Aspects of 2d (0,2) theories

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arXiv:1506.07307

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Exact non-perturbative RG flow in (0,2) theories

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What makes them special?

- 2d (0,2) theories have two supercharges of the same spin
- Naturally, the algebra leads to chiral theories.
- Global symmetries have non-vanishing 't Hooft anomaly, which implies existence of gapless modes
- Autonomous theory of the gapless modes is a conformal field theory.
- Generically, (0,2) theories flow to non-trivial conformal fixed points.

Behavior of the microscopic gauge theory

- In 2 dimensions, gauge coupling is dimensionful. It is classically relevant.
- In addition, there could be other classically relevant interactions (Superpotential terms that lead to Yukawa type interactions)
- Microscopic gauge theory undergoes non-trivial RG flow.
- In fact, RG flow naturally splits into two stages.

Stages of the RG flow

Fast Flow

- During this stage, the dimensionful parameters, including gauge coupling, flow rapidly to infinity.
- A good description of the theory at the end of this stage is in terms of a non-linear sigma model

Slow Flow

- The sigma model has Kahler and complex structure moduli that are classically dimensionless.
- The next stage of RG flow is in this space. At one loop it is logarithmic. Eventually takes the theory to the fixed point.

A picture of the RG flow

Gauge theory

↓ Fast: $g_{\mathrm{YM}} \to \infty$, $m \to \infty$

NLSM

$E \to M$

↓ Slow: t_{Kahler}, t_{Complex}

CFT

- Normalizable vacuum gives rise to state operator correspondence for the CFT
- Global symmetries are promoted to affine Kac-Moody symmetries
- One way to ensure it is to make sure that the sigma model target is compact

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+\Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\bar{\theta}^+\partial_+\phi$$

  - Complex scalar
  - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+\Psi = 0$

$$\Psi = \psi_- - \sqrt{2}\theta^+ G - i\theta^+\bar{\theta}^+\partial_+\psi_-$$

  - Complex left-moving fermion

- Vector multiplet:

  - Gauge invariant d.o.f.: Fermi multiplet Λ

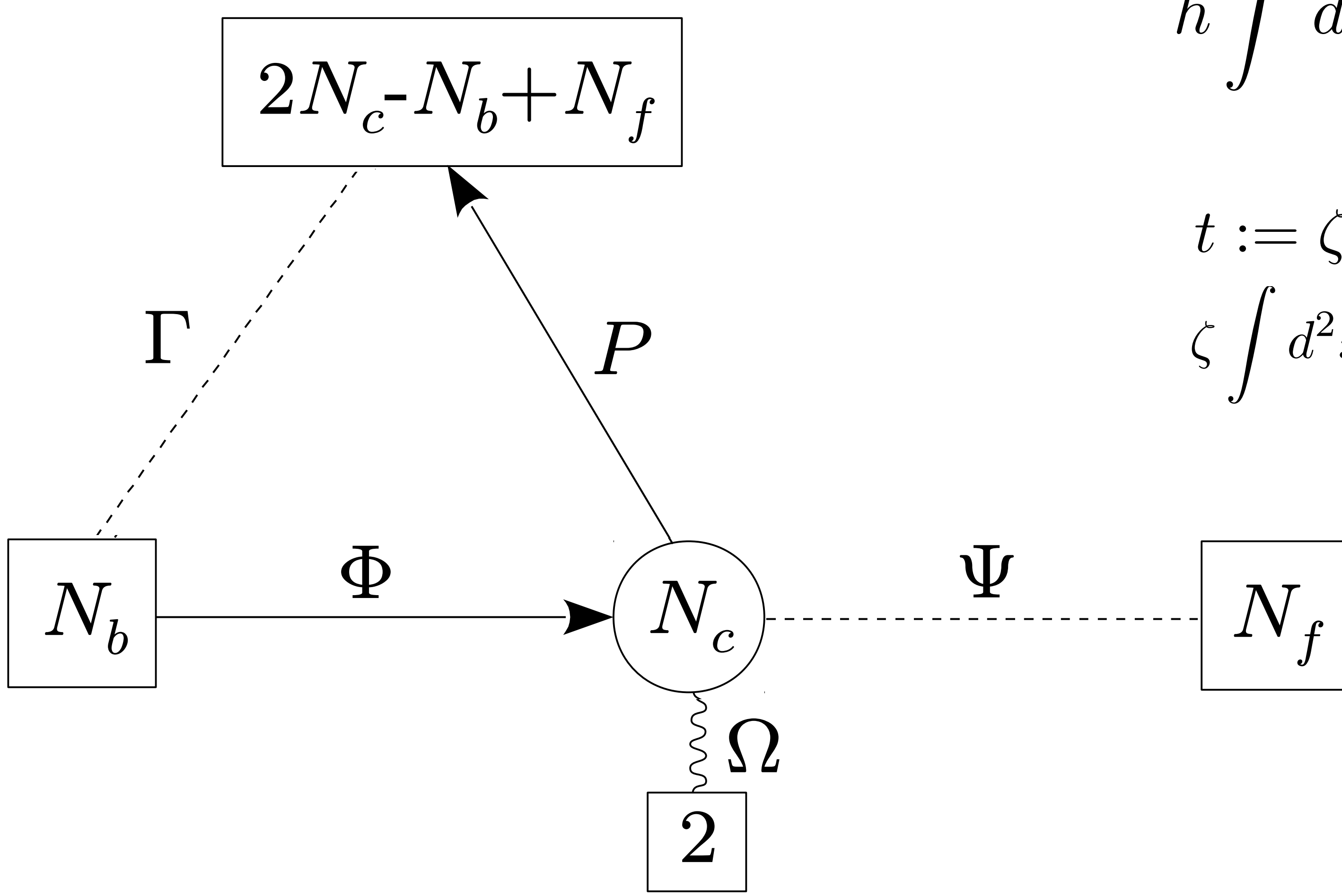
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum

