Noise-Resistant Control for a Spin Qubit Array

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We develop a systematic method of performing corrected gate operations on an array of exchange-coupled singlet-triplet qubits in the presence of both fluctuating nuclear Overhauser field gradients and charge noise. The single-qubit control sequences we present have a simple form, are relatively short, and form the building blocks of a corrected CNOT gate when also implemented on the interqubit exchange link. This is a key step towards enabling large-scale quantum computation in a semiconductor-based architecture by facilitating error reduction below the quantum error correction threshold for both single-qubit and multiqubit gate operations.

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The prospective scalability of spin qubits in semiconductor quantum dots, along with their demonstrated long coherence times and rapid gate operations, make them a leading candidate for quantum computing architectures. Singlet-triplet qubits [1,2], where the quantum information is encoded in the zero-projection spin states of two electrons in a double quantum dot, are particularly promising due to their insensitivity to stray magnetic fields and their purely electrical controllability. However, precise experimental manipulation of singlet-triplet qubits is hindered by two sources of noise invariably present in all laboratory systems: fluctuations in the background nuclear spin bath due to long-range hyperfine-mediated flip-flop processes (Overhauser noise) [3–5], and fluctuations in the electrostatic quantum dot confinement potential due to background electrons hopping on and off nearby impurity sites (charge noise) [6,7]. These fluctuations are slow ($\sim 100\ \mu$s) compared to typical qubit rotation times (~ 0.1 ns), and this highly non-Markovian characteristic can be exploited to suppress their effects by spin echo or related dynamical decoupling protocols when one does not aim to rotate but instead preserve the qubit state, i.e., quantum memory [8–10]. The ability to achieve very long quantum memory times in this way is one of the great advantages of semiconductor spin qubit architectures. The problem is that the dynamical decoupling scheme to preserve coherence does not work during the qubit gate operations.

Similar protocols for robust qubit rotations are, therefore, highly desirable so that quantum coherence is preserved during the gate operations. However, singlet-triplet qubit control is subject to severe physical constraints that make this task quite daunting. A large interdot Overhauser field gradient introduced by nuclear spin pumping [11–13] (or an actual magnetic field gradient due to a proximal micromagnet [14]) performs rotations about the x axis of the Bloch sphere, but this gradient is not tunable on the time scale of an operation. Meanwhile, electrical control of the interdot exchange coupling by tilting the double-well potential [1] leads to rotation about the z axis of the Bloch sphere, but the sign of the coupling is fixed. The only rapidly tunable element is the magnitude of the exchange coupling, and its range is limited by either the singlet-triplet splitting of two electrons on a single dot, or, more restrictively, by keeping the double well near a symmetric configuration to avoid converting the spin qubit into a fragile charge qubit. Thus, one is restricted to positive rotations about a limited range of axes somewhere between $+\hat{x}$ and $+\hat{z}$, and pulses approximating a delta function are not available. This unique set of constraints prohibits straightforward application of control techniques from the NMR literature (e.g., Refs. [15,16]). The high-fidelity gate operations crucial to scalable quantum computation require a totally new approach to the quantum control problem.

Previously, we have shown that there exists a new form of control sequence that respects these constraints and eliminates the leading-order single-qubit error due to Overhauser noise [17]. Recent numerical work has shown that both relevant types of error can be simultaneously addressed [18]. Despite this progress, there remains a need for a protocol that corrects both Overhauser and charge noise errors while being sufficiently simple and flexible for incorporation into multiqubit operations.

In this Letter, we present a method of pulse design that achieves this goal, systematically eliminating both errors to leading order for any quantum circuit. In the language of the NMR community, the task is similar to correcting arbitrary quantum gate operations for both amplitude and detuning errors simultaneously. This task has only recently begun to be addressed in NMR [19,20], and it is remarkable that the far more restricted case of singlet-triplet qubits permits a simple solution, as revealed by our method. Furthermore, our new pulse sequences are an order of magnitude faster than earlier, less-powerful sequences [17]. We begin by showing how to perform universal, robust, single-qubit gates. We then demonstrate how to use these single-qubit pulse sequences to generate a CNOT gate that possesses the same resilience against errors. Finally, we show how to combine the prior two results to generate universal, multiqubit, dynamically corrected operations on a large-scale quantum register. Thus, our work forms a complete prescription for compensating low-frequency noise in ongoing singlet-triplet quantum computation experiments.

