Physics 5456 – Problem set 6

1. Consider scattering in the potential

$$V(r) = \begin{cases} 0 & r > R, \\ V_0 & r < R, \end{cases}$$

for some constant V_0. Use the Born approximation to compute the scattering amplitude $f_k(\theta, \phi)$.

2. **Hard sphere scattering** Consider scattering off a rigid sphere defined by the potential

$$V(r) = \begin{cases} 0 & r > R, \\ \infty & r < R. \end{cases}$$

   (a) By considering the boundary condition on the wavefunction, derive an exact expression for $\tan \delta_\ell$.

   (b) Derive a simple expression for the phase shift δ_ℓ for $\ell = 0$. (HInt: refer to Schwabl section 17.2 for information on spherical Bessel functions.) (For credit, do not merely refer to your answer to the prevous part and say, set $\ell = 0$.)

   (c) Estimate δ_ℓ for small kR, and show that in this regime, the largest phase is δ_0.

   (d) Compute the s-wave contribution to the total scattering cross section, and compare to the geometric cross-section of the rigid sphere in the limit of small kR.