Physics 5456 – Problem set 4

1. **Sudden approximation: α decay** When an atomic nucleus undergoes α decay, its nuclear charge changes suddenly from Z to $Z-2$. Assume the electronic state of the atom can be described by the Coulomb potential eigenstates for a single electron, and use the sudden approximation to compute

   (a) the probability for the atom to remain in the ground state,

   (b) the transition probability from the 1s into the 2s and 2p final states.

2. **Harmonic oscillator transition rates** A particle with mass m and charge q is confined by a one-dimensional harmonic oscillator potential $V(x) = (1/2)m\omega^2 x^2$, and exposed to a weak electric field $E_x(t)$.

   (a) Determine the time-dependent perturbation Hamiltonian $H'(t)$, if

   $$E_x(t) = E_0 \exp(-t^2/\tau^2).$$

   (b) Compute the transition probability induced by this electric field from the ground state (at $t = -\infty$) to any excited state ($t = +\infty$). Discuss the limits $\omega\tau \to 0$, $\omega\tau \to \infty$.

   (c) Consider a weak oscillating field $E_x(t) = 2E_0 \cos(\Omega t)\theta(t)$ (where $\theta(t)$ is the unit step function, nonzero for $t > 0$). Find the transition probability from the ground state to any excited state at short times, and determine the corresponding transition rates as $t \to \infty$.