Physics 5714 – Problem set 6

1. (AWH 2.2.20) If A^{-1} has elements

$$(A^{-1})_{ij} \;=\; \frac{1}{\det A}(-)^{i+j}M_{ji}$$

(where the M_{ji} are the minors, transposed), then show that $A^{-1}A = 1$. (To be clear, since this is A^{-1}, of course $A^{-1}A = 1$. What I want you to do is to check that this follows from the expression above for A^{-1}, to better understand minors and cofactors. Answers stating simply that "A^{-1} is the inverse of A" will receive zero credit.)

2. Show that $\det(A^{-1}) = (\det A)^{-1}$.

3. Let V be the vector space of polynomials of degree ≤ 2, *i.e.*

$$c_0 \;+\; c_1 x \;+\; c_2 x^2$$

with dot product defined by

$$f(x)\cdot g(x) \;=\; \int_0^1 f(x)g(x)dx$$

Use the Gram-Schmidt procedure to construct an orthonormal basis from the linearly independent vectors

$$\left\{1,\; x\,+\,1,\; x^2\,+\,x\right\}$$