An introduction to decomposition

Symmetry seminar
July 5, 2022

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An overview of hep-th/0502027, 0502044, 0502053, 0606034, … (many …),
& recently arXiv: 2101.11619, 2106.00693, 2107.12386, 2107.13552, 2108.13423,
2204.09117, 2204.13708, 2206.14824 & to appear w/ T. Pantev, D. Robbins, T. Vandermeulen

My talk today concerns **decomposition**,
a new notion in quantum field theory (QFT).

Briefly, decomposition is the observation that some local QFTs
are secretly equivalent to
sums of other local QFTs, known as 'universes.'

When this happens, we say the QFT `decomposes.'
Decomposition of the QFT can be applied to give insight
into its properties.

What does it mean for one local QFT to be a sum of other local QFTs?

(Hellerman et al ’06)

1) Existence of projection operators

The theory contains topological operators Π_i such that

$$\Pi_i \Pi_j = \delta_{i,j} \Pi_j \qquad \sum_i \Pi_i = 1 \qquad [\Pi_i, \mathcal{O}] = 0$$

Operators Π_i simultaneously diagonalizable; state space = $\mathcal{H} = \oplus_i \mathcal{H}_i$

Correlation functions:

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_m \rangle = \sum_i \langle \Pi_i \mathcal{O}_1 \cdots \mathcal{O}_m \rangle = \sum_i \langle (\Pi_i \mathcal{O}_1) \cdots (\Pi_i \mathcal{O}_m) \rangle = \sum_i \langle \tilde{\mathcal{O}}_1 \cdots \tilde{\mathcal{O}}_m \rangle_i$$

2) Partition functions decompose

$$Z = \sum_{\text{states}} \exp(-\beta H) = \sum_i \sum \exp(-\beta H_i) = \sum_i Z_i$$

(on a connected spacetime)

When does this happen?

There are many examples of decomposition !

Finite gauge theories in 2d (orbifolds): we'll see examples later. (T Pantev, ES '05; D Robbins, ES, T Vandermeulen '21)

Common thread: a subgroup of the gauge group acts trivially.

Example: If $K \subset \text{center}(\Gamma) \subset \Gamma$ acts trivially, then $[X/\Gamma] = \coprod_{\text{irreps } K} [X/(\Gamma/K)]_{\hat{\omega}}$

Gauge theories:

- 2d $U(1)$ gauge theory with nonmin' charges = sum of $U(1)$ theories w/ min charges (Hellerman et al '06)
- 2d G gauge theory w/ center-invt matter = sum of $G/Z(G)$ theories w/ discrete theta (ES '14)

Ex: $SU(2)$ theory (w/ center-invt matter) = $SO(3)_+ \coprod SO(3)_-$ (w/ same matter)

- 2d pure G Yang-Mills = sum of trivial QFTs indexed by irreps of G (Nguyen, Tanizaki, Unsal '21) (U(1): Cherman, Jacobson '20)

$$\text{Ex: pure } SU(2) = \coprod_{\text{irreps } SU(2)} (\text{sigma model on pt})$$

- 4d Yang-Mills w/ restriction to instantons of deg' divisible by k (Tanizaki, Unsal '19)

= union of ordinary 4d Yang-Mills w/ different θ angles

More examples ....

There are many examples of decomposition !

More examples :

TFTs: 2d unitary TFTs w/ semisimple local operator algebras decompose to invertibles

Examples: (Implicit in Durhuus, Jonsson '93; Moore, Segal '06)

(Also: Komargodski et al '20, Huang et al 2110.02958)

- 2d abelian BF theory at level k = disjoint union of k invertibles (sigma models on pts)

(Hellerman, ES, 1012.5999)

- 2d G/G model at level k = disjoint union of invertible theories as many as integrable reps of the Kac-Moody algebra (Komargodski et al 2008.07567)

- 2d Dijkgraaf-Witten = sum of invertible theories, as many as irreps (In fact, is a special case of orbifolds discussed later in this talk.)

Sigma models on gerbes = disjoint union of sigma models on spaces w/ B fields

Solves tech issue w/ cluster decomposition. (T Pantev, ES '05)

What do these examples have in common?....

What do the examples have in common?
When is one local QFT a sum of other local QFTs ?

Answer: in d spacetime dimensions,
a theory decomposes when it has a $(d-1)$-form symmetry.

(2d: Hellerman et al '06;
d>2: Tanizaki-Unsal '19, Cherman-Jacobson '20)

Decomposition & higher-form symmetries go hand-in-hand.

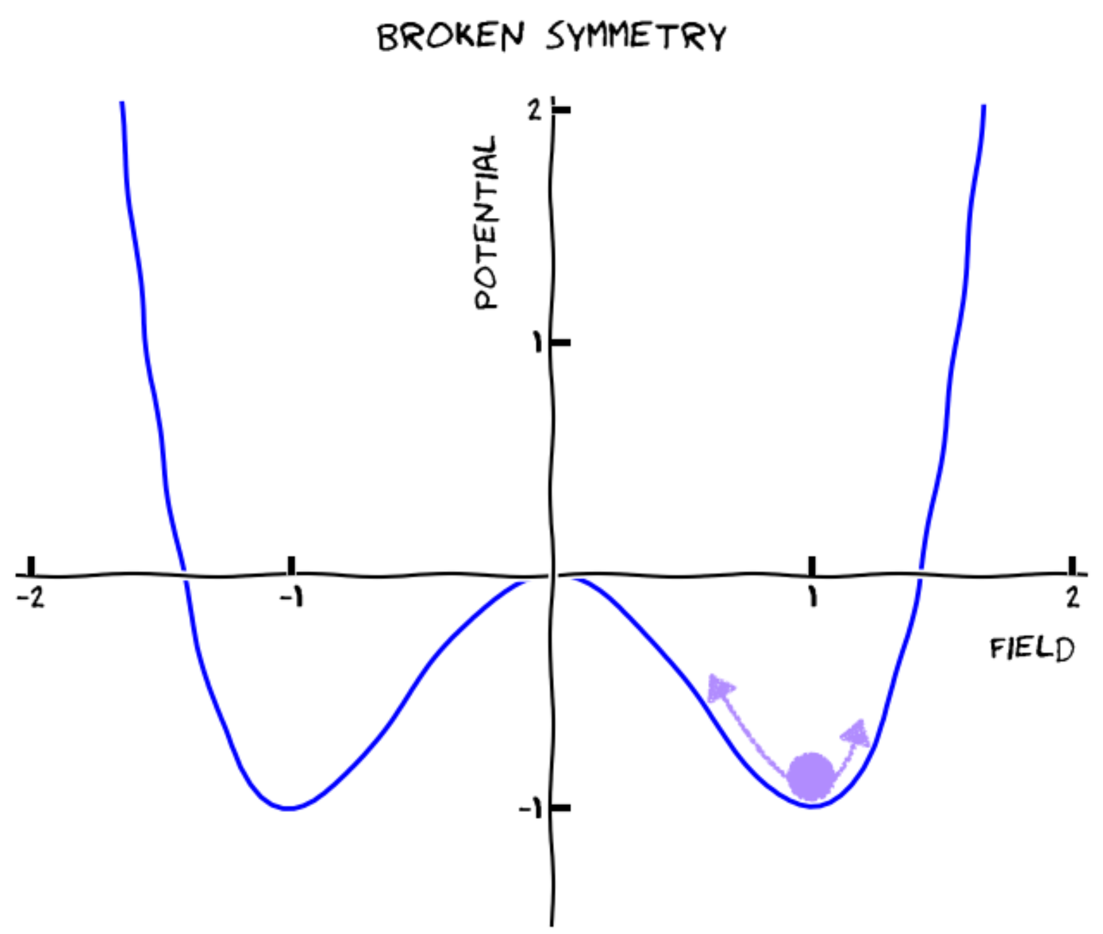
Decomposition $\neq$ spontaneous symmetry breaking

