Recent developments in 2d (0,2) theories

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In this talk, I'll give not only a summary of my own recent work, but also a broader overview of existing results on perturbative heterotic compactifications & (0,2) SCFT's.

To that end, what are some of the larger issues ?

- Nontrivial IR fixed points in 2d GLSM's

Likely: CY compactifications, but even these are subtle (eg, stability)

Non-Kahler constructions (Adams et al, Melnikov, Quigley, Sethi, Stern, …)

Last week, GGP gave exs of non-CY GLSM's with nontriv' IR.

Related: susy breaking? Most work over the last decade or so has focused on IR nontriv', but, recently some have started looking at susy breaking in 2d theories.

What are some of the larger issues in (0,2) ?

- Dualities

(0,2) mirror symmetry — some significant progress made (Adams-Basu-Sethi, Melnikov-Plesser), but mostly still an open subject.

Gauge dualities motivated by or inherited from 4d (eg Gadde-Gukov-Putrov, Kutasov-Lin)

Other 2d dualities not motivated by 4d, incl. abelian/nonabelian dualities, decomposition.

We'll look at these in more detail later.

What are some of the larger issues in (0,2) ?

- Nonperturbative corrections

What's the analogue of Gromov-Witten invariants in (0,2) ?

Analogue computed by *quantum sheaf cohomology*
(Katz, ES, Donagi, Guffin, McOrist, Melnikov, …)

Computations exist in many cases,
but more remains to be done.

Unfortunately neither std GW computational techniques nor
susy localization are applicable.

In particular, understanding q.s.c. surely important if we're to
ever properly understand (0,2) mirrors.

What are some of the larger issues in (0,2) ?

- Moduli

— non-Kahler moduli: much progress (Melnikov-ES, Anderson-Gray-ES, de la Ossa-Svanes), but still questions.

— surprises in infinitesimal CY moduli: complex/bundle intertwined (Anderson-Gray-Lukas-Ovrut …)

— some moduli are obstructed by eg nonpert' corrections

In today's talk, I'm going to walk through many of these issues in more detail….

Today I'll outline progress in a few of these areas:

Review of quantum sheaf cohomology

Dualities

- (0,2) mirror symmetry
- Gauge bundle dualization duality
- Geometry of Seiberg(-like) dualities
- Abelian/nonabelian dualities & more complicated ex's
- Decomposition in 2d: $SU(2) = SO(3)_+ + SO(3)_-$

Brief overview of moduli

Review of quantum sheaf cohomology

Quantum sheaf cohomology is the heteroti version of quantum cohomology.

Encodes nonperturbative corrections to charged matter couplings.

Example: (2,2) compactification on CY 3-fold

Gromov-Witten invariants encoded in $\overline{\mathbf{27}}^3$ couplings

Off the (2,2) locus, Gromov-Witten inv’ts no longer relevant.

Mathematical GW computational tricks no longer apply.

No known analogue of periods, Picard-Fuchs equations.

New methods needed….

… and a few have been developed.

Review of quantum sheaf cohomology

Quantum sheaf cohomology is the heterotic version of quantum cohomology.

Ex: ordinary quantum cohomology of $\mathbb{P}^n$

$$\mathbb{C}[x]/(x^{n+1}-q)$$

Ex: quantum sheaf cohomology of $\mathbb{P}^n \times \mathbb{P}^n$

with bundle

$$0 \to O \oplus O \stackrel{*}{\to} O(1,0)^{n+1} \oplus O(0,1)^{n+1} \to E \to 0$$

where $* = \begin{bmatrix} Ax & Bx \\ C\tilde{x} & D\tilde{x} \end{bmatrix}$ $x, \tilde{x}$ homog' coord's on $\mathbb{P}^n$'s

is given by $\mathbb{C}[x,y]/(\det(Ax+By)-q_1, \det(Cx+Dy)-q_2)$

Check: When E=T, this becomes $\mathbb{C}[x,y]/(x^{n+1}-q_1, \mathrm{y}^{n+1}-q_2)$

Review of quantum sheaf cohomology

Ordinary quantum cohomology

= OPE ring of the A model TFT in 2d

The A model is obtained by twisting along $U(1)_V$

In a heterotic (0,2) NLSM, if $\det E^* \cong K_X$

then there is a nonanomalous U(1) we can twist along.

Result: a pseudo-topological field theory, ``A/2 model''

Quantum sheaf cohomology

= OPE ring of the A/2 model

Review of quantum sheaf cohomology

Quantum sheaf cohomology

= OPE ring of the A/2 model

When does that OPE ring close into itself?

(2,2) susy **not** required.

For a SCFT, in a neighborhood of the (2,2) locus,
can use combination of

- worldsheet conformal invariance
- right-moving N=2 algebra

to argue closure.

(Adams-Distler-Ernebjerg, ’05)

The mathematics can be defined in greater generality….

