases,

because of decomposition.

Not really new compactifications, but instead: GW predictions, GLSM phases.

Mathematical interpretation:

Finally, let me conclude with a schematic of gauging BG, and why it should result in an ordinary theory:

$$\left[\frac{X \times \cancel{BG}}{\cancel{BG}}\right] \cong X$$

Summary:

- Brief overview of 1-form symmetries in 2d theories
- Brief review of decomposition of 2d theories w/ 1-form symmetries

Some simple concrete examples:

1. An orbifold with a 1-form symmetry
   - Explicit description of decomposition
   - Explicitly gauge the 1-form symmetry

2. Pure nonsusy SU(2) Yang-Mills
   - Explicit description of decomposition
   - Explicitly gauge the 1-form symmetry

Last but not least, we're running an online workshop on GLSMs

on August 17-21, 2020:

https://indico.phys.vt.edu/e/glsms2020

Thank you!

In what sense is the `group' of $K-$bundles, BK, a group? (K abelian)

Let P, Q be two $K-$bundles with transition functions $g_{\alpha\beta},\ h_{\alpha\beta}$

Product: $P \otimes Q \ \sim \ g_{\alpha\beta} h_{\alpha\beta}$

Well-defined? $g_{\alpha\beta} \ \sim \ s_\alpha (g_{\alpha\beta}) s_\beta^{-1}$

so $g_{\alpha\beta} h_{\alpha\beta} \ \sim \ s_\alpha (g_{\alpha\beta}) s_\beta^{-1} (h_{\alpha\beta})$

$= \ s_\alpha (g_{\alpha\beta} h_{\alpha\beta}) s_\beta^{-1}$ if K is abelian.

So, as long as K abelian, have a well-defined product.

Inverses, identity, etc follow similarly.

Just one catch: everything only holds **up to isomorphism**.

$P \otimes P^{-1} \cong I, \quad P \otimes I \cong P,$ etc

Not quite an ordinary group; instead, is ``2-group''

Aside:

More general 2-groups than just BK exist.

Example: extensions

$$1 \longrightarrow BU(1) \longrightarrow \widehat{SU(2)} \longrightarrow SU(2) \longrightarrow 1$$

Possible extensions $\widehat{SU(2)}$ classified by $k \in H^3(SU(2))$

This is the 2-group underlying WZW models.

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