SSB:

Superselection sectors:

- separated by dynamical domain walls
- only genuinely disjoint in IR
- only one overall QFT

Prototype:

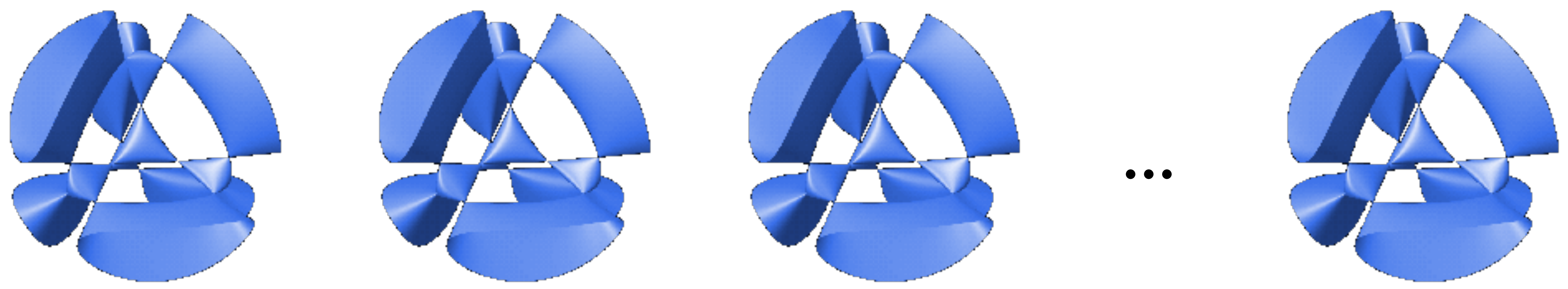


Decomposition:

Universes:

- separated by *non*dynamical domain walls
- disjoint at *all* energy scales
- *multiple* different QFTs present

Prototype:



(see e.g. Tanizaki-Unsal 1912.01033)

Since 2005, decomposition has been checked in many examples in many ways. Examples:

- GLSM's: mirrors, quantum cohomology rings (Coulomb branch) (T Pantev, ES '05; Gu et al '18-'20)
- Orbifolds: partition f'ns, massless spectra, elliptic genera (T Pantev, ES '05; Robbins et al '21)
- Open strings, K theory (Hellerman et al hep-th/0606034)
- Susy gauge theories w/ localization (ES 1404.3986)
- Nonsusy pure Yang-Mills ala Migdal (ES '14; Nguyen, Tanizaki, Unsal '21)
- Adjoint QCD_2 (Komargodski et al '20)
- Numerical checks (lattice gauge thy) (Honda et al '21)
- Versions in d-dim'l theories w/ (d-1)-form symmetries (Tanizaki, Unsal, '19; Cherman, Jacobson '20)

This list is incomplete; apologies to those not listed.

Applications include:

- Sigma models with target stacks & gerbes (T Pantev, ES '05)
- Predictions for Gromov-Witten theory (checked by H-H Tseng, Y Jiang, E Andreini, etc starting '08)
- Nonperturbative constructions of geometries in GLSMs (Caldararu et al 0709.3855, Hori '11, ..., Romo et al '21)
- Elliptic genera (Eager et al '20)
- Anomalies in orbifolds (Robbins et al '21)

In d spacetime dimensions,
a theory decomposes when it has a global $(d-1)$-form symmetry.

Today we'll produce exs by gauging a trivially-acting $(d-2)$-form symmetry
(<-> non-complete charge spectrum)

This is equivalent to

- Theory w/ restriction on instantons
- Sigma models on gerbes
  = fiber bundles with fibers = `groups' of form symmetries $G^{(d-1)} = B^{d-1}G$
- Algebra of topological local operators

Decomposition (into 'universes') relates these pictures.

Examples:

restriction on instantons = "multiverse interference effect"

form symmetry of QFT = translation symmetry along fibers of gerbe

trivial group action b/c $BG = [\mathrm{point}/G]$

Goal for today: a (hopefully pedagogical) introduction to decomposition

Outline:

- **Decomposition in 2d orbifolds, from a perspective that will motivate later cases**

  Global 1-form symmetry from gauging trivially-acting 0-form symmetry

  Aside on gauge theory examples

- Decomposition in 3d orbifolds

  Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Decomposition in 3d Chern-Simons

  Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Application to condensation defects (work in progress)

Let's first construct a family of examples in $d = 2$ spacetime dimensions.

We'll gauge a noneffectively-acting $(d-2) = 0$-form symmetry,
to get a global 1-form symmetry (& hence a decomposition).

Specifically, consider the orbifold $[X/\Gamma]$, where

$$1 \longrightarrow K \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad \sim \ \omega \in H^2(G, K)$$

is a central extension, and K, Γ, G are finite, K abelian, and K acts trivially.
(Decomposition exists more generally, but today I'll stick w/ easy cases.)

The orbifold $[X/\Gamma]$ has a global $BK \ = \ K^{(1)}$ symmetry, & should decompose.

I'm going to outline one way to see that

$$\mathrm{QFT}\left([X/\Gamma]\right) \ = \ \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)} \right)$$

where $$\begin{array}{ccc} H^2(G, K) & \longrightarrow & H^2(G, U(1)) \\ \omega & \mapsto & \rho(\omega) \end{array}$$ gives the discrete torsion on universe ρ

Claim: $\mathrm{QFT}([X/\Gamma]) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)}\right)$

Let's establish this in partition functions on T^2.

Universally, for any Γ orbifold on T^2,

$$Z_{T^2}([X/\Gamma]) = \frac{1}{|\Gamma|} \sum_{\gamma_1 \gamma_2 = \gamma_2 \gamma_1} Z_{\gamma_1, \gamma_2}(X)$$

where $Z_{g,h} = \Big(g$  $\longrightarrow X \Big)$

h

("twisted sectors")

(Think of $Z_{g,h}$ as sigma model to X with branch cuts g, h.)

We need to count commuting pairs of elements in Γ ....