Review of quantum sheaf cohomology

Quantum sheaf cohomology

= OPE ring of the A/2 model

A model:

Operators $b_{i_1 \cdots i_p \bar{\imath}_1 \cdots \bar{\imath}_q} \chi^{\bar{\imath}_1} \cdots \chi^{\bar{\imath}_q} \cdots \chi^{i_1} \cdots \chi^{i_p} \leftrightarrow H^{p,q}(X)$

A/2 model:

Operators: $b_{\bar{\imath}_1 \cdots \bar{\imath}_n a_1 \cdots a_p} \psi_+^{\bar{\imath}_1} \cdots \psi_+^{\bar{\imath}_n} \lambda_-^{a_1} \cdots \lambda_-^{a_p} \leftrightarrow H^n(X, \wedge^p E^*)$

On the (2,2) locus, A/2 reduces to A.

For operators, follows from

$$H^q(X, \wedge^p T^* X) = H^{p,q}(X)$$

Review of quantum sheaf cohomology

Quantum sheaf cohomology

= OPE ring of the A/2 model

A model: Classical contribution:

$$\langle O_1 \cdots O_n \rangle = \int_X \omega_1 \wedge \cdots \wedge \omega_n = \int_X (\text{top-form})$$

$$\omega_i \in H^{p_i, q_i}(X)$$

A/2 model: Classical contribution:

$$\langle O_1 \cdots O_n \rangle = \int_X \omega_1 \wedge \cdots \wedge \omega_n$$

Now, $\omega_1 \wedge \cdots \wedge \omega_n \in H^{\text{top}}(X, \wedge^{\text{top}} E^*) = \mathrm{H}^{\text{top}}(X, K_X)$

using the anomaly constraint

Again, a top form, so get a number.

Review of quantum sheaf cohomology

Quantum sheaf cohomology

= OPE ring of the A/2 model

Instanton sectors have the same form,
except X replaced by moduli space M of instantons,
E replaced by induced sheaf F over moduli space M.

Must compactify M,
and extend F over compactification divisor.

$$\left.\begin{array}{l} \wedge^{\mathrm{top}} E^* \cong K_X \\ \mathrm{ch}_2(\mathrm{E}) = \mathrm{ch}_2(\mathrm{TX}) \end{array}\right\} \stackrel{\mathrm{GRR}}{\Longrightarrow} \wedge^{\mathrm{top}} F^* \cong K_M$$

Review of quantum sheaf cohomology

Outline of mathematical computations:

$$0 \to W \otimes O \to Z^* \to F \to 0$$

Correlation functions are maps

$$\mathrm{Sym}^{\mathrm{m}}(\mathrm{H}^1(\mathrm{F}^*))(=\mathrm{Sym}^{\mathrm{m}}\mathrm{W}) \to \mathrm{H}^{\mathrm{m}}(\wedge^{\mathrm{top}}\mathrm{F}^*) = \mathbb{C}$$

for m = rank F.

That map is induced by a class in

$$\mathrm{Ext}^{\mathrm{m}}(\mathrm{Sym}^{\mathrm{m}}\mathrm{W} \otimes O_M, \wedge^{\mathrm{top}} F^*)$$

corresponding to the Koszul res'n

$$0 \to \wedge^m F^* \to \wedge^m Z \to \wedge^{m-1} Z \otimes W \to \wedge^{m-2} Z \otimes \mathrm{Sym}^2 W$$
$$\cdots \to Z \otimes \mathrm{Sym}^{m-1} W \to \mathrm{Sym}^m W \otimes O_M \to 0$$

The map we want is induced by the sequence above….

Review of quantum sheaf cohomology

Outline of mathematical computations:

$$0 \to \wedge^m F^* \to \wedge^m Z \to \wedge^{m-1} Z \otimes W \to \wedge^{m-2} Z \otimes \mathrm{Sym}^2 W$$
$$\cdots \to Z \otimes \mathrm{Sym}^{m-1} W \to \mathrm{Sym}^m W \otimes O_M \to 0$$

This factors into short exact sequences of the form

$$0 \to S_i \to \wedge^i Z \otimes \mathrm{Sym}^{m-i} W \to S_{i-1} \to 0$$

and the corresponding coboundary maps

$$\delta : H^i(S_i) \to H^{i+1}(S_{i+1})$$

factor the map determining correlation functions:

$$H^0(\mathrm{Sym}^m W \otimes O) \to H^1(S_1) \xrightarrow{\delta} H^2(S_2) \xrightarrow{\delta} \cdots \xrightarrow{\delta} H^{m-1}(S_{m-1}) \xrightarrow{\delta} \mathrm{H}^m(\wedge^{\mathrm{top}} F^*)$$

So, to evaluate correlation functions,
compute coboundary maps.