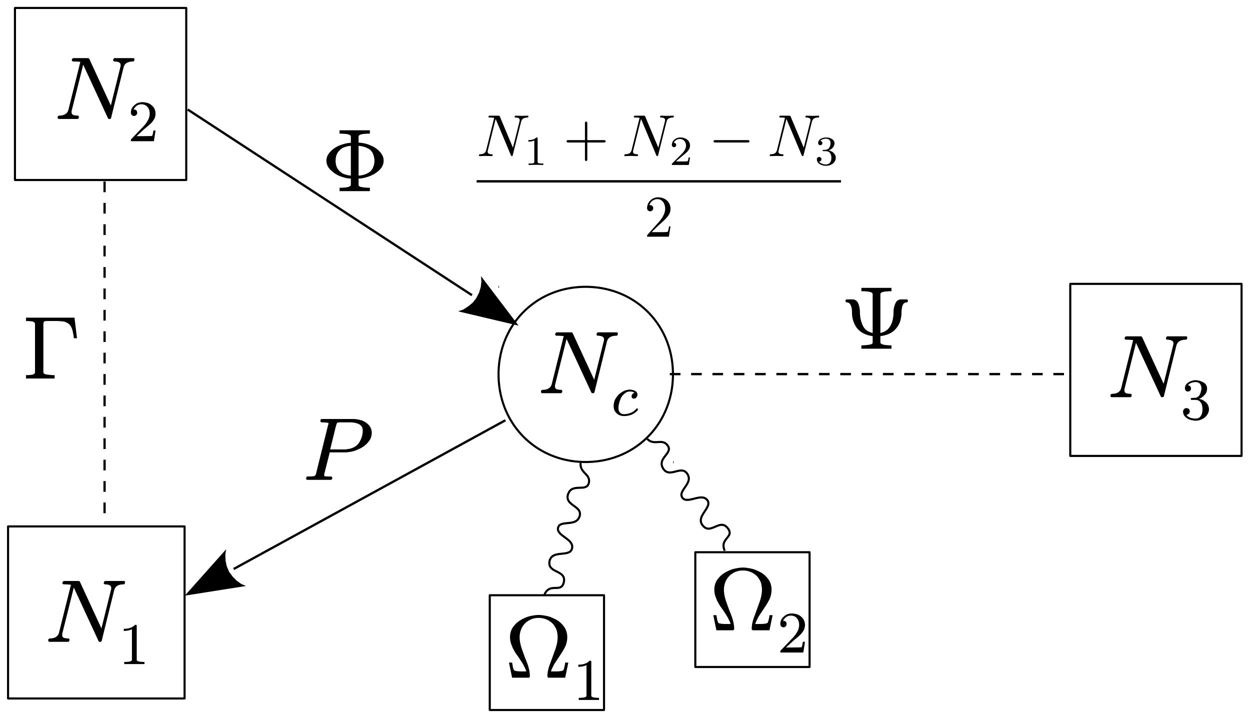
$$h \int d\theta \, \Gamma P \Phi + t \int d\theta \, \Lambda$$

$$t := \zeta + i\theta$$

$$\zeta \int d^2x \, \mathrm{tr} \, D + \theta \int \mathrm{tr} \, F$$



(0,2) SQCD



$=: \mathcal{T}_{N_1, N_2, N_3}$

- Triality is invariance of the fixed point under permutations of N's
- The support is obtained by computing the elliptic genus
- The IR fixed point can be solved explicitly to prove triality!
- Two possible IR CFTs related to each other by charge conjugation