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We need to count commuting pairs of elements in Γ ....

$$1 \longrightarrow K \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad \sim \ \omega \in H^2(G,K)$$

Write $\gamma \in \Gamma$ as $\gamma = (g \in G, k \in K)$ where $\gamma_1\gamma_2 = (g_1 g_2, k_1 k_2 \omega(g_1, g_2))$

Then, $\gamma_1\gamma_2 = \gamma_2\gamma_1 \ \Leftrightarrow \ g_1 g_2 = g_2 g_2$ and $\omega(g_1, g_2) = \omega(g_2, g_1)$

commuting pairs in G such that $\omega(g_1, g_2) = \omega(g_2, g_1)$

Restriction on nonperturbative sectors

(In an orbifold, nonperturbative sectors = twisted sectors)

Claim: $$\mathrm{QFT}([X/\Gamma]) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)}\right)$$

Let's establish this in partition functions on T^2.

Universally, for any Γ orbifold on T^2, $$Z_{T^2}([X/\Gamma]) = \frac{1}{|\Gamma|} \sum_{\gamma_1\gamma_2=\gamma_2\gamma_1} Z_{\gamma_1,\gamma_2}(X)$$

We need to count commuting pairs of elements in Γ .... $$1 \longrightarrow K \longrightarrow \Gamma \longrightarrow G \longrightarrow 1$$

These are commuting pairs in G such that $\omega(g_1, g_2) = \omega(g_2, g_1)$

So: $$Z_{T^2}([X/\Gamma]) = \frac{1}{|\Gamma|} \sum_{\gamma_1\gamma_2=\gamma_2\gamma_1} Z_{\gamma_1,\gamma_2}(X) = \frac{|K|^2}{|\Gamma|} \sum_{g_1g_2=g_2g_1} \delta\left(\frac{\omega(g_1, g_2)}{\omega(g_2, g_1)} - 1\right) Z_{g_1,g_2}$$

where we have used $Z_{\gamma_1,\gamma_2} = Z_{g_1,g_2}$ since K acts trivially.

Claim: $\mathrm{QFT}([X/\Gamma]) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)}\right)$

Let's establish this in partition functions on T^2.

So far:

$$Z_{T^2}([X/\Gamma]) = \frac{1}{|\Gamma|} \sum_{\gamma_1\gamma_2=\gamma_2\gamma_1} Z_{\gamma_1,\gamma_2}(X) = \frac{|K|^2}{|\Gamma|} \sum_{g_1g_2=g_2g_1} \delta\left(\frac{\omega(g_1,g_2)}{\omega(g_2,g_1)} - 1\right) Z_{g_1,g_2}$$

Next, write

$$\delta\left(\frac{\omega(g_1,g_2)}{\omega(g_2,g_1)} - 1\right) = \frac{1}{|\hat{K}|} \sum_{\rho \in \hat{K}} \frac{\rho \circ \omega(g_1,g_2)}{\rho \circ \omega(g_2,g_1)}$$

where $\rho \circ \omega \in H^2(G, U(1))$

(discrete torsion!)

so that, after rearrangement,

$$Z_{T^2}([X/\Gamma]) = \frac{|G||K|^2}{|\Gamma||\hat{K}|} \sum_{\rho \in \hat{K}} Z_{T^2}\left([X/G]_{\rho\circ\omega}\right) = \sum_{\rho \in \hat{K}} Z_{T^2}\left([X/G]_{\rho\circ\omega}\right)$$

consistent with decomposition !

Adding the universes projects out some sectors — interference effect.

So far we have demonstrated that for T^2 partition functions,

$$\mathrm{QFT}\left([X/\Gamma]\right) \;=\; \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)} \right)$$

which is the statement of decomposition in this case ($K \subset \Gamma$ central).

Similar computations can be done at any genus,
and for local operators, etc.

Next, we'll walk through details in a simple example....

To make this more concrete, let's walk through an example, where everything can be made completely explicit.

Example: Orbifold $[X/D_4]$ in which the $\mathbb{Z}_2$ center acts trivially.

(T Pantev, ES '05)

— has $B\mathbb{Z}_2$ (1-form) symmetry

$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ so this is closely related to a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Decomposition predicts

$$\mathrm{QFT}\left([X/\Gamma]\right) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho(\omega)}\right)$$

which here means

$$\mathrm{QFT}\left([X/D_4]\right) = \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \mathrm{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

Let's check this explicitly....

Example, cont'd

$$\mathrm{QFT}\left([X/D_4]\right) \;=\; \mathrm{QFT}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \mathrm{QFT}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{d.t.}}\right)$$

At the level of operators, one reason for this is that the theory admits projection operators:

Let $\hat{z}$ denote the (dim 0) twist field associated to the trivially-acting $\mathbb{Z}_2$:

$\hat{z}$ obeys $\hat{z}^2 \;=\; 1.$

Using that relation, we form projection operators:

$$\Pi_\pm \;=\; \frac{1}{2}(1\pm\hat{z}) \qquad (\text{ = specialization of general formula})$$

$$\Pi_\pm^2 = \Pi_\pm \qquad \Pi_\pm\Pi_\mp = 0 \qquad \Pi_+ + \Pi_- = 1$$

Note: untwisted sector lies in both universes; universes = lin' comb's of twisted & untwisted.

Next: compare partition functions....

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$D_4 \; = \; \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

Take the (1+1)-dim'l spacetime to be T^2.

The partition function of any orbifold $[X/\Gamma]$ on T^2 is

$$Z_{T^2}([X/\Gamma]) \; = \; \frac{1}{|\Gamma|} \sum_{gh=hg} Z_{g,h} \qquad \text{where } \; Z_{g,h} \; = \; \left(g \underset{h}{\square} \longrightarrow X \right)$$

("twisted sectors")

(Think of $Z_{g,h}$ as sigma model to X with branch cuts g, h.)

We're going to see that

$$Z_{T^2}\left([X/D_4]\right) \; = \; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \; + \; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\rm d.t.}\right)$$

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$D_4 = \{1, z, a, b, az, bz, ab, ba = abz\}$$

where z generates the $\mathbb{Z}_2$ center.

$$D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \overline{a}, \overline{b}, \overline{ab}\}$$ where $\overline{a} = \{a, az\}$ etc

$$Z_{T^2}\left([X/D_4]\right) = \frac{1}{|D_4|} \sum_{g,h \in D_4,\ gh=hg} Z_{g,h}$$ where $$Z_{g,h} = \left(g \underset{h}{\blacksquare} \longrightarrow X \right)$$

Since z acts trivially,

$Z_{g,h}$ is symmetric under multiplication by z

$$Z_{g,h} = g \underset{h}{\blacksquare} = gz \underset{h}{\blacksquare} = g \underset{hz}{\blacksquare} = gz \underset{hz}{\blacksquare}$$

This is the $B\mathbb{Z}_2$ 1-form symmetry.