Review of quantum sheaf cohomology

Outline of mathematical computations:

The map determining correlation functions:

$$H^0(\mathrm{Sym}^m W \otimes O) \rightarrow H^1(S_1) \stackrel{\delta}{\rightarrow} H^2(S_2) \stackrel{\delta}{\rightarrow} \cdots \stackrel{\delta}{\rightarrow} H^{m-1}(S_{m-1}) \stackrel{\delta}{\rightarrow} \mathrm{H}^m(\wedge^{\mathrm{top}} F^*)$$

To evaluate correlation functions, compute coboundary maps.

OPE's emerge as the kernel of those maps.

So, briefly, OPE's determined where coboundaries fail to be isomorphisms.

Review of quantum sheaf cohomology

So far, I've outlined mathematical computations of quantum sheaf cohomology, but GLSM-based methods also exist:

- Quantum cohomology ((2,2)):

  Morrison-Plesser '94

- Quantum sheaf cohomology ((0,2)):

  McOrist-Melnikov '08

Let's quickly review these methods….

Review of quantum sheaf cohomology

GLSM-based quantum sheaf cohomology computations:

(McOrist-Melnikov '08)

(0,2) Fermi superfields defined by $\bar{D}_+ \Lambda^i = \Sigma_a \left[A^a_{(\alpha)} \right]^i_j \Phi^j_{(\alpha)}$

or equiv'ly bundle E defined by

$$0 \to W \otimes O \xrightarrow{(A^a_{(\alpha)})^i_j \Phi^j_{(\alpha)}} \oplus_\alpha V_\alpha \otimes O(\vec{q}_\alpha) \to E \to 0$$

Define $M_{(\alpha)} = \sum_a \Sigma_a A^a_{(\alpha)}$

then quantum correction to effective action of form

$$L_{\text{eff}} = \int d\theta^+ \sum_a \Upsilon_a \log \left[\prod_\alpha (\det M_{(\alpha)})^{Q^a_\alpha} / q_a \right]$$

from which one derives $\prod_\alpha \left(\det M_{(\alpha)} \right)^{Q^a_\alpha} = q_a$

— these are q.s.c. rel'ns — match math' computations

Review of quantum sheaf cohomology

State of the art: computations on toric varieties

To do: compact CY's

Intermediate step: Grassmannians (work in progress)

Briefly, what we need are better computational methods.

Conventional GW tricks seem to revolve around idea that A model is independent of complex structure, not necessarily true for A/2.

- McOrist-Melnikov '08 have argued an analogue for A/2
- Despite attempts to check (Garavuso-ES '13), still not well-understood

Outline:

Review of quantum sheaf cohomology

Dualities

- (0,2) mirror symmetry
- Gauge bundle dualization duality
- Geometry of Seiberg(-like) dualities
- Abelian/nonabelian dualities & more complicated ex's
- Decomposition in 2d: $SU(2) = SO(3)_+ + SO(3)_-$

Brief overview of moduli

(0,2) mirror symmetry ((0,2) susy)

Let's begin our discussion of dualities with one of the oldest conjectured (0,2) dualities: (0,2) mirrors.

Nonlinear sigma models with (0,2) susy defined by space X, with gauge bundle $E \to X$

S'pose (0,2) mirror defined by space Y, w/ gauge bundle F.

dim X = dim Y

rk E = rk F

A/2(X, E) = B/2(Y, F)