The model Hamiltonian within the logical subspace of a singlet-triplet qubit is written in terms of the Pauli operators σ as

$$H(t) = \frac{h}{2}\sigma_x + \frac{J[\epsilon(t)]}{2}\sigma_z, \tag{1}$$

with $h = g\mu_B \Delta B_z$ the energy associated with the average magnetic field gradient across the double dot and J the positive, bounded exchange coupling. The exchange is a function of the energy difference between balanced and imbalanced singlet charge states, ϵ, which can be controlled dynamically [1]. Since both Overhauser and charge noise are typically several orders of magnitude slower than gate times, the resulting perturbations about h and $\epsilon(t)$, δh and $\delta\epsilon$, respectively, are treated as random constants.

A single-qubit rotation of angle ϕ about axis $h\hat{x} + J\hat{z}$, $R(h\hat{x} + J\hat{z}, \phi)$, naïvely performed by holding the exchange constant over some time interval, results in errors Δ_i (see Supplemental Material for the explicit formulas [21]),

$$\begin{aligned} U(J,\phi) &\equiv \exp\left[-i\left(\frac{h+\delta h}{2}\sigma_x + \frac{J+\delta J}{2}\sigma_z\right)\frac{\phi}{\sqrt{h^2+J^2}}\right] \\ &= \exp\left[-i\left(\frac{h}{2}\sigma_x + \frac{J}{2}\sigma_z\right)\frac{\phi}{\sqrt{h^2+J^2}}\right]\left(I - i\sum_i \Delta_i\sigma_i\right). \end{aligned} \tag{2}$$

Here h and δh are assumed to be independent of J, and $\delta J = \delta\epsilon \frac{\partial J(\epsilon)}{\partial\epsilon}|_{J(\epsilon)=J}$ arises from fluctuations in the background impurity potential and hence in detuning, $\delta\epsilon$.

Our general strategy is to construct an identity operation such that the error in its implementation exactly cancels the leading order error in the original rotation. For example, a 2π rotation interrupted by a 2π rotation about a different axis (the SUPCODE identity of Ref. [17]) allows three degrees of freedom with which to tune the error: the two axes and the point of interruption. Here we use a similar concept in a more general form to compensate for all error sources. First, note that a $2m_n\pi$ rotation interrupted by a general zeroth order identity gives a new zeroth order identity with different first order errors,

$$\begin{aligned} &U(j_n, m_n\pi + \theta_n)\left(I - i\sum_i \delta_i^{(n-1)}\sigma_i\right)U(j_n, m_n\pi - \theta_n) \\ &\quad = I - i\sum_i \delta_i^{(n)}\sigma_i. \end{aligned} \tag{3}$$

This defines a recursion relation for the error of a "level-n" parameterized identity,

$$\begin{aligned} &U(j_n, m_n\pi + \theta_n)\ldots U(j_1, m_1\pi + \theta_1)U(j_0, 2m_0\pi) \\ &\quad \times U(j_1, m_1\pi - \theta_1)\ldots U(j_n, m_n\pi - \theta_n). \end{aligned} \tag{4}$$

The recursion equations are straightforward to generate [21], but their explicit form is lengthy and unnecessary for the discussion here.