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$D_4 \;=\; \{1, z, a, b, az, bz, ab, ba = abz\}$$

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$$Z_{T^2}\left([X/D_4]\right) \;=\; \frac{1}{|D_4|} \sum_{g,h\in D_4,\; gh=hg} Z_{g,h}$$ where $$Z_{g,h} \;=\; \left(g \underset{h}{\square} \longrightarrow X \right)$$

Each D_4 twisted sector ($Z_{g,h}$) that appears is the same as a $D_4/\mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$ twisted sector,

appearing with multiplicity $|\mathbb{Z}_2|^2 = 4$,

except for the sectors $\overline{a}\underset{\overline{b}}{\square}$ $\quad \overline{a}\underset{\overline{ab}}{\square}$ $\quad \overline{b}\underset{\overline{ab}}{\square}$ which do **not** appear.

Restriction on nonperturbative sectors

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$Z_{T^2}\left([X/D_4]\right) \; = \; \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \; - \; (\text{some twisted sectors}) \right)$$

$$= \; 2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \; - \; (\text{some twisted sectors}) \right)$$

Different theory than $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Physics knows when we gauge even a trivially-acting group!

Example, cont'd

Compute the partition function of $[X/D_4]$

(T Pantev, ES '05)

$$Z_{T^2}\left([X/D_4]\right) \;=\; \frac{|\mathbb{Z}_2\times\mathbb{Z}_2|}{|D_4|}\,|\mathbb{Z}_2|^2\left(Z_{T^2}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]\right) \;-\; \left(\text{some twisted sectors}\right)\right)$$

$$= \;2\left(Z_{T^2}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]\right) \;-\; \left(\text{some twisted sectors}\right)\right)$$

Fact: given any one partition function $\quad Z_{T^2}([X/G]) \;=\; \frac{1}{|G|}\sum_{gh=hg} Z_{g,h}$

we can multiply in $SL(2,\mathbb{Z})$-invariant phases $\epsilon(g,h)$

to get another consistent partition function (for a different theory)

$$Z' \;=\; \frac{1}{|G|}\sum_{gh=hg}\epsilon(g,h)\,Z_{g,h}$$

There is a universal choice of such phases, determined by elements of $H^2(G,U(1))$

This is called "discrete torsion."

Example, cont'd

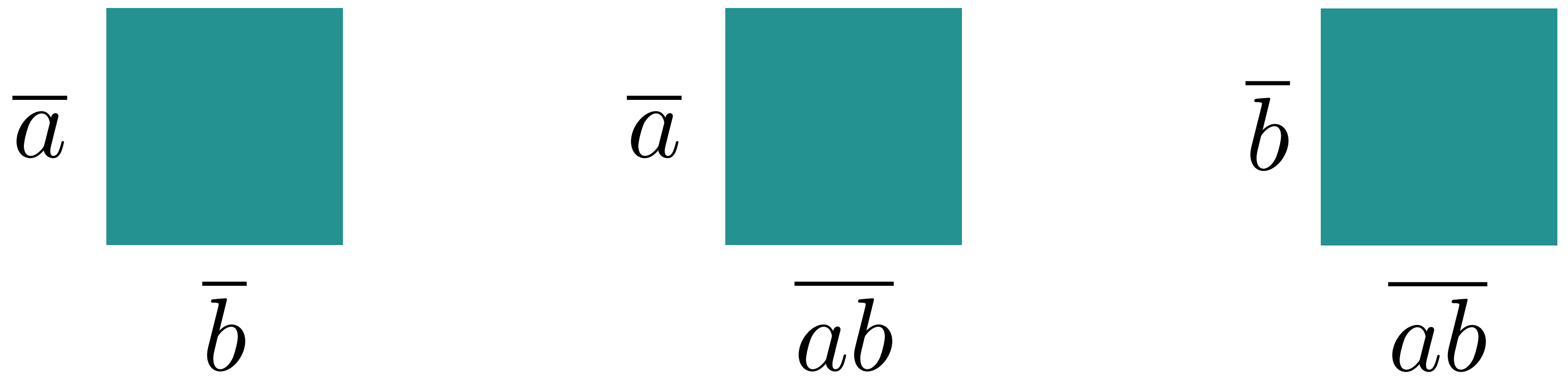
Compute the partition function of $[X/D_4]$ (T Pantev, ES '05)

$$Z_{T^2}\left([X/D_4]\right) \;=\; \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \;-\; (\text{some twisted sectors}) \right)$$

$$= \; 2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \;-\; (\text{some twisted sectors}) \right)$$

In a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, discrete torsion $\in \; H^2(\mathbb{Z}_2 \times \mathbb{Z}_2, U(1)) \; = \; \mathbb{Z}_2$,

and the nontrivial element acts as a sign on the twisted sectors



the same sectors which were omitted above.

$$Z_{T^2}\left([X/D_4]\right) \;=\; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \;+\; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

Adding the universes projects out some sectors — interference effect.

Example, cont'd

Compute the partition function of $[X/D_4]$

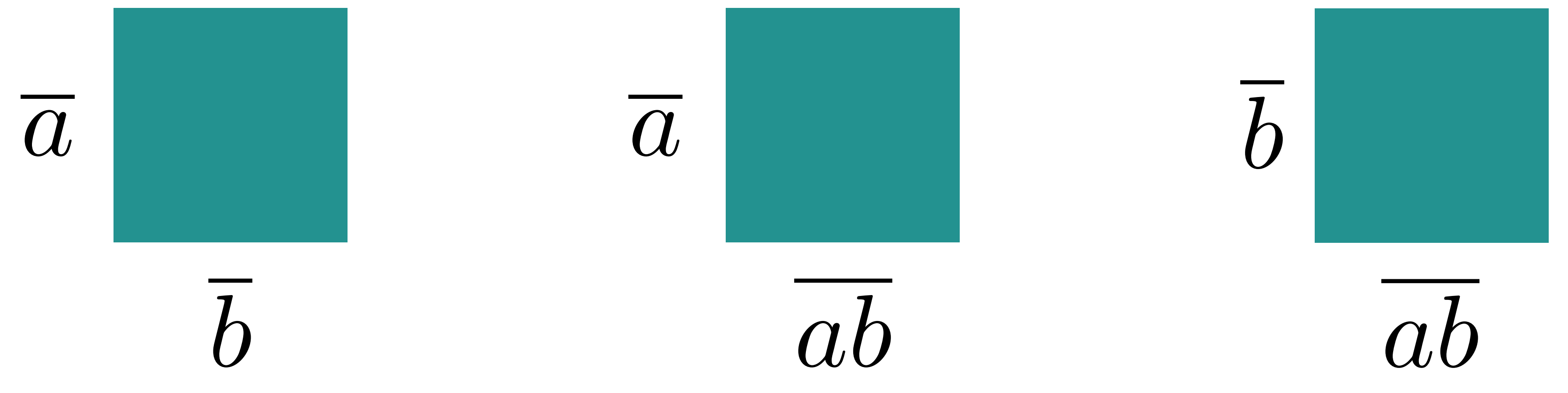
(T Pantev, ES '05)

$$Z_{T^2}\left([X/D_4]\right) \;=\; \frac{|\mathbb{Z}_2 \times \mathbb{Z}_2|}{|D_4|} |\mathbb{Z}_2|^2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \;-\; (\text{some twisted sectors}) \right)$$

$$= \; 2 \left(Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]\right) \;-\; (\text{some twisted sectors}) \right)$$

