Physics 5455 – Problem set 11

1. **Perturbed two-dimensional oscillator.** Consider a symmetric two-dimensional harmonic oscillator

$$H_0 \;=\; \frac{p_x^2 + p_y^2}{2m} \;+\; \frac{1}{2}m\omega^2(x^2+y^2) \;=\; \hbar\omega(a^\dagger a \;+\; b^\dagger b \;+\; 1)$$

with perturbation $H_1 = \lambda xy$. Calculate the first-order correction to the energy eigenvalues of the triply degenerate second excited state with $n = n_x + n_y = 2$, and find the corresponding "good" eigenstates.

2. **Quadratic Stark effect in the hydrogen ground state.** A hydrogen-like atom or ion in the ground state is placed in a homogeneous electric field. Treat the electric field potential energy

$$H_1 \;=\; eE_0 z$$

as a small perturbation.

   (a) Determine the first-order correction to the ground state wavefunction directly from the relation

$$(H_0 - E_n^0)|n^1\rangle \;=\; -(H_1 - E_n^1)|n^0\rangle$$

   Hint: You are being asked to solve a differential equation. Use (and motivate) the following solution ansatz:

$$\psi^1_{100}(r,\theta,\phi) \;=\; \psi^0_{100}(r)f(r)Y_{1,0}(\theta,\phi)$$

   with $f(r) = A + Br + Cr^2$.

   (b) Using your wavefunction from (a), directly show that the second-order shift of the ground state energy is

$$\langle 100^0|H_1|100^1\rangle \;=\; -\frac{9a^3E_0^2}{4Z^4}$$

3. **WKB approximation for perturbed infinite square well.** Find the approximate energy eigenvalues for the perturbed infinite potential well

$$V(x) \;=\; \begin{cases} V_0 > 0 & 0 \le x \le a/2 \\ 0 & a/2 < x \le a \\ \infty & x < 0, x > a \end{cases}$$

within the WKB approximation, assuming $E > V_0$, and compare with the corresponding result from first-order perturbation theory.

Hint: you don't need to repeat the derivation from class, just use the Bohr-Sommerfeld quantization condition.