We find that a level-5 identity contains enough flexibility to obtain errors which exactly cancel those of the naïve rotation $U(J, \phi)$ to leading order in both δh and $\delta\epsilon$. This is the central new result of our work, and we choose a simple form for this compensating identity,

$$\begin{aligned} &U\left(J, \pi + \frac{\phi}{2}\right)U(j_4, \pi)U(j_3, \pi)U(0, \pi)U(j_1, \pi)U(j_0, 4\pi) \\ &\quad \times U(j_1, \pi)U(0, \pi)U(j_3, \pi)U(j_4, \pi)U\left(J, \pi + \frac{\phi}{2}\right) \\ &\quad = \exp\left[-i\left(\frac{h}{2}\sigma_x + \frac{J}{2}\sigma_z\right)\frac{\phi}{\sqrt{h^2+J^2}}\right] + \mathcal{O}[(\delta h + \delta\epsilon)^2]. \end{aligned} \tag{5}$$

Here we have taken $j_5 = J$, $\theta_{1,2,3,4} = 0$, and $\theta_5 = -\phi/2$ so that the overall sequence is symmetric and $\Delta_y + \delta_y^{(5)} = 0$. We have arbitrarily set $j_2 = 0$ for simplicity. The remaining parameters j_0, j_1, j_3, j_4 are determined by numerical solution of the four coupled, nonlinear equations that set the coefficients of δh and $\delta\epsilon$ in both $\Delta_x + \delta_x^{(5)}$ and $\Delta_z + \delta_z^{(5)}$ to be zero [21].

An example of this sequence, along with the dependence of the parameters on the rotation angle, is shown in Fig. 1. While there are multiple possible solutions, we have constrained our numerical solution to give only positive values of the exchange coupling within a typically experimentally accessible region. In all cases, our restrictions to physically practical sequences still permit error compensation. The suppression of error is evident in Fig. 2. The corrected sequence infidelity scales as the fourth (instead of second) power of the fluctuations. (Hence, if one reinterprets δh and $\delta\epsilon$ as standard deviations of Gaussian distributions, the plotted corrected sequence infidelities should be multiplied by a factor of three.) For fluctuations on the order of a few percent, our pulse sequence suppresses gate error by two orders of magnitude.

The results shown here are for the usual empirical model at negative ϵ, $J(\epsilon) \propto J_0 + J_1\exp(\epsilon/\epsilon_0)$ with $J_0 \sim 0$ so $\delta J/J = \delta\epsilon/\epsilon_0$ [22,23]. However, our method does not require this assumption, and is equally valid for other models of $J(\epsilon)$, as we have also verified explicitly,

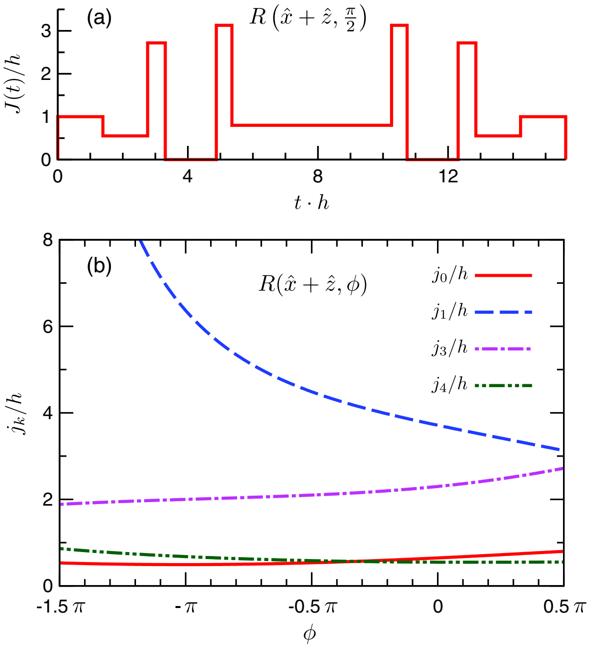

FIG. 1 (color online). Rotations about $\hat{x}+\hat{z}$. (a) Example pulse sequence for $\pi/2$ rotation. (b) Parameters vs angle, ϕ.

including models where the exchange cannot be tuned all the way to zero. The important requirement is simply that there is a model so that the correlations between the δJs at different values of J are known. Also, although we have taken the pulse sequence to be piecewise constant, we have checked that including a finite rise time only introduces a perturbation about the parameter values plotted. In an experimental context, then, an optimization of the actual parameters around the ideal pulse described here should quickly converge to the adjusted sequence for that particular real setup.