Discrete torsion is $H^2(\mathbb{Z}_2 \times \mathbb{Z}_2, U(1)) = \mathbb{Z}_2$,

and acts as a sign on the twisted sectors



which were omitted above.

$$Z_{T^2}\left([X/D_4]\right) \;=\; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \;+\; Z_{T^2}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

Matches prediction of decomposition

$$\text{QFT}\left([X/D_4]\right) \;=\; \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{w/o d.t.}}\right) \coprod \text{QFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\right)$$

Example, cont'd

$$Z_{T^2}\left([X/D_4]\right) \; = \; Z_{T^2}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{w/o d.t.}}\right) \; + \; Z_{T^2}\left([X/\mathbb{Z}_2\times\mathbb{Z}_2]_{\text{d.t.}}\right)$$

Matches prediction of decomposition

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The computation above demonstrated that the partition function on T^2
has the form predicted by decomposition.
The same is also true of partition functions at higher genus
— just more combinatorics.
(see hep-th/0606034, section 5.2 for details)

Only slightly novel aspect: in gen'l, one finds dilaton shifts,
which mostly I'll suppress in this talk.

Example, cont'd

Massless states of $[X/D_4]$ for $X = T^6$ (T Pantev, ES '05)

Massless states of $[T^6/D_4]$

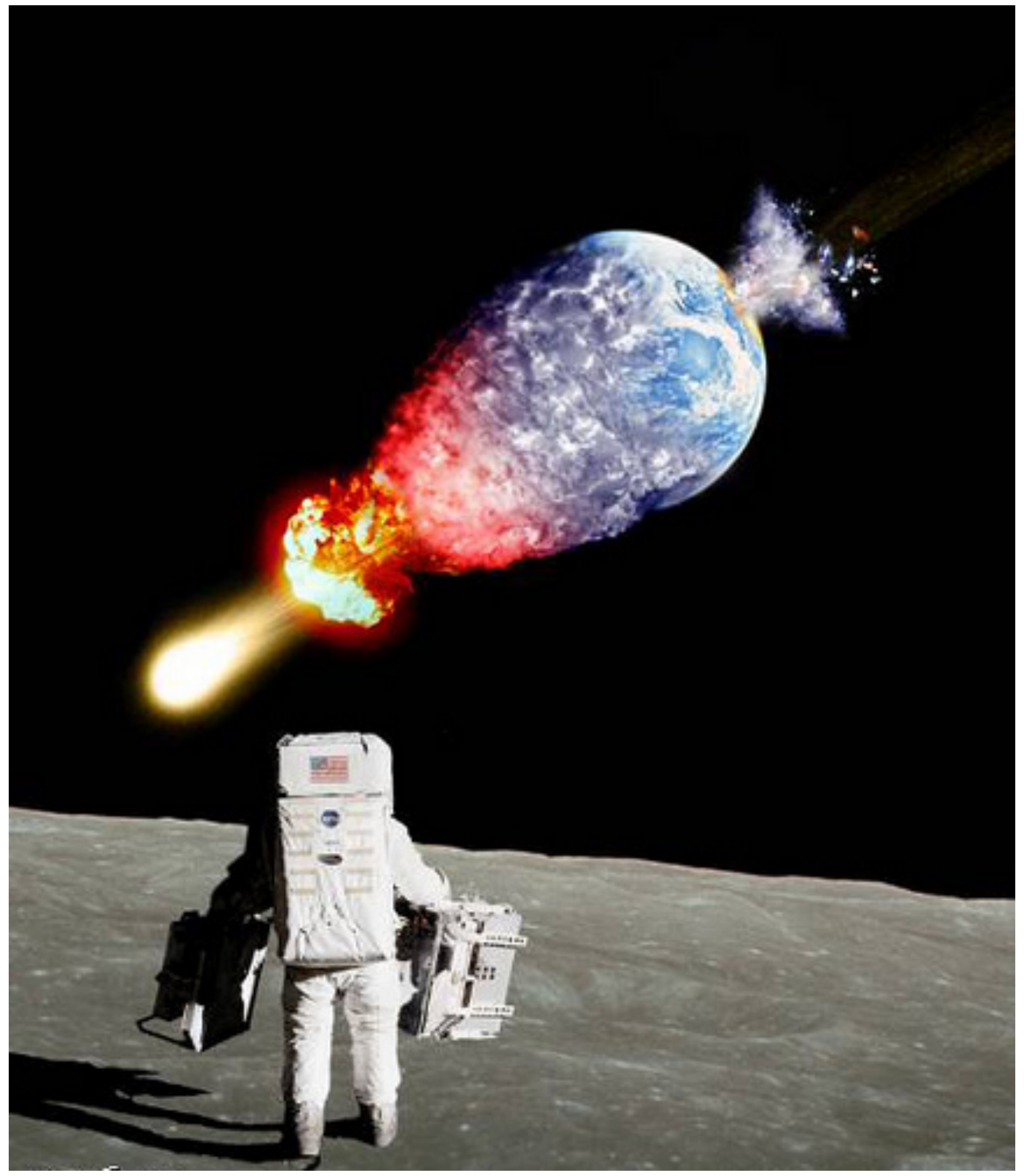
$$\begin{array}{ccccccc} & & & 2 & & & \\ & & 0 & & 0 & & \\ & 0 & & 54 & & 0 & \\ 2 & & 54 & & 54 & & 2 \\ & 0 & & 54 & & 0 & \\ & & 0 & & 0 & & \\ & & & 2 & & & \end{array}$$

Signals mult' components / cluster decomp' violation

If we didn't know about decomposition, the 2's in the corners would be a problem...

A big problem!

They signal a violation of cluster decomposition, the same axiom that's violated by restricting instantons.



Ordinarily, I'd assume that the computation was wrong.

However, decomposition saves the day....

Example, cont'd

Massless states of $[X/D_4]$ for $X = T^6$ (T Pantev, ES '05)

Massless states of $[T^6/D_4]$

$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 54 & & 0 & \\
2 & & 54 & & 54 & & 2 \\
 & 0 & & 54 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 2 & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 51 & & 0 & \\
1 & & 3 & & 3 & & 1 \\
 & 0 & & 51 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
+
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & 3 & & 0 & \\
1 & & 51 & & 51 & & 1 \\
 & 0 & & 3 & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Signals mult' components /

cluster decomp' violation

spectrum of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orb' w/o d.t.

spectrum of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orb' w/ d.t.

matching the prediction of decomposition

$$
\mathrm{CFT}\left([X/D_4]\right) = \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{w/o\,d.t.}}\right) \coprod \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)
$$

This computation was not a one-off, but in fact verifies a prediction in Hellerman et al '06 regarding QFTs in (1+1)-dims with 1-form symmetry.

