HUYGENS' PRINCIPLE AND FEYNMAN'S PATH INTEGRAL

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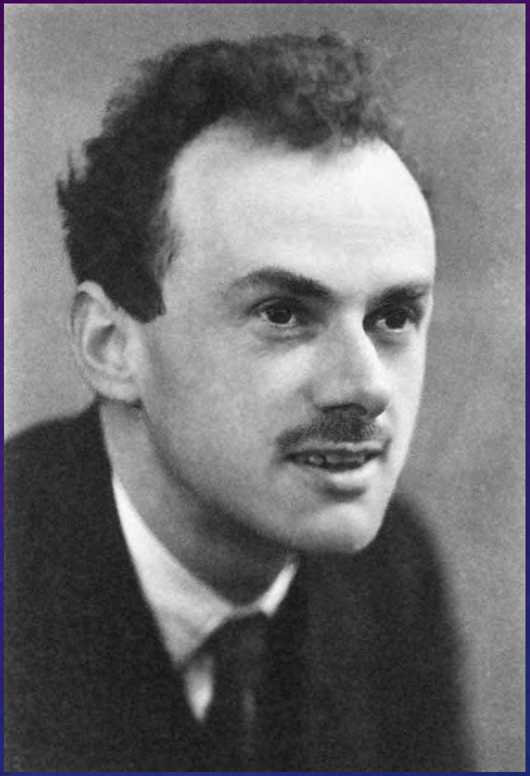
SPRING MEETING OF THE CHESAPEAKE SECTION OF THE AAPT
JAMES MADISON UNIVERSITY, HARRISONBURG, VA

VIRGINIA TECH

TODAY'S TALK WILL BE ABOUT THE WORK BY:

Christiaan Huygens by Casper Netscher,
Museum Boerhaave, Leiden, Netherlands



From the Nobel Prize website

From the Nobel Prize website

Christiaan Huygens
(1629 – 1695)

relevant work in 1678

Paul Adrien Maurice Dirac
(1902 – 1984)

relevant work in 1933

Richard Phillips Feynman
(1918 – 1988)

relevant work in 1948

REFERENCES:

- Christiaan Huygens
  "Treatise on Light"
  drafted 1678; published in Leyden by Van der Aa, 1690

- P. A. M. Dirac
  "The Lagrangian in Quantum Mechanics"
  Physikalische Zeitschrift der Sowjetunion. **3**: 64–72 (1933)

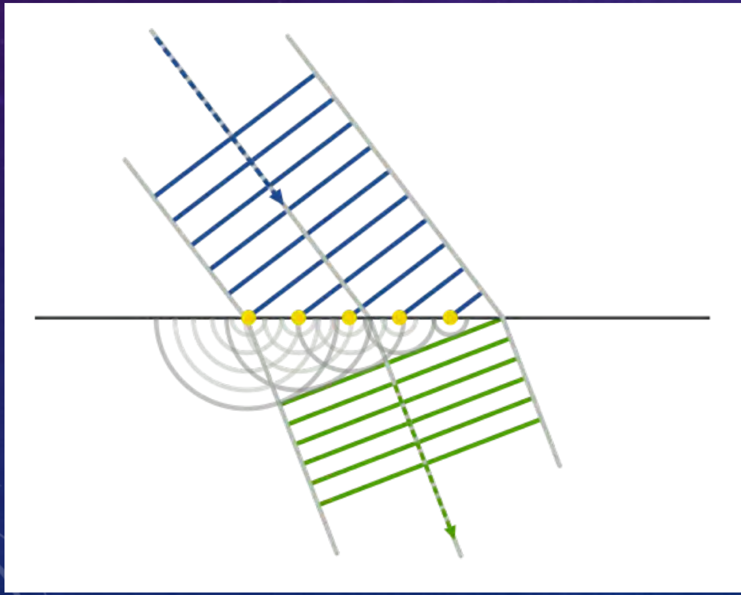
- R. P. Feynman
  "Space-Time Approach to Non-Relativistic Quantum Mechanics"
  Reviews of Modern Physics. **20** (2): 367–387 (1948)
  See also: R. P. Feynman and A. R. Hibbs
  "Quantum Mechanics and Path Integrals" (Dover, 2010)