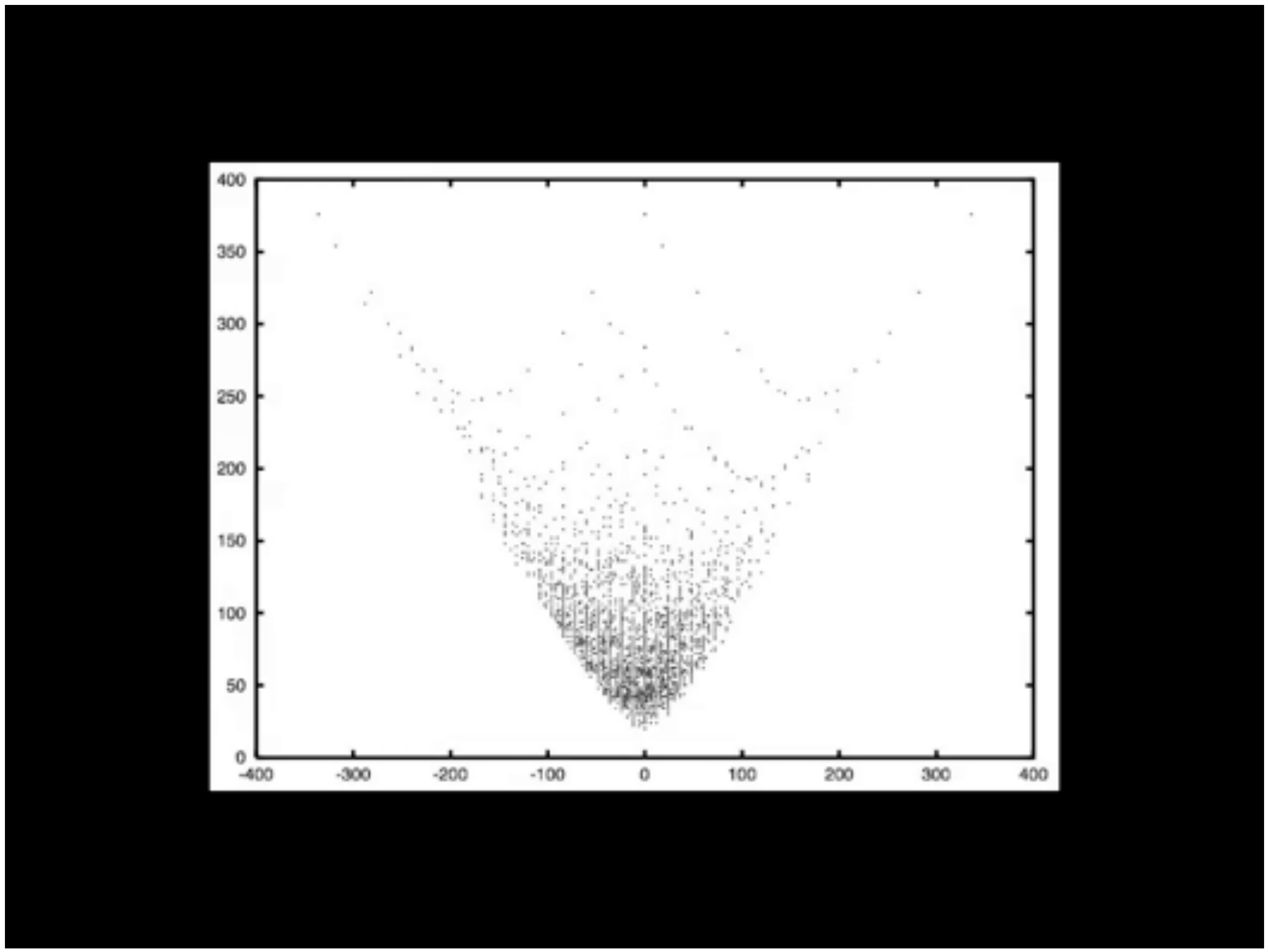
$$H^p(X, \wedge^q E^*) = H^p(Y, \wedge^q F)$$

(moduli) = (moduli)

When E=TX, should reduce to ordinary mirror symmetry.

(0,2) mirror symmetry ((0,2) susy)

Numerical evidence:



Horizontal:

$$h^1(E) - h^1(E^*)$$

Vertical:

$$h^1(E) + h^1(E^*)$$

(E rank 4)

(Blumenhagen-Schimmrigk-Wisskirchen, NPB 486 ('97) 598-628)

(0,2) mirror symmetry **((0,2) susy)**

Constructions include:

- Blumenhagen-Sethi '96 extended Greene-Plesser orbifold construction to (0,2) models — handy but only gives special cases
- Adams-Basu-Sethi '03 repeated Hori-Vafa-Morrison-Plesser-style GLSM duality in (0,2)
- Melnikov-Plesser '10 extended Batyrev's construction & monomial-divisor mirror map to include def's of tangent bundle, for special ('reflexively plain') polytopes

Lots of progress, but still don't have a general construction.

Gauge bundle dualization duality ((0,2) susy)

(Nope, not a typo….)

Nonlinear sigma models with (0,2) susy defined by space X, with gauge bundle $E \rightarrow X$

Duality: CFT(X, E) = CFT(X, E^*)

ie, replacing the gauge bundle with its dual seems to be an invariance of the theory.

(A/2-B/2 in ES ’06; complete in Gadde-Gukov-Putrov ’13, Jia-ES-Wu ’14)

Let’s outline some checks….

Gauge bundle dualization duality ((0,2) susy)

Check that (0,2) theory invariant under $E \leftrightarrow E^*$:

- Action invariant:

$$L = \frac{1}{2} g_{\mu\nu} \partial\phi^\mu \overline{\partial}\phi^\nu + \frac{i}{2} g_{\mu\nu}\psi_+^\mu D_{\overline{z}}\psi_+^\nu + \frac{i}{2} h_{\alpha\beta}\lambda_-^\alpha D_z \lambda_-^\beta + F_{i\overline{j}a\overline{b}}\psi_+^i\psi_+^{\overline{j}}\lambda_-^a\lambda_-^{\overline{b}}$$

Under $E \leftrightarrow E^*$, $\lambda_-^a \leftrightarrow \lambda_-^{\overline{b}}$ & $F \leftrightarrow -F$

so we see the Lagrangian is invariant.