Another example: Triv'ly acting subgroup **not** in center

Consider $[X/\mathbb{H}]$, $\mathbb{H} =$ eight-element gp of unit quaternions,

where $\langle i \rangle = \mathbb{Z}_4 \subset \mathbb{H}$ acts trivially.

Decomposition predicts

$$\mathrm{QFT}\left([X/\Gamma]\right) = \mathrm{QFT}\left(\left[\frac{X \times \hat{K}}{G} \right]_{\hat{\omega}} \right)$$

(Hellerman et al '06)

where $\hat{K} =$ irreps of K

$\hat{\omega} =$ discrete torsion on universes

Here, $G = \mathbb{H}/\langle i \rangle = \mathbb{Z}_2$ acts nontriv'ly on $\hat{K} = \mathbb{Z}_4$, interchanging 2 elements,

so $$\mathrm{QFT}\left([X/\mathbb{H}]\right) = \mathrm{QFT}\left(X \coprod [X/\mathbb{Z}_2] \coprod [X/\mathbb{Z}_2] \right)$$

(Hellerman et al, hep-th/0606034, sect. 5.4)

— different universes; $X \neq [X/\mathbb{Z}_2]$

— easily checked

Quick note: applications of decomposition in 2d orbifolds

One recent application was to understand Wang-Wen-Witten's work on anomaly resolution. (Robbins et al '21)

Briefly, given an orbifold $[X/G]$ with a gauge anomaly,
Wang-Wen-Witten abstractly construct a related orbifold $[X/\Gamma]_B$,
with a trivially-acting $K \subset \Gamma$,
which in principle is anomaly free.

However, it was shown using decomposition in (Robbins et al '21) that

$$[X/\Gamma]_B \; = \; \coprod [X/\,\text{anomaly-free subgp of } G]$$

which gives a simple way to understand why WWW's procedure works.

So far we've discussed orbifolds, but analogous statements hold in gauge theories.

Decomposition:

$$\mathrm{QFT}(G-\text{gauge theory}) = \coprod_{\text{char's } \hat{K}} \mathrm{QFT}\left(G/K-\text{gauge theory w/ discrete theta angles}\right)$$

Example: pure $SU(2)$ gauge theory = sum $SO(3)_+ + SO(3)_-$ pure gauge theories

where $\pm$ denote discrete theta angles (w_2)

$SU(2)$ instantons (bundles) $\subset SO(3)$ instantons (bundles)

The discrete theta angles weight the non-$SU(2)$ $SO(3)$ instantons so as to cancel out of the partition function of the disjoint union.

Summing over the $SO(3)$ theories projects out some instantons, giving the $SU(2)$ theory.

Restriction on nonperturbative sectors,

implemented by a sum over universes.

Before going on, let's quickly check these claims for pure $SU(2)$ Yang-Mills in 2d.

The partition function Z, on a Riemann surface of genus g, is

(Migdal, Rusakov)

$$Z(SU(2)) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps

$$Z(SO(3)_+) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SO(3) reps

(Tachikawa '13)

$$Z(SO(3)_-) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$

Sum over all SU(2) reps

that are not SO(3) reps

Result: $Z(SU(2)) = Z(SO(3)_+) + Z(SO(3)_-)$ as expected.

Aside: for pure 2d YM, there exists a more extreme decomposition to invertible field theories. (Nguyen, Tanizaki, Unsal '21)

Aside: a common feature of these theories:
violation of cluster decomposition

As Weinberg taught us years ago,
restricting instantons violates cluster decomposition,
and as we have seen, instanton restriction is a common feature in these theories.

A disjoint union of QFTs also violates cluster decomposition,
but in a trivially controllable fashion.

Lesson: restricting instantons OK,
so long as one has a disjoint union.

(Hellerman, Henriques, T Pantev, ES, M Ando, hep-th/0606034)

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Outline:

- Decomposition in 2d orbifolds, from a perspective that will motivate later cases

  Global 1-form symmetry from gauging trivially-acting 0-form symmetry

  Aside on gauge theory examples

- **Decomposition in 3d orbifolds**

  Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Decomposition in 3d Chern-Simons

  Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Application to condensation defects (work in progress)

Three-dimensional examples

Let's construct an example of a decomposition in 3d.

We need a theory with a global 2-form symmetry.

One way to get that is by gauging a trivially-acting one-form symmetry, by which we mean, for example, line operators have no braiding.

Three-dimensional examples

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

Example: Consider an orbifold $[X/\Gamma]$ where

$$1 \longrightarrow BK \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad [\omega] \in H^3(G,K)$$

G, K finite; K abelian; BK acts trivially.

Since BK acts trivially, this theory should have a global 2-form symmetry, & so decompose.

Let's see that explicitly.

Projectors: Projectors are constructed from monopole operators associated to the BK, which generate K-gerbes on surrounding S^2's.

For example, if $K = \mathbb{Z}_k$, then as $\mathbb{Z}_k$-gerbes on S^2 have one generator, there is one generating monopole operator, call it $\hat{z}$, with the property $\hat{z}^k = 1$.

$$\Pi_n = \frac{1}{k} \sum_{m=0}^{k-1} \xi^{mn} \hat{z}^m \qquad \text{where } \xi = \exp(2\pi i/k)$$

Three-dimensional examples

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

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Since BK acts trivially, this theory should have a global 2-form symmetry, & so decompose.

We find:

$$\mathrm{QFT}\left([X/\Gamma]\right) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho \circ \epsilon} \right)$$

(closely analogous to 2d orbifolds with trivially-acting K)

Three-dimensional examples

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

Example: Consider an orbifold $[X/\Gamma]$ where

$$1 \longrightarrow BK \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad [\omega] \in H^3(G, K)$$

G, K finite; K abelian; BK acts trivially. Claim $[X/\Gamma]$ decomposes.

Partition function:

In general terms, the path integral for the orbifold $[X/\Gamma]$ involves a sum over

- principal Γ-bundles E over the 3-manifold M_3
- Maps $E \to X$

just like an ordinary orbifold.

Also, since BK acts trivially, the twisted sectors will be those of a G orbifold.

However, those G-twisted sectors are restricted....

Three-dimensional examples

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

Example: Consider an orbifold $[X/\Gamma]$ where

$$1 \longrightarrow BK \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad\qquad [\omega] \in H^3(G,K)$$

G, K finite; K abelian; BK acts trivially. Claim $[X/\Gamma]$ decomposes.