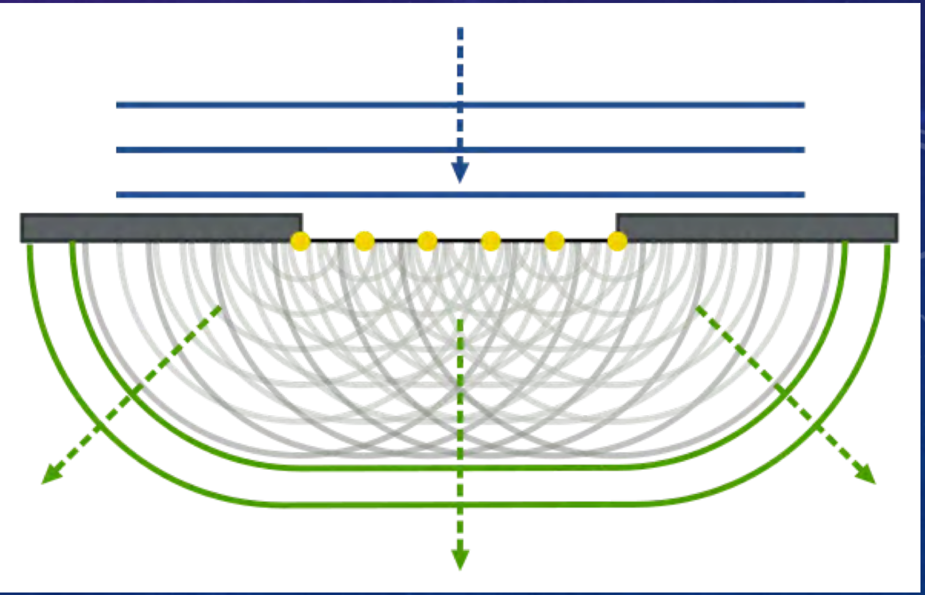
KNOWLEDGE TENDS TO BE COMPARTMENTALIZED:

- Huygens' Principle is a concept we learn about in Optics
- Feynman's Path Integral is a concept we learn about in Quantum Mechanics (if at all. Not in many intro textbooks.)
- The two concepts are actually very closely related but students (and many of us instructors) do not often see the connection and keep the knowledge in separate compartments (labeled Optics and QM, respectively) in our brains
- Seeing the connection will help us understand QM (and also Classical Mechanics) better!

HUYGENS' PRINCIPLE

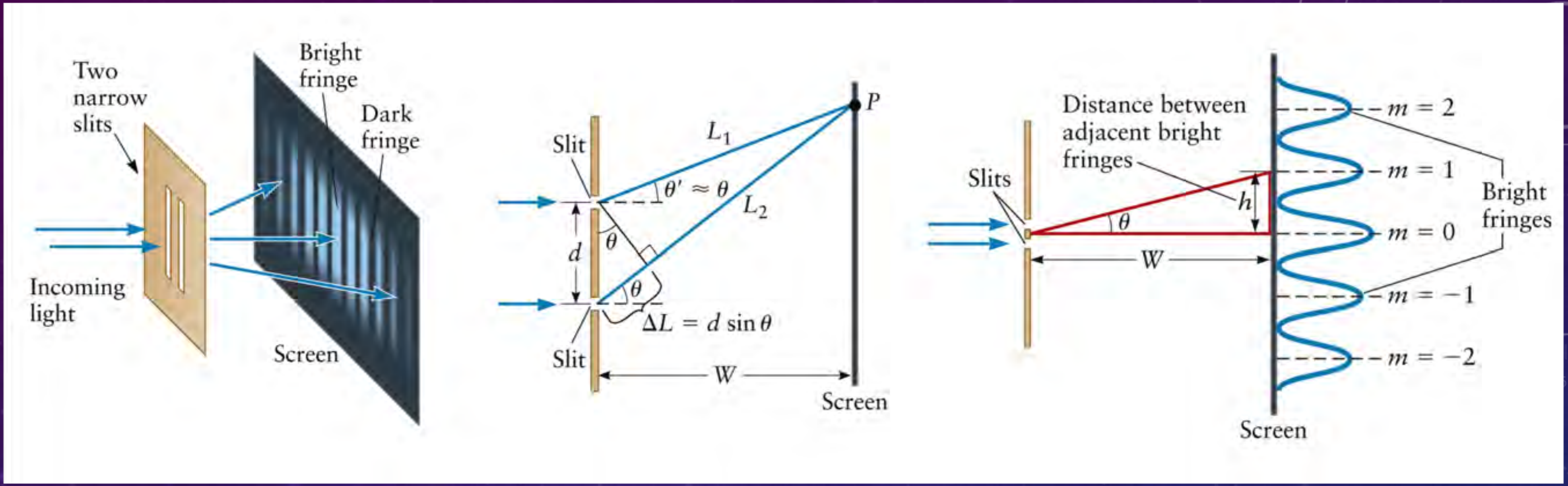
- Each point on a wave front acts like a point source of a spherical wave
- The subsequent wave front will be the superposition of all these spherical waves





