Physics 5455 – Problem set 8

1. **Charged particle in a homogeneous electric field.** A particle with mass m and charge q is placed in a homogeneous electric field $\vec{E} = E_0\hat{x}$ (in three dimensions).

   (a) What is its potential energy? What sort of energy eigenstate spectrum (*e.g.* discrete, ...) would you expect for this problem?

   (b) Find the Hamiltonian in the momentum representation and solve the time-independent Schrödinger equation for eigenfunctions $\varphi_E(\vec{p})$. Normalize them.

   (c) From the result in (b), obtain the corresponding spatial wavefunctions. Hint: you should arrive at Airy functions

$$\mathrm{Ai}(z) \;=\; \frac{1}{\pi}\int_0^\infty \cos\left(\frac{s^3}{3} \;+\; sz\right) ds$$

2. **Cylindrical quantum wire.** Consider a quantum wire along the x axis with a confining potential

$$V(\vec{r}) \;=\; \begin{cases} -V_0 & 0 \le y^2+z^2 \le a^2 \\ 0 & y^2+z^2 > a^2 \end{cases}$$

   (V_0 constant, $V_0 > 0$). Through separation of variables, obtain the form of the bound states in this potential. Write down the conditions that would determine the binding energies, but do not attempt a solution. Hint: you should arrive at Bessel functions. Second hint: asymptotically,

$$I_n(x) \;\sim\; \frac{e^x}{\sqrt{2\pi x}}, \quad K_n(x) \;\sim\; \frac{e^{-x}}{\sqrt{2\pi x}}$$

   as $x \to \infty$.

3. **Spherical delta shell potential.** Consider a quantum particle in three dimensions, interacting with a spherical delta shell potential

$$V(r) \;=\; -\lambda\delta(r-a)$$

   ($a, \lambda > 0$).

   (a) Obtain its s-wave ($\ell = 0$) scattering states and the reflection amplitude and coefficient for an incoming radial wave $\exp(-ikr)$.

   (b) Derive the condition relating λ to E for the existence of a bound state with $\ell = 0$. Determine its normalized wavefunction.