

**PHYSICS 4455 — QUANTUM MECHANICS**  
**Problem Set 7 — due 10/20/2005, in class.**

**The finite potential well in one dimension.**

Consider a structureless, non-relativistic particle of mass  $m$  in one dimension, moving in the presence of a finite square well potential:

$$V(x) = \begin{cases} -V_0 & |x| < L \\ 0 & |x| \geq L \end{cases} .$$

Just like the semi-infinite square well, there are bound and scattering states associated with this one, naturally for  $E < 0$  and  $E > 0$ , respectively.

1. For this problem, find the wavefunctions and energies associated with bound states and discuss how many such states there are for a given  $V_0 L^2$ .

Note that this is a very old and standard problem. Indeed, you will find the entire problem laid out in section 8.1 of Liboff. The point of this HW is that you should try to do the problem by yourself (as opposed to just blindly copying line after line from Liboff). In particular, instead of dealing with determinants of  $4 \times 4$  matrices, try exploiting the idea of parity and see if you can follow my track (without determinants of any matrices, as presented in class for the semi-infinite well).

2. As a prelude to the scattering problem, find the “time delay” (which will be negative here!!! so that “speed-up” would have been a better term) for a *classical* particle, coming into the potential with speed  $v$ . To be precise, let the particle’s motion be described by

$$x = vt - 2L; \quad t \geq 0$$

for the free case ( $V_0 \equiv 0$ ). In other words, at  $t = 0$ , the particle is located a distance  $L$  to the left of the (zero depth) well and moves uniformly to the right. With  $V_0 > 0$ , the particle should have traversed the well region faster, so that, after it emerges from the well, its trajectory should read

$$x = vt - 2L + a.$$

Find  $a$  in terms of  $V_0$  and  $L$ . Expressing  $a$  as  $-vt_d$ , you can write this as

$$x = v(t - t_d) - 2L$$

where  $t_d$  is the (negative) “time delay.”

**You should review:**

Liboff, Chapters 6, 7.1 and 8.1.