DOUBLE SLIT EXPERIMENT

Image from Giordano



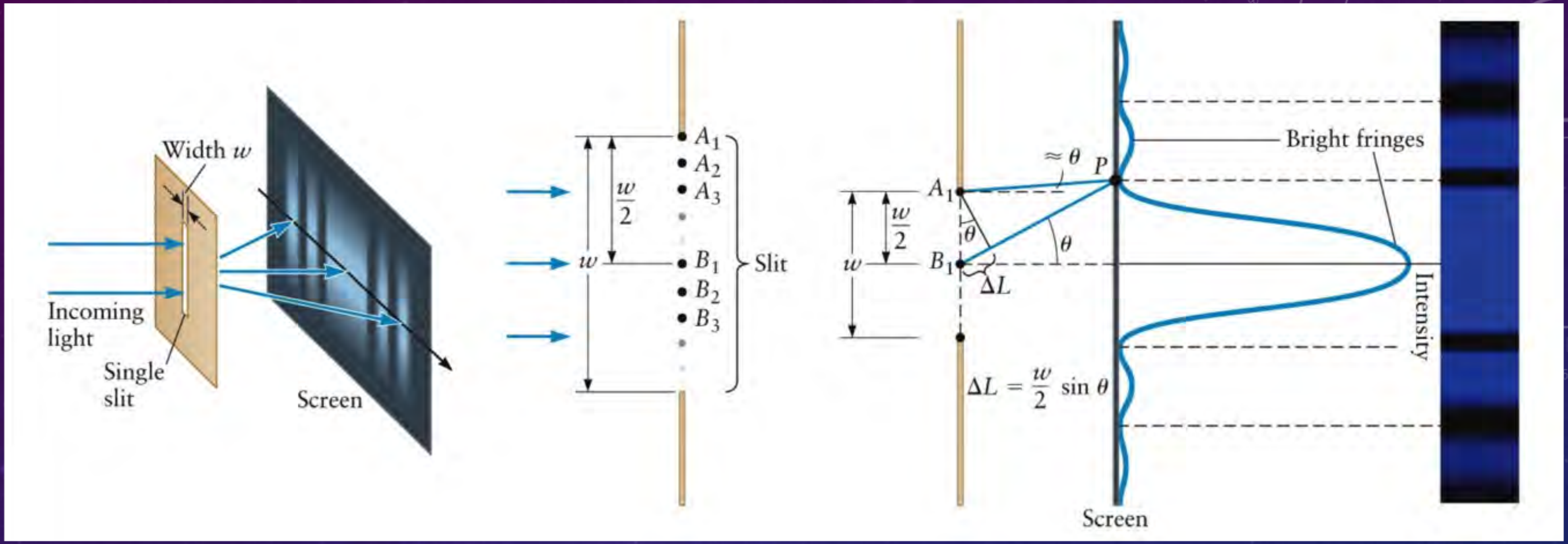
- The two slits act like two sources of spherical waves
- The two waves must be superposed on the screen:

$$\begin{cases} \text{from slit 1: } \exp[i(kL_1 - \omega t)] \\ \text{from slit 2: } \exp[i(kL_2 - \omega t)] \end{cases}$$