Partition function:

On T^3, the sum over Γ-twisted sectors maps to a sum over G-twisted sectors such that

$$\epsilon(g_1,g_2,g_3) \;=\; \frac{\omega(g_1,g_2,g_3)}{\omega(g_2,g_1,g_3)}\frac{\omega(g_3,g_1,g_2)}{\omega(g_1,g_3,g_2)}\frac{\omega(g_2,g_3,g_1)}{\omega(g_3,g_2,g_1)} \;=\; 1 \;\in\; K$$

— restriction on nonperturbative sectors

We can implement that restriction by inserting a delta function

$$\delta(\epsilon - 1) \;=\; \frac{1}{|K|}\sum_{\rho\in\hat{K}} \rho\circ\epsilon$$

Partition function....

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

Three-dimensional examples

Example: Consider an orbifold $[X/\Gamma]$ where

$$1 \longrightarrow BK \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad\qquad [\omega] \in H^3(G,K)$$

G, K finite; K abelian; BK acts trivially. Claim $[X/\Gamma]$ decomposes.

Partition function on T^3:

Delta f'n, enforcing constraint

$$Z_{T^3}([X/\Gamma]) = \frac{|H^0(T^3,K)|}{|H^1(T^3,K)|} \frac{1}{|H^0(T^3,G)|} \sum_{z_{1-3}\in K} \sum_{g_{1-3}\in G} \delta(\epsilon - 1)\, Z(g_1,g_2,g_3)$$

$$= \frac{1}{|K|^2 |G|} |K|^3 \sum_{g_{1-3}\in G} \frac{1}{|K|} \sum_{\rho\in\hat{K}} (\rho\circ\epsilon)(g_1,g_2,g_3)\, Z(g_1,g_2,g_3)$$

$$= \sum_{\rho\in\hat{K}} Z_{T^3}\left([X/G]_{\rho\circ\epsilon}\right)$$

where $\rho\circ\epsilon$ defines C-field-analogue of discrete torsion

Adding the universes projects out some sectors — interference effect.

Three-dimensional examples

(T Pantev, D Robbins, T Vandermeulen, ES 2204.13708)

Example: Consider an orbifold $[X/\Gamma]$ where

$$1 \longrightarrow BK \longrightarrow \Gamma \longrightarrow G \longrightarrow 1 \qquad [\omega] \in H^3(G,K)$$

G, K finite; K abelian; BK acts trivially. Claim $[X/\Gamma]$ decomposes.

Partition function on T^3:

$$Z_{T^3}([X/\Gamma]) = \sum_{\rho \in \hat{K}} Z_{T^3}\left([X/G]_{\rho\circ\epsilon} \right)$$

where $\rho \circ \epsilon$ defines C-field-analogue of discrete torsion

consistent with

$$\mathrm{QFT}\left([X/\Gamma]\right) = \coprod_{\rho \in \hat{K}} \mathrm{QFT}\left([X/G]_{\rho\circ\epsilon} \right)$$

Decomposition

Similar results arise on other 3-manifolds.

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- **Decomposition in 3d Chern-Simons**

  Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Application to condensation defects (work in progress)

Three-dimensional examples

(T Pantev, ES, 2206.14824)

Example: Chern-Simons theories

Chern-Simons theories are particularly interesting for these ideas.

For example, classically AdS_3 is Chern-Simons for $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$,
so understanding decomposition in Chern-Simons theories
may give toy models of issues in gravity theories
such as Marolf-Maxfield factorization.

So, what's the decomposition in Chern-Simons ?

Three-dimensional examples

(T Pantev, ES, 2206.14824)

Example: Chern-Simons theories

Consider Chern-Simons(H) / BA for A finite & abelian.

There is an associated `crossed module'

$$1 \longrightarrow K(=\ker d) \longrightarrow A \xrightarrow{d} H \longrightarrow G(=H/\mathrm{im}\, d) \longrightarrow 1$$

Similar remarks apply: only restricted G bundles can appear.

To implement that restriction, must sum over universes....

Conjecture:

$$\text{Chern-Simons}(H) \,/\, BA \;=\; \coprod_{\rho \in \hat{K}} \text{Chern-Simons}(G)_{\omega(\rho)}$$

Decomposition

Three-dimensional examples

(T Pantev, ES, 2206.14824)

Example: Chern-Simons theories

Consider Chern-Simons(H) / BA for A finite & abelian.

Conjecture: $$\text{Chern-Simons}(H) / BA = \coprod_{\rho \in \hat{K}} \text{Chern-Simons}(G)_{\omega(\rho)}$$

Example: Chern-Simons($SU(2)$) / $B\mathbb{Z}_2$ where the $B\mathbb{Z}_2$ acts via the center

$$1 \longrightarrow K(=1) \longrightarrow \mathbb{Z}_2 \xrightarrow{d} SU(2) \longrightarrow SO(3)\,(=SU(2)/\mathrm{im}\,d) \longrightarrow 1$$

so predict

$$\text{Chern-Simons}(SU(2)) / B\mathbb{Z}_2 = \text{Chern-Simons}(SO(3))$$

which is a standard result.

(T Pantev, ES, 2206.14824)

Three-dimensional examples

Example: Chern-Simons theories

Consider Chern-Simons(H) / BA for A finite & abelian.

Conjecture: $$\text{Chern-Simons}(H) / BA = \coprod_{\rho\in\hat{K}} \text{Chern-Simons}(G)_{\omega(\rho)}$$

Example: Chern-Simons($SU(2)$) / $B\mathbb{Z}_4$ where the $B\mathbb{Z}_4$ maps to the center

$$1 \longrightarrow K\,(=\mathbb{Z}_2) \longrightarrow \mathbb{Z}_4 \xrightarrow{d} SU(2) \longrightarrow SO(3)\,(=SU(2)/\mathrm{im}\,d) \longrightarrow 1$$

so predict

$$\text{Chern-Simons}(SU(2)) / B\mathbb{Z}_4 = \coprod_{\rho\in\hat{\mathbb{Z}}_2} \text{Chern-Simons}(SO(3))_{\omega(\rho)}$$

where here ω couples to third Stiefel-Whitney class.

(T Pantev, ES, 2206.14824)

Three-dimensional examples

Example: Chern-Simons theories

Consider Chern-Simons(H) / BA for A finite & abelian.

Conjecture:
$$\text{Chern-Simons}(H) \,/\, BA = \coprod_{\rho \in \hat{K}} \text{Chern-Simons}(G)_{\omega(\rho)}$$

How to check?

For example, boundaries. Above becomes

$$\text{WZW}(H)/A = \coprod_{\rho \in \hat{K}} \text{WZW}(G)_{\theta(\rho)}$$

where the boundary discrete theta angle related to bulk via transgression.

Can show, in fact, boundary discrete theta angle = discrete torsion,
and the predicted boundary decomposition = standard 2d orbifold decomposition.

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- **Application to condensation defects** (work in progress)

Condensation defects

Work in progress

Let's now apply these ideas to condensation defects.

Basic idea:

Let's work in 3d. Suppose I have a theory with a global 1-form symmetry.
In 3d, no decomposition — would need a 2-form symmetry
— but the restriction to a two-submanifold Σ **does** decompose.

Then, gauge the global 1-form symmetry along Σ.