- Consistency conditions:

$\mathrm{ch}_2(\mathrm{E}) = \mathrm{ch}_2(\mathrm{TX})$ invariant under $E \leftrightarrow E^*$

- Massless spectra:

$h^\bullet(X, \wedge^\bullet E)$, $h^\bullet(X, \mathrm{End}\, E)$ invariant under $E \leftrightarrow E^*$

using $h^p(X, \wedge^q E^*) \cong h^{n-p}(X, \wedge^{r-q} E)$ (Serre duality on CY)

Gauge bundle dualization duality ((0,2) susy)

Check that (0,2) theory invariant under $E \leftrightarrow E^*$:

- Bundle must be `stable': $g^{i\bar{j}} F_{i\bar{j}} = 0$

  Math result: a bundle is stable iff its dual is stable.

- Elliptic genera:

$$\mathrm{Tr}\ (-)^{F_R} z^{J_L} q^{L_0} \overline{q}^{\overline{L}_0}$$

$$= q^{-(2n+r)/24} \int_X \mathrm{Td}(TX) \wedge \mathrm{ch}\Big(\bigotimes_{k=1,2,3,\cdots} S_{q^k}((TX)^{\mathbb{C}}) \bigotimes_{k=1/2,3/2,5/2,\cdots} \wedge_{q^k} ((zE)^{\mathbb{C}}) \Big)$$

where $(zE)^{\mathbb{C}} = zE \oplus \overline{z}E^*$

Manifestly invariant under $E \leftrightarrow E^*$, so long as also $z \leftrightarrow \overline{z}$.

- Worldsheet instantons: A/2(X, E) = B/2(X, E^*) **(ES '06)**

Gauge bundle dualization duality ((0,2) susy)

Invariant understanding:

Physics only cares about the gauge field,
ie, underlying principal bundle,
not about an associated vector bundle.

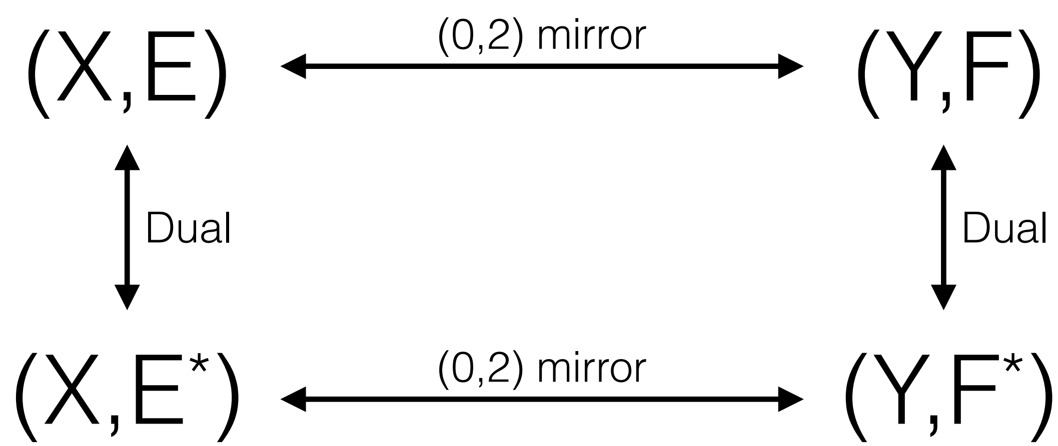
What we're really doing is changing the associated vector bundle (dual = bundle associated to dual rep),
and physics shouldn't care about that,
hence CFT unchanged.

If we had CFT realizations of other associated vector bundles,
those CFT's should be the same too.

Gauge bundle dualization duality ((0,2) susy)

How is this related to (0,2) mirrors?

Maybe orthogonal:



Both exchange A/2, B/2 models, both flip sign of left U(1)…

Maybe notion of (0,2) mirrors is richer,
& more variations exist to be found:

$$\longleftrightarrow (X_1,E_1) \longleftrightarrow (X_2,E_2) \longleftrightarrow (X_3,E_3) \longleftrightarrow (X_4,E_4) \longleftrightarrow$$

Gadde-Gukov-Putrov's triality ('13) seems to be in this spirit.

Dualities ((0,2) & (2,2) susy)

So far we've discussed dualities that act nontrivially on target-space geometries.

Next: gauge dualities

Ex: Gadde-Gukov-Putrov triality

Ex: Kutasov-Lin reduction of 4d N=1 to 2d (0,2) dualities

At least sometimes, give different presentations of **same** geometry.

We'll first discuss how they present the same geometry, then turn to some examples specific to 2d, **not** inherited from / analogous to 4d gauge dualities.