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

- Target space for $\zeta > 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_2} \to Gr(N_c, N_1)$
- Target space for $\zeta < 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_1} \to Gr(N_c, N_2)$

$Gr(k, N) \sim Gr(N-k, N)$

$$\wr$$

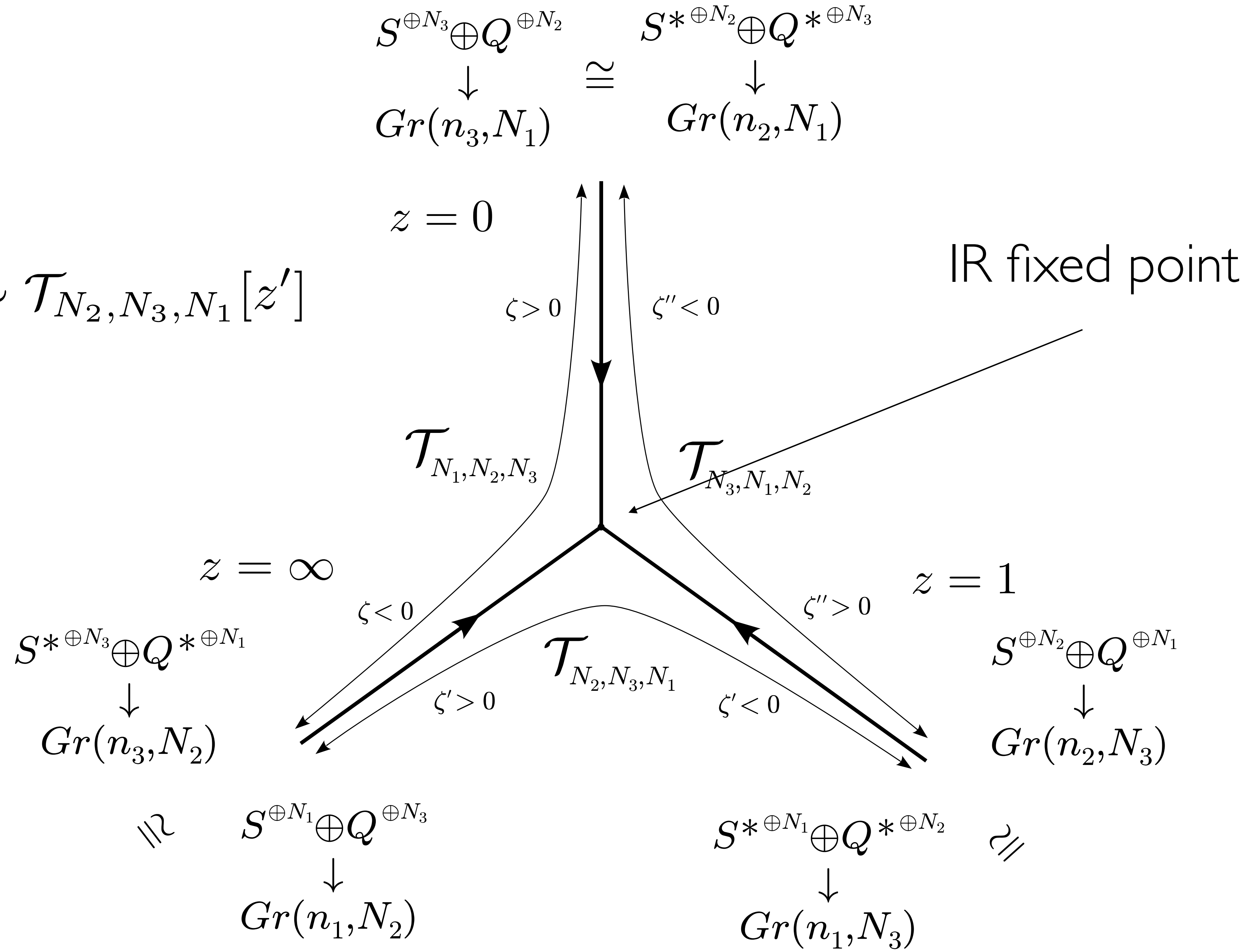
$$Q^{\oplus N_3} \oplus S^{\oplus N_2} \to Gr(N_2 - N_c, N_2)$$