$$\text{where: } \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f$$

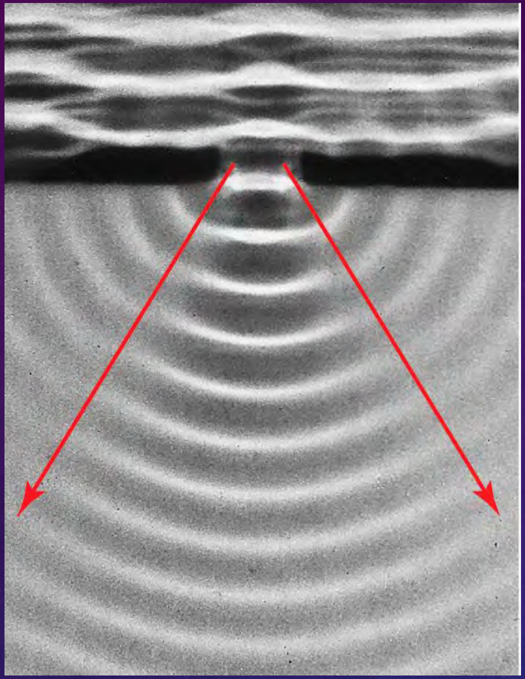
SINGLE SLIT INTERFERENCE

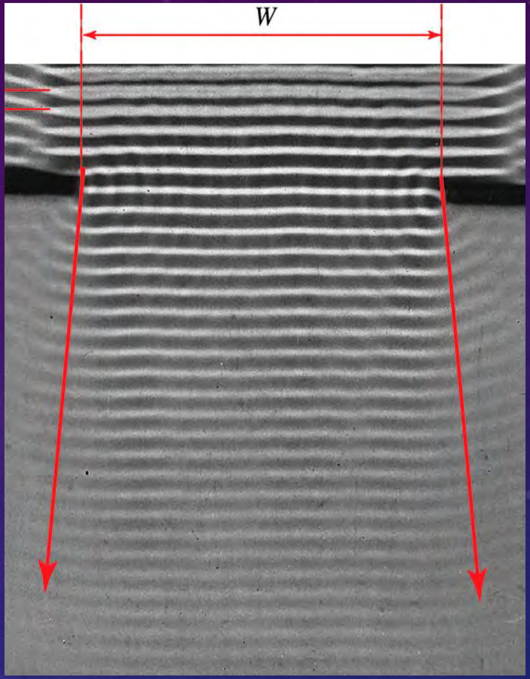
Image from Giordano



- When the slit width is large compared to the wavelength, consider all points in the slit to be point sources of spherical waves
- The important message here is NOT that you can see interference effects with a single slit, but that the width of the central bright fringe decreases as the width of the single slit is increased
- Diffraction becomes negligible when the width of the slit is large compared to the wavelength of the light

IMPORTANCE OF INTERFERENCE





Images from
Cutnell & Johnson

- Interference collimates the beam and prevents the waves from spreading out
- Effective interference requires the light to be monochromatic and coherent
    - This is why flash lights do not make good pointers but lasers do