Geometric dualities ((2,2) susy)

U(k) gauge group,
matter: n chirals in fund' **k**, n>k,
A chirals in antifund' $\mathbf{k}^*$, A<n

Seiberg ⟷

U(n-k) gauge group,
matter: n chirals Φ in fund' **k**,
A chirals P in antifund' $\mathbf{k}^*$,
nA neutral chirals M,
superpotential: $W = M \Phi P$

↓ ↓

NLSM on $\mathrm{Tot}\left(S^A \rightarrow G(k,n)\right) \quad = \mathrm{Tot}\left((Q^*)^A \rightarrow G(n\text{-}k,n)\right)$

generalizing G(k,n) = G(n-k,n)

Build GLSM for RHS using $0 \rightarrow S \xrightarrow{\Phi} \mathcal{O}^n \rightarrow Q \rightarrow 0$

(Physical duality at top proposed by Benini-Cremonesi '12)

So, 2d analogue of Seiberg duality has geometric description.

Very handy in cases where few global symmetries exist.

Geometric dualities ((2,2) susy)

The trick I've just outlined,
implicitly assumes the geometries are either

- positively-curved (Fano)
- Calabi-Yau

If the space is negatively-curved, then in gen'l expect add'l contributions from discrete Coulomb vacua.
I'll restrict to Fano & CY for the next several slides….

Abelian/nonabelian dualities ((2,2) susy)

A fun example is motivated by the geometry
$G(2,4)$ = degree 2 hypersurface in $\mathbb{CP}^5$

Result:

U(2) gauge theory,
matter: 4 chirals ϕ_i in fundamental **2**

is Seiberg dual to

U(1) gauge theory,
matter: 6 chirals $z_{ij} = -z_{ji}$, $i,j=1\ldots4$, of charge +1,
one chiral P of charge -2,
superpotential

$$W = P(z_{12}\, z_{34} - z_{13}\, z_{24} + z_{14}\, z_{23})$$

Abelian/nonabelian dualities ((2,2) susy)

G(2,4) = degree 2 hypersurface in $\mathbb{CP}^5$

U(2) gauge theory,
matter: 4 chirals ϕ_i in **2**

⟷

U(1) gauge theory,
6 chirals $z_{ij} = -z_{ji}$, i,j=1…4, of charge +1,
one chiral P of charge -2,
superpotential

Relation: $z_{ij} = \epsilon_{\alpha\beta}\phi_i^\alpha \phi_j^\beta$

Compare symmetries: GL(4) action

$\phi_i^\alpha \mapsto V_i^{\ j}\phi_j^\alpha$ $z_{ij} \mapsto V_i^{\ k} V_j^{\ \ell} z_{k\ell}$

Chiral rings, anomalies, Higgs moduli space match automatically.

Can also show elliptic genera match, applying computational methods of Benini-Eager-Hori-Tachikawa '13, Gadde-Gukov '13.

Abelian/nonabelian dualities ((2,2) susy)

This extends to hypersurfaces & complete intersections:

$$G(2,4)[d_1,d_2,\cdots] = \mathbb{P}^5[2,d_1,d_2,\cdots]$$

U(2) gauge theory,
matter: 4 chirals ϕ_i in **2**
chirals p_a of charge $-d_a$
under det U(2)
superpotential

$$W = \sum_a p_a f_a(\epsilon_{\alpha\beta}\phi_i^\alpha \phi_j^\beta)$$

U(1) gauge theory,
6 chirals $z_{ij} = -z_{ji}$, i,j=1…4, of charge +1,
one chiral P of charge -2,
chirals P_a of charge $-d_a$,
superpotential

$$W = P(z_{12}z_{34} - z_{13}z_{24} + z_{14}z_{23}) + \sum_a P_a f_a(z_{ij})$$

$$\epsilon_{\alpha\beta}\phi_i^\alpha \phi_j^\beta = z_{ij}$$

Straightforward extrapolation of previous duality,
as one might hope.

Abelian/nonabelian dualities ((2,2) susy)

$$G(2,4)[d_1,d_2,\cdots] = \mathbb{P}^5[2,d_1,d_2,\cdots]$$

Examples of this form illustrate the usefulness of geometry, in those cases in which it's applicable.

Specifically: **superpotentials break global symmetries**.

W/o global symmetries as a guide, finding dualities would be far more difficult.

In these examples, can instead use geometry to locate otherwise obscure dualities.

Abelian/nonabelian dualities ((0,2) susy)

Let's build on our previous duality

$$G(2,4)[d_1,d_2,\cdots] = \mathbb{P}^5[2,d_1,d_2,\cdots]$$

by extending to heterotic cases.

Example:

Bundle $0 \to E \to \oplus^8 O(1,1) \to O(2,2) \oplus^2 O(3,3) \to 0$

on the CY G(2,4)[4].