Triality of sigma models

- Let $z := e^{-t}$

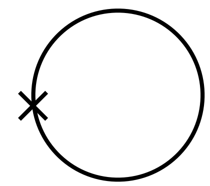
  $\mathcal{T}_{N_1,N_2,N_3}[z] \sim \mathcal{T}_{N_2,N_3,N_1}[z']$

  $$z' = \frac{1}{1-z}$$



One loop beta function

- The FI parameter runs due to the coupling $D|\phi|^2 - D|P|^2$
- Feynman diagram contributing to its one loop beta function is



$$\sim \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}$$

- Only scalars run in the loop but when (say) Φ gets a vev P is massive and vice versa

- For $\zeta \to \infty$ $$\frac{d\zeta}{d\log\Lambda} = \frac{N_1}{2\pi} \qquad d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$
- For $\zeta \to -\infty$ $$\frac{d\zeta}{d\log\Lambda} = -\frac{N_2}{2\pi} \qquad d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

Limits of beta function

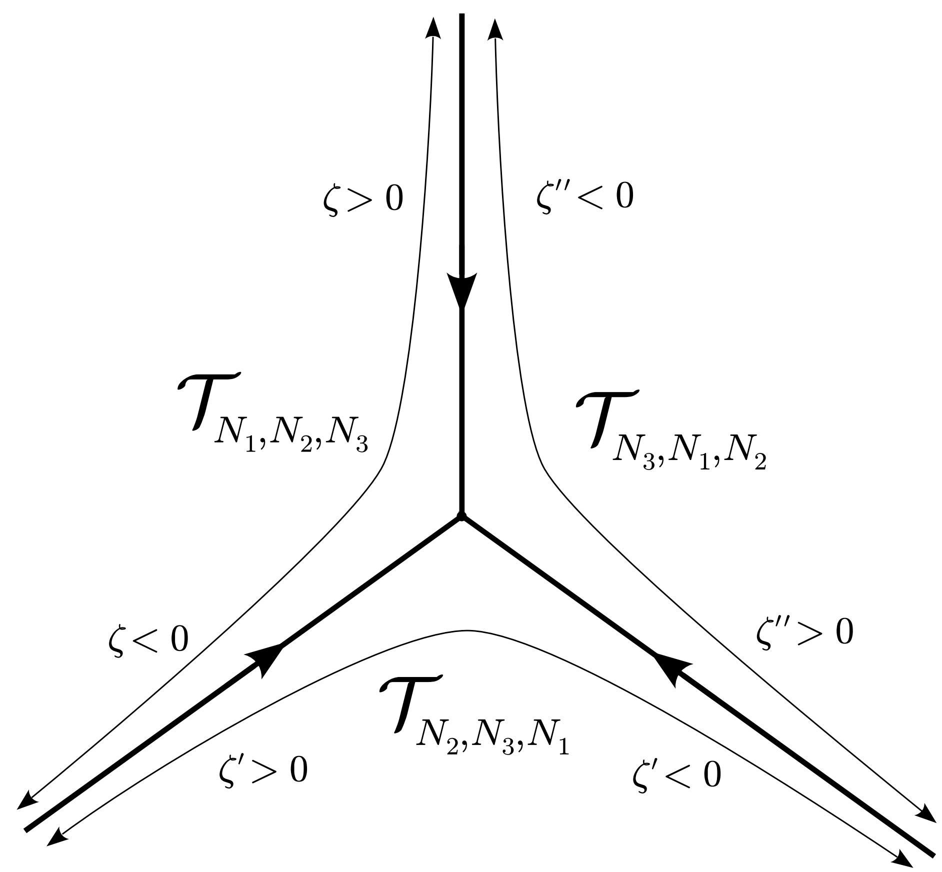
- Three UV fixed points
- Poles are at

  $$0, \infty, 1$$

- With residues

  $$\frac{1}{N_1}, \frac{1}{N_2}, \frac{1}{N_3}$$

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$



$$d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

$$d\log\Lambda = \frac{1}{N_3}\frac{dz}{z-1}$$

- Two IR fixed points
- Total residue must be

  $$-\Big(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\Big)$$

The exact beta function

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1}$$
$$-\frac{1}{2}\Big(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\Big)\Big(\frac{(1+x)dz}{z-a} + \frac{(1-x)dz}{z-b}\Big)$$

- Require consistency with triality
- ie Invariance under $z \to \frac{1}{1-z}$ and cyclic permutation of N's
- x=1 and a, b are fixed points of equation $z = \frac{1}{1-z}$

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1}$$
$$-\frac{1}{2}\Big(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\Big)\Big(\frac{dz}{z-e^{\frac{i\pi}{3}}} + \frac{dz}{z-e^{-\frac{i\pi}{3}}}\Big)$$

Renormalization group flow lines