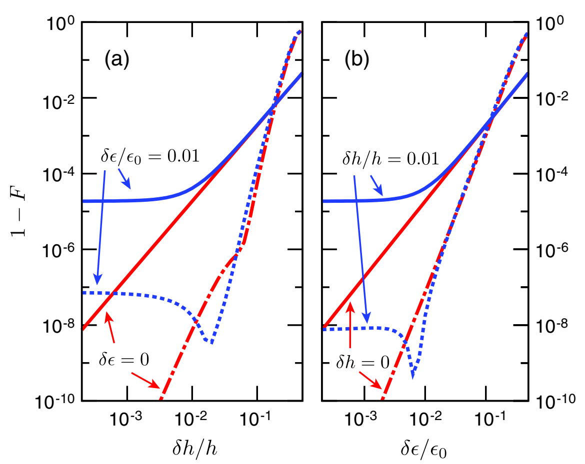

FIG. 2 (color online). Gate infidelity of naïve (solid lines) and corrected (dashed lines) $\pi/2$ rotations about $\hat{x}+\hat{z}$ vs (a) Overhauser field gradient fluctuations, $\delta h/h$, and (b) detuning fluctuations, $\delta\epsilon/\epsilon_0$.

The particular realization of the identity above is not a unique solution—we had more parameters than equations and simply chose values for which solutions were easy to find. Further optimization may yield an even shorter solution. However, this identity uses only 14π of total rotation and is already quite efficient. (Note that an $\hat{x}+\hat{z}$ rotation with the original SUPCODE sequence [17], only correcting δh, naïvely required 120π of total rotation in the identities, making the current protocol more than an order of magnitude more efficient than the original.)

So far, we have only discussed correcting rotations about axes of the form $h\hat{x}+J\hat{z}$. Given the experimental constraints on h and J, this only allows direct access to a segment of the positive x-z quarter-plane. Clearly, one can build an arbitrary corrected single-qubit rotation from a string of the corrected rotations discussed above. More efficiently, though, one could also build it from a string of uncorrected rotations about the accessible axes and then correct the total error with a single compensating identity rather than correcting each segment individually. We have found that more general rotations can be corrected using a level-6 identity [21].

Now we turn to the construction of corrected two-qubit gates on a pair of singlet-triplet qubits. Two-qubit gates have been demonstrated experimentally via capacitive coupling [22], and proposed theoretically via exchange coupling [24,25]. We will consider the latter case with neighboring qubits A, consisting of dots 1 and 2, and B, consisting of dots 3 and 4. The qubits interact via an exchange link between dots 2 and 3. In recent work, Ising gates are constructed in the absence of noise such that phases due to static local Overhauser fields cancel, and only a state-dependent phase from the exchange pulse survives [24],

$$\begin{aligned} U_{xx}(\alpha) &\equiv R^{(A)}(\hat{z},\pi)R^{(B)}(\hat{z},\pi)C_{23}\left(\frac{\alpha}{2}\right) \\ &\quad\times R^{(A)}(\hat{z},\pi)R^{(B)}(\hat{z},\pi)C_{23}\left(\frac{\alpha}{2}\right) \\ &= \exp\left(i\frac{\alpha}{2}\sigma_x^{(A)}\sigma_x^{(B)}\right)+\mathcal{O}(\delta h,\delta\epsilon), \qquad (6) \end{aligned}$$

where $R^{(A/B)}(\hat{r},\theta)$ denotes a rotation of qubit A/B by θ about $\hat{r}$ on the singlet-triplet Bloch sphere. $C_{23}(\alpha/2)$ denotes the application of a pulse to the interqubit link such that in an $S_z=0$ subspace it would act as a 2π rotation about some axis. This is required to avoid swapping antialigned spins on dots 2 and 3, which would cause leakage out of the logical qubit subspace. The choice of axis is determined by fixing the total interqubit pulse area, $\int dt J_{23}(t)=\alpha/2$, which causes the desired relative phase to be acquired by two-qubit states with aligned electron spins on dots 2 and 3.