Described by

U(2) gauge theory

4 chirals in fundamental

1 Fermi in (-4,-4) (hypersurface)

8 Fermi's in (1,1) (gauge bundle E)

1 chiral in (-2,-2) (gauge bundle E)

2 chirals in (-3,-3) (gauge bundle E)

plus superpotential

rep' of U(2)

Abelian/nonabelian dualities ((0,2) susy)

Example:

Bundle $$0 \to E \to \oplus^8 O(1,1) \to O(2,2) \oplus^2 O(3,3) \to 0$$

on the CY G(2,4)[4].

Geometry predicts this is dual to

Bundle $$0 \to E \to \oplus^8 O(1) \to O(2) \oplus^2 O(3) \to 0$$

on the CY $\mathbb{P}^5[2,4]$

Checks:
- both satisfy anomaly cancellation
- elliptic genera match

To make the duality work, we used the fact that reps defining bundle all live in det U(2)….

More complicated examples ((2,2) susy)

Duality: G(2,n) = Pfaffian

Mathematically,

G(2,n) = rank 2 locus of nxn matrix A over $\mathbb{P}^{\binom{n}{2}-1}$

$$A(z_{ij}) = \begin{bmatrix} z_{11}=0 & z_{12} & z_{13} & \cdots \\ z_{21}=-z_{12} & z_{22}=0 & z_{23} & \cdots \\ z_{31}=-z_{13} & z_{32}=-z_{23} & z_{33}=0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Proposal:

U(2) gauge theory, n chirals in fundamental

dual to

U(n-2)xU(1) gauge theory,
n chirals X in fundamental of U(n-2),
n chirals P in antifundamental of U(n-2),
(n choose 2) chirals $z_{ij} = - z_{ji}$ of charge +1 under U(1),
W = tr PAX

Even more complicated possibilities exist.

Dualities ((0,2) & (2,2) susy)

How do these gauge dualities relate to (0,2) mirrors?

As we've seen, gauge dualities often relate different presentations of the same geometry,
whereas (0,2) mirrors exchange different geometries.

Existence of (0,2) mirrors seems to imply that there ought to exist more `exotic' gauge dualities,
that present different geometries.

On to a different duality….

More games in 2d you can't play in 4d

Decomposition: (Hellerman et al '06)

In a 2d orbifold or abelian gauge theory,
if a finite subgroup of the gauge group acts trivially on all matter, the theory decomposes as a disjoint union.

Ex: $\mathrm{CFT}([X/\mathbb{Z}_2]) = \mathrm{CFT}\left(X \coprod X\right)$

On LHS, the $\mathbb{Z}_2$ acts triv'ly on X,
hence there are dim' zero twist fields.
Projection ops are lin' comb's of dim 0 twist fields.

Ex: $\mathrm{CFT}([X/D_4])$ where $\mathbb{Z}_2 \subset D_4$ acts trivially on X

$$= \mathrm{CFT}\left([X/\mathbb{Z}_2 \times \mathbb{Z}_2] \coprod [X/\mathbb{Z}_2 \times \mathbb{Z}_2]_{\mathrm{d.t.}}\right)$$

where $D_4 / \mathbb{Z}_2 = \mathbb{Z}_2 \times \mathbb{Z}_2$

More games in 2d you can't play in 4d

Decomposition: **(Hellerman et al '06)**

In a 2d orbifold or abelian gauge theory, if a finite subgroup of the gauge group acts trivially on all matter, the theory decomposes as a disjoint union.

Ex in abelian gauge theory: variation of $\mathbb{CP}^{N-1}$ model

U(1)xU(1) gauge theory

Fields x_i, z with charges

x_i	z
1	-n
0	k

For suitable FI, second U(1) almost eliminates z, except for residual $\mathbb{Z}_k$

Result is $\mathbb{CP}^{N-1}$ with trivial $\mathbb{Z}_k$ action

More games in 2d you can't play in 4d

Decomposition: (Hellerman et al '06)

Ex in abelian gauge theory: variation of $\mathbb{CP}^{N-1}$ model

U(1)xU(1) gauge theory

Fields x_i, z with charges

x_i	z
1	-n
0	k

Result is $\mathbb{CP}^{N-1}$ with trivial $\mathbb{Z}_k$ action

Can show quantum cohomology ring is

$$\mathbb{C}[x,y]/(x^N = qy^n, y^k = 1)$$

— Decomposition manifest.

More games in 2d you can't play in 4d

Decomposition:

In that previous example, implicitly:

U(1) gauge theory with nonminimal charges

$\neq$ U(1) gauge theory with minimal charges

Why?

Answer: nonperturbative effects

Noncompact worldsheet: distinguish via θ periodicity

Compact worldsheet: define charged field via specific bundle

(Adams-Distler-Plesser, Aspen '04)

More games in 2d you can't play in 4d

Extension of decomposition to nonabelian gauge theories:

Since 2d gauge fields don't propagate,
analogous phenomena should happen in nonabelian cases.