DIFFRACTION GRATING

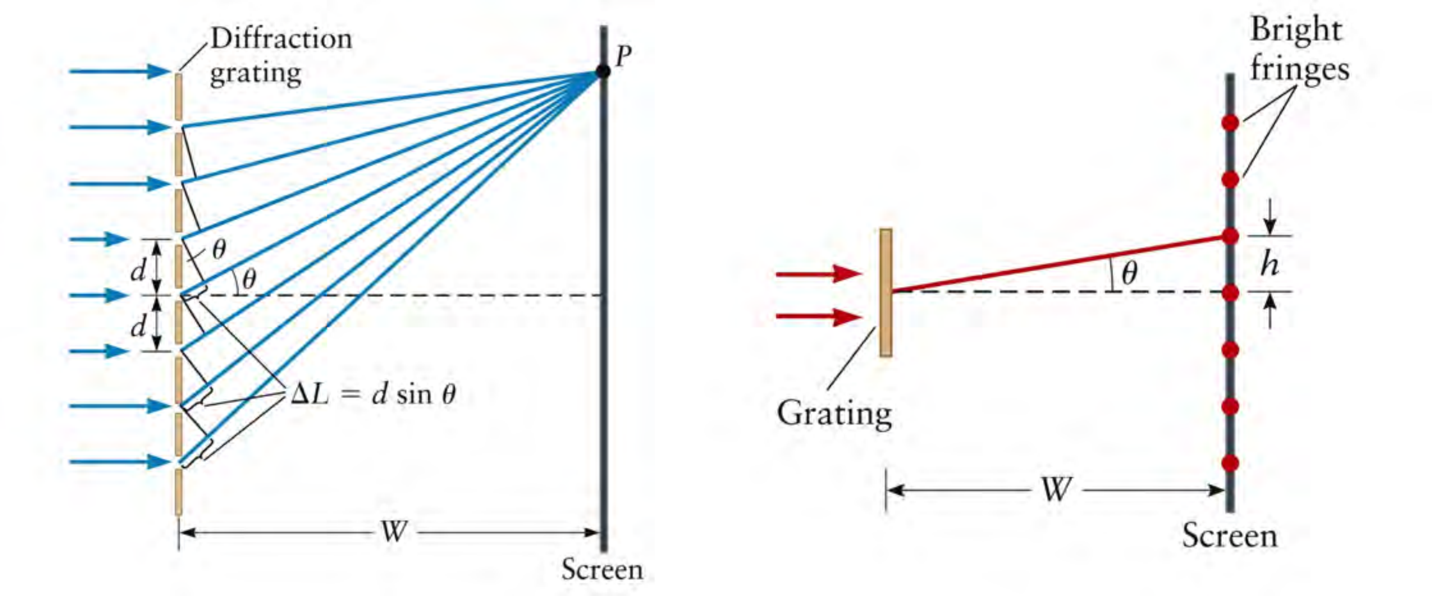


Image from Giordano

- The wide single slit can be considered the limit of a diffraction grating in which the spacing of the slits is taken to zero → only the central bright fringe will remain

THE PATH INTEGRAL

- According to Quantum Mechanics, particles are also waves (wave-particle duality)

- Huygens' Principle should apply to Quantum Waves as well as EM waves!

- NOT to be confused with Contour Integrals in complex analysis!

QUANTUM WAVES

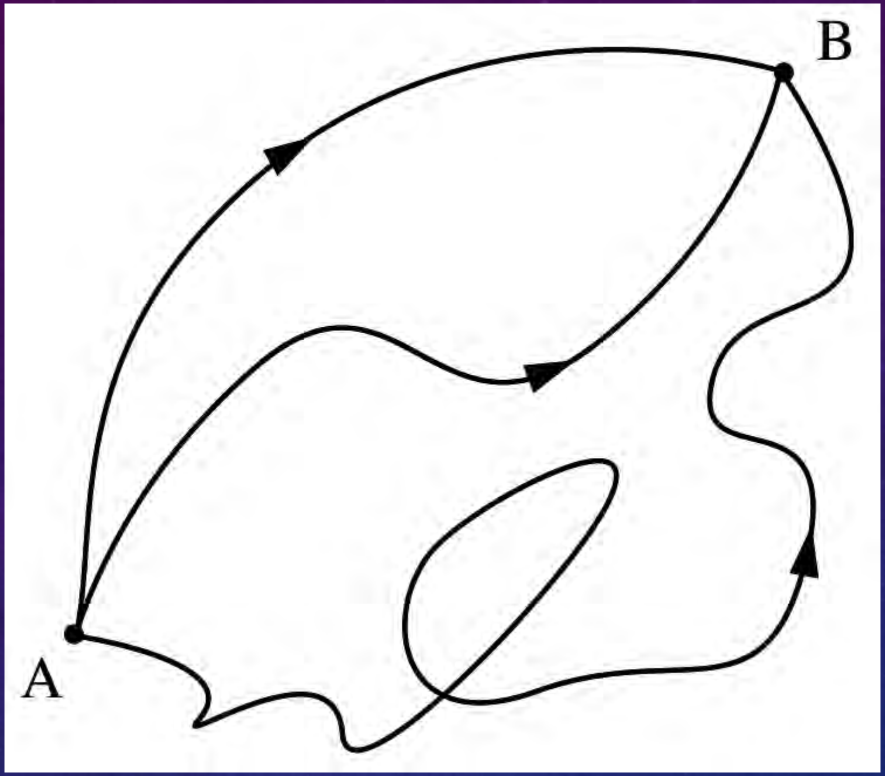
$$p = \hbar k = \frac{2\pi\hbar}{\lambda} = \frac{h}{\lambda}, \qquad E = \hbar\omega = \hbar(2\pi f) = hf$$

$$(kL - \omega t) = \frac{1}{\hbar}\left(pL - Et\right) = \frac{1}{\hbar}\left(\int_{x_i}^{x_f} p\,dx - \int_{t_i}^{t_f} E\,dt\right)$$

$$= \frac{1}{\hbar}\int_{t_i}^{t_f}\left(p\frac{dx}{dt} - E\right)dt = \frac{1}{\hbar}\int_{t_i}^{t_f} L\,dt = \frac{S}{\hbar}$$