By using our compensated pulse sequences to perform the $R^{(A/B)}(\hat{z},\pi)$ operations—and performing them on A and B with equal durations such that both qubits are idle for

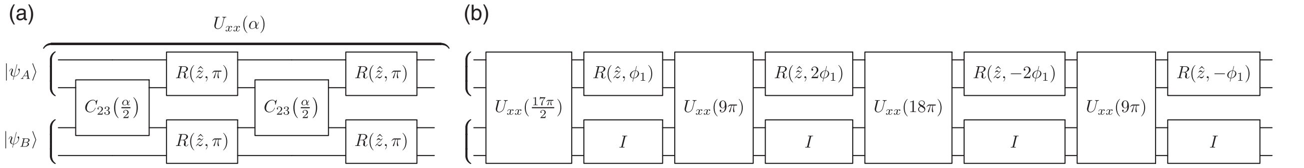

FIG. 3. Quantum circuits for (a) Overhauser-corrected and (b) fully corrected two-qubit Ising gates. Each block represents a rotation carried out by a composite pulse.

the same amount of time—single-qubit errors on the right-hand side of Eq. (6) are eliminated to first order. For equal gradients on both qubits, the equal time condition is trivially satisfied. More generally, the π rotations on A and B would have different durations and one would pad them with corrected 2π rotations about different axes (i.e., having different durations) to compensate [21].

Also implementing the $C_{23}(\alpha/2)$ with our pulse sequences suppresses leakage error, but the angle α is still sensitive to charge noise induced exchange amplitude error. This is because our sequences only correct rotations within the $S_z = 0$ subspace of the $SU(2)^{\otimes 2}$ space on which C_{23} acts. The problem at this stage, though, is one familiar from NMR contexts—pulse length error in Ising-coupled spins [26]. Defining a tilted version of the Ising gate,

$$U'_{xx}(\alpha, \phi) \equiv R^{(A)}(\hat{z}, -\phi) U_{xx}(\alpha) R^{(A)}(\hat{z}, \phi), \tag{7}$$

the interqubit exchange amplitude error can be corrected using a BB1 sequence [15,26]. Typically we only report angles modulo 2π, but because the error is proportional to α and using our composite pulses results in a large angle $\alpha = 8\pi + \mathrm{mod}\,(\alpha, 2\pi)$, we must use a slightly modified version of the BB1 sequence:

$$\begin{aligned} &U'_{xx}(\theta_1, \phi_1) U'_{xx}(2\theta_1, 3\phi_1) U'_{xx}(\theta_1, \phi_1) U_{xx}(\alpha) \\ &\quad = \exp\left(i \frac{\alpha}{2} \sigma_x^{(A)} \sigma_x^{(B)}\right) + \mathcal{O}[(\delta h + \delta\epsilon)^2], \end{aligned} \tag{8}$$

where $\theta_1 = 8\pi + \pi$, $\phi_1 = \arccos[-\alpha/4(8\pi + \pi)]$ [27], and all rotations are corrected composites. For $\alpha = 8\pi + \pi/2$, this gate is equivalent to a CNOT gate up to single-qubit operations [24,25]. In practice, we also perform corrected identities on qubit B during the rotations on A in Eq. (7) to avoid reintroducing single-qubit errors.

We sketch the implementation of the Ising $\pi/2$ gate in Fig. 3. We have displayed the case of a linear gradient, after trivial contractions of sequential single-qubit operations in Eq. (8). Each line represents a quantum dot, which may be linked to its neighbor by an exchange pulse. A singlet-triplet qubit then is denoted by a pair of lines which are understood to have total spin projection zero. The error suppression for a CNOT gate is shown in Fig. 4. For the sake of display, we have reduced the number of parameters by assuming a uniformly fluctuating linear field gradient across the four dots, as well as identical detuning fluctuations on each link.