Result: (ES, '14)

For G semisimple, with center-inv't matter,
G gauge theories decompose:

Ex: $SU(2) = SO(3)_+ + SO(3)_-$

— SO(3)'s have different discrete theta angles

More games in 2d you can't play in 4d

Extension of decomposition to nonabelian gauge theories:

Aside: discrete theta angles

(Gaiotto-Moore-Neitzke '10, Aharony-Seiberg-Tachikawa '13, Hori '94)

Consider 2d gauge theory, group $G = \tilde{G} / K$

$\tilde{G}$ compact, semisimple, simply-connected

K finite subgroup of center of $\tilde{G}$

The theory has a degree-two K-valued char' class w

For λ any character of K, can add a term to the action

$$\lambda(w)$$

— discrete theta angles, classified by characters

Ex: $SO(3) = SU(2) / \mathbb{Z}_2$ has 2 discrete theta angles

More games in 2d you can't play in 4d

Ex: SU(2) = $SO(3)_+$ + $SO(3)_-$

Let's see this in pure nonsusy 2d QCD.

$$Z(SU(2)) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$ Sum over all SU(2) reps

$$Z(SO(3)_+) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$ Sum over all SO(3) reps

(Tachikawa '13)

$$Z(SO(3)_-) = \sum_R (\dim R)^{2-2g} \exp(-AC_2(R))$$ Sum over all SU(2) reps that are not SO(3) reps

Result: $Z(SU(2)) = Z(SO(3)_+) + Z(SO(3)_-)$

More games in 2d you can't play in 4d

More general statement of decomposition for 2d nonabelian gauge theories with center-invariant matter:

For G semisimple, K a finite subgp of center of G,

$$G = \sum_{\lambda \in \hat{K}} (G/K)_{\lambda}$$

$\lambda \in \hat{K}$ → indexes discrete theta angles

Outline:

Review of quantum sheaf cohomology

Dualities

- (0,2) mirror symmetry
- Gauge bundle dualization duality
- Geometry of Seiberg(-like) dualities
- Abelian/nonabelian dualities & more complicated ex's
- Decomposition in 2d: $SU(2) = SO(3)_+ + SO(3)_-$

Brief overview of moduli

Brief overview of moduli

It was known historically that for large-radius NLSM's on the (2,2) locus, there were three classes of infinitesimal moduli:

$H^1(X, T^*X)$ Kahler moduli

$H^1(X, TX)$ Complex moduli

$H^1(X, \mathrm{End}\, \mathrm{E})$ Bundle moduli

where, on (2,2) locus, $E = TX$

For many years it was falsely assumed that this would still be the case for CY compactification off the (2,2) locus, just change E.

Nowadays, we know differently….

Brief overview of moduli

For Calabi-Yau (0,2) compactifications off the (2,2) locus, moduli are as follows:

(Anderson-Gray-Lukas-Ovrut, ‘10)

$H^1(X, T^* X)$ Kahler moduli

$H^1(Q)$ where

$$0 \to \mathrm{End}\,\mathrm{E} \to Q \to TX \to 0 \qquad (F)$$

(Atiyah sequence)

There remained for a long time the question of moduli of non-Kahler compactifications….

Brief overview of moduli

For non-Kahler (0,2) compactifications,
in the **formal** $\alpha' \to 0$ limit,

(Melnikov-ES, '11)

$H^1(S)$ where

$$0 \to T^* X \to S \to Q \to 0 \qquad (H,\ dH = 0)$$

$$0 \to \mathrm{End}\,\mathrm{E} \to Q \to TX \to 0 \qquad (F)$$

Now, we also need α' corrections….

Brief overview of moduli

Through first order in α',
the moduli are *overcounted* by

(Anderson-Gray-ES ’14; de la Ossa-Svanes ’14)

$H^1(S)$ where

$$0 \to T^*X \to S \to Q \to 0 \qquad (H, \text{Green-Schwarz})$$

$$0 \to \mathrm{End}\,E \oplus \mathrm{End}\,TX \to Q \to TX \to 0 \qquad (F, R)$$

Current state-of-the-art

WIP to find correct counting, & extend to higher orders

Brief overview of moduli

So far I've outlined infinitesimal moduli — marginal operators.

These can be obstructed by eg nonperturbative effects.

Dine-Seiberg-Wen-Witten '86 observed that a single worldsheet instanton can generate a superpotential term obstructing def's off (2,2) locus….

… but then Silverstein-Witten '95, Candelas et al '95, Basu-Sethi '03, Beasley-Witten '03 observed that for polynomial moduli in GLSM's, the contributions of all pertinent worldsheet instantons cancel out.

— those moduli are unobstructed; math not well-understood.

Moduli w/o such a description can still be obstructed, see for example Aspinwall-Plesser '11, Braun-Kreuzer-Ovrut-Scheidegger '07

Summary:

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Brief overview of moduli