This produces a condensation defect,
and also selects a universe from the decomposition of the restriction to Σ
(using choice of theta angle for 1-form symm).

So, here: condensation defect = universe in decomposition on Σ

Condensation defects

Work in progress

Let's apply this to orbifolds. Consider an orbifold in 3d, with target $[X/\Gamma]$, where $K \subset \Gamma$ acts trivially (and is in the center, for simplicity).

This theory has a global BK symmetry — but we're in 3d, so no decomposition.

Restrict theory to 2d manifold Σ — now, we get a decomposition, label defects $S_i(\Sigma)$.

Gauge the BK:

$$Z\left(S_i(\Sigma = T^2)\right) = \frac{1}{|K|} \sum_{z \in H^2(\Sigma, K) = K} \epsilon_i(z) \left(\frac{1}{|\Gamma|} \sum_{gh = hgz} g \quad h \right)$$

(ES, 1911.05080)

Fusion rule: For covariance, join the defects by a 3d `Wilson membrane', just as in OPEs.

$$Z\left(S_{\ell_1}(\Sigma = T^2) \times S_{\ell_2}(\Sigma = T^2)\right) = \gcd(p,k) \frac{1}{|\mathbb{Z}_{\mathrm{lcm}(p,k)}|} \frac{1}{|\Gamma|^2} \sum_{z \in \mathbb{Z}_{\mathrm{lcm}(p,k)}} \sum_{g_1 h_1 = h_1 g_1 z} \sum_{g_2 h_2 = h_2 g_2 z} \sum_{\gamma \in \Gamma} \epsilon_{\ell_1}(z) \epsilon_{\ell_2}(z) \quad \gamma$$

where $\quad g_i h_i = h_i g_i z, \quad g_1 = \gamma g_2 \gamma^{-1}, \quad h_1 = \gamma h_2 \gamma^{-1}$

Condensation defects

Work in progress

Example: 3d $\mathbb{Z}_2$ Dijkgraaf-Witten theory = orbifold $[\mathrm{point}/\mathbb{Z}_2]$

(Roumpedakis et al 2204.02407)

Consider a 3d orbifold $[\mathrm{point}/\mathbb{Z}_2]$

Since the $\mathbb{Z}_2$ acts trivially, this has a global $B\mathbb{Z}_2$ symmetry.

No decomposition in 3d,

but the restriction to any 2d submfld $\Sigma \subset$ spacetime decomposes,

to two identical universes.

Gauge $B\mathbb{Z}_2$ along Σ

Get two condensation defects $S_{e,1}(\Sigma) \cong S_{e,2}(\Sigma)$ (depending upon one-form theta angle)

Call either $S_e(\Sigma)$; can show $S_e(\Sigma) \times S_e(\Sigma) = \underbrace{2}\, S_e(\Sigma)$

(Roumpedakis et al 2204.02407)

Coeff' can be described as a 2d unitary TFT, but equiv to 2 copies.

Next, we'll see a more intricate example where those universes can be distinguished....

Condensation defects

Work in progress

Example: 3d orbifold $[X/D_4]$

Consider a 3d orbifold $[X/D_4]$ where $\mathbb{Z}_2 \subset D_4$ acts trivially.

— has a global $B\mathbb{Z}_2$ symmetry

No decomposition in 3d, but if restrict to a Riemann surface $\Sigma \subset$ spacetime, get a decomposition:

$$[X/D_4]|_\Sigma = [X/\mathbb{Z}_2 \times \mathbb{Z}_2] \coprod [X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}$$

Gauge $B\mathbb{Z}_2$ along $\Sigma \subset$ spacetime to get condensation defects:

$$S_0(\Sigma) = [X/\mathbb{Z}_2 \times \mathbb{Z}_2]|_\Sigma \qquad S_1(\Sigma) = [X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}|_\Sigma$$

Condensation defects

Work in progress

Example: 3d orbifold $[X/D_4]$

Consider a 3d orbifold $[X/D_4]$ where $\mathbb{Z}_2 \subset D_4$ acts trivially.

Gauge $B\mathbb{Z}_2$ along $\Sigma \subset$ spacetime to get condensation defects:

$$S_0(\Sigma) \;=\; [X/\mathbb{Z}_2 \times \mathbb{Z}_2]\,|_\Sigma \qquad\qquad S_1(\Sigma) \;=\; [X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\text{d.t.}}\,|_\Sigma$$

Can show:

$$S_0(\Sigma) \times S_0(\Sigma) \;=\; 2\, S_0(\Sigma)$$

$$S_0(\Sigma) \times S_1(\Sigma) \;=\; 2\, S_1(\Sigma)$$

$$S_1(\Sigma) \times S_1(\Sigma) \;=\; \underbrace{2}\, S_0(\Sigma)$$

Coeff' can be described as a 2d unitary TFT, but equiv to 2 copies.

Condensation defects

Work in progress

Example: 3d orbifold $[X/\mathbb{H}]$

Consider a 3d orbifold $[X/\mathbb{H}]$ where $\mathbb{Z}_4 \subset \mathbb{H}$ acts trivially.

No decomposition in 3d, but if restrict to a Riemann surface $\Sigma \subset$ spacetime, get a decomposition:

$$[X/\mathbb{H}]|_\Sigma = X|_\Sigma \coprod [X/\mathbb{Z}_2]|_\Sigma \coprod [X/\mathbb{Z}_2]|_\Sigma$$

Gauge $B\mathbb{Z}_2$ along Σ to get condensation defects

$$S_0(\Sigma) = [X/\mathbb{Z}_2]|_\Sigma \coprod [X/\mathbb{Z}_2]|_\Sigma \qquad S_1(\Sigma) = X|_\Sigma$$

Condensation defects

Work in progress

Example: 3d orbifold $[X/\mathbb{H}]$

Consider a 3d orbifold $[X/\mathbb{H}]$ where $\mathbb{Z}_4 \subset \mathbb{H}$ acts trivially.

Gauge $B\mathbb{Z}_2$ along Σ to get condensation defects

$$S_0(\Sigma) \;=\; [X/\mathbb{Z}_2]|_\Sigma \coprod [X/\mathbb{Z}_2]|_\Sigma \qquad\qquad S_1(\Sigma) \;=\; X|_\Sigma$$

Can show:

$$S_0(\Sigma) \times S_0(\Sigma) \;=\; 2\, S_0(\Sigma)$$

$$S_0(\Sigma) \times S_1(\Sigma) \;=\; 2\, S_1(\Sigma)$$

$$S_1(\Sigma) \times S_1(\Sigma) \;=\; \underbrace{2}\, S_0(\Sigma)$$

Coeff' can be described as a 2d unitary TFT, but equiv to 2 copies.

Summary

Decomposition: `one' theory = disjoint union of several

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    Aside on gauge theory examples

- Decomposition in 3d orbifolds

    Global 2-form symmetry from gauging trivially-acting 1-form symmetry

- Decomposition in 3d Chern-Simons

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- Application to condensation defects (work in progress)