As is always the case with composite pulse schemes, the cost of the correction is longer gate time. Currently, our shortest implementation of the corrected CNOT gate requires 20 composite pulses, corresponding to $\sim 300\pi$ of total rotation. While certainly challenging, this is well within the realm of possibility given that our sequence suppresses decoherence during its operation. Recall that statistical fluctuations of the control Hamiltonian lead to rapid decoherence on the free induction time scale, T_2^*. Dynamical decoupling can extend coherence to a much longer time T_2, though, where dynamical fluctuations become important. Roughly speaking, gate error scales like the ratio of gate time to coherence time. Since typically $T_2 \geq 10^4 T_2^*$ in semiconductor spin systems near the charge degeneracy point [1,2,8,28], if one works in that regime it is well worth using longer gate implementations that simultaneously perform dynamical decoupling such that the relevant coherence time is close to T_2 rather than T_2^*. (Although note that if the qubit is operated in a region where J is more sensitive to ϵ, as done intentionally in Ref. [22], T_2 will decrease in the absence of charge noise correction.) Stated differently, the ratio of the time scale on which dynamical fluctuations can be expected to the time duration of a simple π rotation can be roughly estimated as $\sim 10^6$, so it is logical to sacrifice some rotation time to gain additional precision.

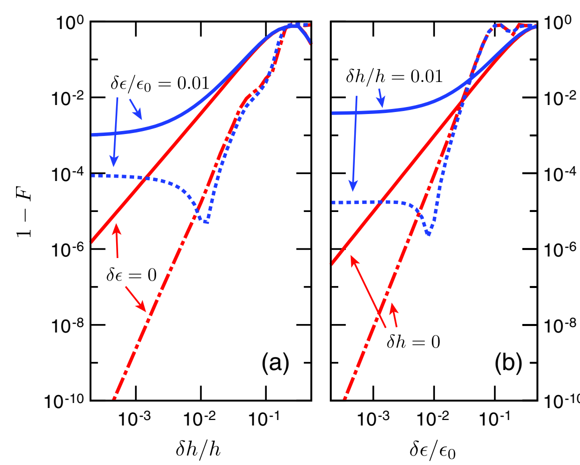

FIG. 4 (color online). Gate infidelity [21] of uncorrected (solid lines) and corrected (dashed lines) CNOT operations vs (a) Overhauser field gradient fluctuations, $\delta h/h$, and (b) detuning fluctuations, $\delta\epsilon/\epsilon_0$.

Since each experiment will have its own preferred region of control space, further optimization would be premature, but future work will almost certainly reduce the length in any case. Also, it may well be possible to construct a "one-shot" correction for the CNOT gate, as we have done for the composite single-qubit rotations, rather than correcting at each intermediate stage. These extensions, however, are beyond the scope of the present work.

Finally, we discuss application to an array of exchange-coupled qubits. Our method facilitates operations below the quantum error correction threshold, a vital requirement for scaling up. Arbitrary corrected multiqubit circuits can be implemented similarly to Fig. 3 as long as there exist single-qubit corrected identities of sufficiently variable duration to protect the many idle qubits during the application of nontrivial gates to other qubits. We find that level-6 identities can be used to cover the entire range of relevant idling times [21]. An additional benefit of using our sequences in large systems is that errors due to small spatial inhomogeneities in the control Hamiltonian (1) are also suppressed.

In summary, we have shown above that the relevant types of error in experimental singlet-triplet spin qubit manipulation can be eliminated to leading order by a new composite pulse sequence. We show how to apply this sequence for single-qubit, two-qubit, and multiqubit systems, opening a path towards scalable, universal quantum computation in a noisy solid-state environment.

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