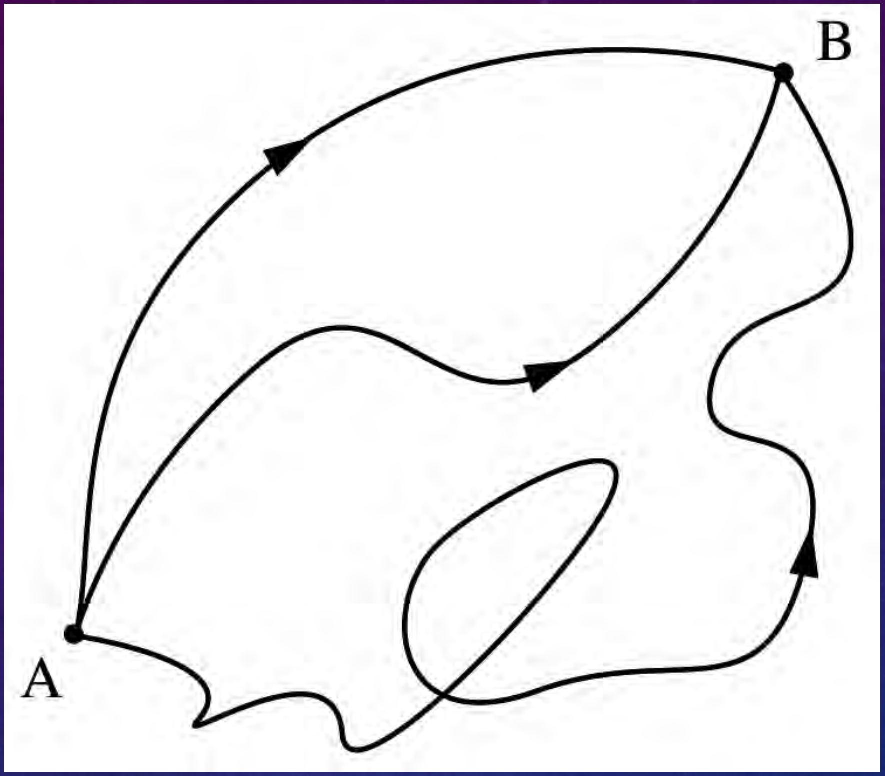
$$E \leftrightarrow H = p\frac{dx}{dt} - L \quad \rightarrow \quad L = p\frac{dx}{dt} - E$$

THE PATH INTEGRAL



- To calculate the probability amplitude of a particle which was at point A at time t_1 to be at point B at time t_2, interfere all the waves propagating along ALL possible paths connecting A at time t_1 to B at time t_2

THE PATH INTEGRAL



$$\langle B(t_2)|A(t_1)\rangle = \int Dx(t)\exp[iS(x(t))/\hbar]$$

THE PATH INTEGRAL

- Note that the Path Integral is over ALL paths:
  - Paths do not have to be straight
  - Includes those in which the particle goes backwards
  - Can go faster than the speed of light

- Huygens' Principle actually applies better to Quantum Waves than EM Waves because the Schrödinger equation is a first order differential equation in time, whereas the wave equation is second order
  - In the case of EM Waves, you cannot let the wave propagate backward

THE PATH INTEGRAL

- Paths in which the particle is moving faster than the speed of light are necessary to obtain the correct answer

- This is not just in non-relativistic Quantum Mechanics

- In Relativistic Quantum Mechanics (and in Lorentz Covariant Quantum Field Theory) the Path Integral must include paths that go outside the light-cone for the Path Integral to give the correct answer!

THE PATH INTEGRAL

- Which Path gives the dominant contribution?

- Recall single slit interference/diffraction grating
  - Many waves with slightly different phases interfering tend to cancel each other out
  - Contributions from paths where $S(x(t))$ is rapidly changing would not contribute
  - The dominant contribution would come from paths where $S(x(t))$ is not changing → $\delta S=0$
  - Classical Mechanics!!

CLOSING THOUGHTS

- When a particle moves from point A to point B, its quantum wave is probing the entire Universe at superluminal speeds!
- The apparent (and rather mundane) phenomenon of the particle moving from point A to point B is a result of the interference of all these quantum waves
- As you walk home today, look up at the stars and think about what your quantum wave is doing