9.11 Three charges are located along the z axis, a charge $+2q$ at the origin, and charges $-q$ at $z = \pm a\cos\omega t$.
Determine the lowest nonvanishing multipole moment, the angular distribution of radiation, and the total power radiated.
Assume $ka \ll 1$.

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$$\rho = q\left[2\delta(z) - \delta(z - a\cos\omega t) - \delta(z + a\cos\omega t)\right]\delta(x)\delta(y)$$

$$\bar{J} = \hat{z} q a\omega \sin\omega t\left[\delta(z - a\cos\omega t) - \delta(z + a\cos\omega t)\right]\delta(x)\delta(y)$$

$$\bar{p} = \int d^3x\, \bar{x}\rho = -q\hat{z}\left[a\cos\omega t - a\cos\omega t\right] = 0$$

$$\bar{m} = \frac{1}{2}\int d^3x\, \bar{x}\times\bar{J} \propto \hat{z}\times\hat{z} = 0$$

$$Q_{ij} = \int d^3x\, \rho(3x_i x_j - r^2\delta_{ij})$$

$$= -q\, a^2\cos^2\omega t\,(3\delta_{i3}\delta_{j3} - \delta_{ij})\,(2)$$

$$\Rightarrow \boxed{\begin{aligned} Q_{33} &= -4q\,a^2\cos^2\omega t \\ Q_{11} = Q_{22} &= +2qa^2\cos^2\omega t \end{aligned}}$$

Write $\cos^2\omega t = \frac{1}{2}(1 + \cos 2\omega t)$

The zero-frequency part will not contribute to the radiation, so henceforth we focus on the frequency $2\omega$ component.

Write (the relevant part of)

$$Q_{33} = \text{Re}\left(-2qa^2 e^{-2i\omega t}\right)$$

$$Q_{11} = Q_{22} = \text{Re}\left(+qa^2 e^{-2i\omega t}\right)$$

9.11, cont'd

We can then apply standard results for radiation from harmonically-varying electric quadrupoles:

$$\boxed{\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^6}{512\pi^2} \underbrace{(Q_{33})^2}_{(-2qa^2)^2} \sin^2\theta \cos^2\theta} \qquad (9.51)$$

$$\boxed{= \frac{c^2 Z_0 k^6 q^2 a^4}{128\pi^2} \sin^2\theta \cos^2\theta} \qquad \text{for } \underline{k = \frac{2\omega}{c}}$$

$$\boxed{P = \int d\Omega \frac{dP}{d\Omega}}$$

$$= \frac{c^2 Z_0 k^6 q^2 a^4}{128\pi^2} (2\pi) \underbrace{\int_{-1}^{1} d(\cos\theta) \cos^2\theta (1 - \cos^2\theta)}_{\frac{1}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta \Big]_{-1}^{1}} = 2\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{4}{15}$$

$$\boxed{= \frac{c^2 Z_0 k^6 q^2 a^4}{240\pi}}$$