Physics 5714 – Problem set 1

(AWH = Arfken-Weber-Harris, 7th edition)

1. Verify the expansion of the triple vector product

$$\vec{A} \times (\vec{B} \times \vec{C}) \;=\; \vec{B}(\vec{A} \cdot \vec{C}) \;-\; \vec{C}(\vec{A} \cdot \vec{B})$$

2. (AWH 3.5.1) If $S(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$, find
   (a) ∇S at the point $(1, 2, 3)$
   (b) the magnitude of the gradient of S, $|\nabla S|$, at $(1, 2, 3)$
   (c) the direction cosines of ∇S at $(1, 2, 3)$

3. (AWH 3.5.2)
   (a) Find a unit vector perpendicular to the surface

$$x^2 + y^2 + z^2 \;=\; 3$$

   at the point $(1, 1, 1)$

   (b) Derive the equation of the plane tangent to the surface at $(1, 1, 1)$

4. (AWH 3.5.6) For a particle moving in a circular orbit $\vec{r} = \hat{x} r \cos(\omega t) + \hat{y} r \sin(\omega t)$, ($r$, ω constant)
   (a) evaluate $\vec{r} \times \dot{\vec{r}}$
   (b) Show that

$$\frac{d^2}{dt^2}\vec{r} \;+\; \omega^2 \vec{r} \;=\; 0$$

5. (AWH 3.5.9) Show

$$\nabla \cdot (\vec{A} \times \vec{B}) \;=\; \vec{B} \cdot (\nabla \times \vec{A}) \;-\; \vec{A} \cdot (\nabla \times \vec{B})$$

   (Hint: treat as a triple scalar product.)

6. (AWH 3.6.5) Verify the vector identity

$$\nabla \times (\vec{A} \times \vec{B}) \;=\; (\vec{B} \cdot \nabla)\vec{A} \;-\; (\vec{A} \cdot \nabla)\vec{B} \;-\; \vec{B}(\nabla \cdot \vec{A}) \;+\; \vec{A}(\nabla \cdot \vec{B})$$