Evidence for (Infinitely Diverse) Non-Convex Mirrors

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@ Southeastern Regional Mathematical String Theory Meeting
V-Tech University, Blacksburg VA; 2017.10.07

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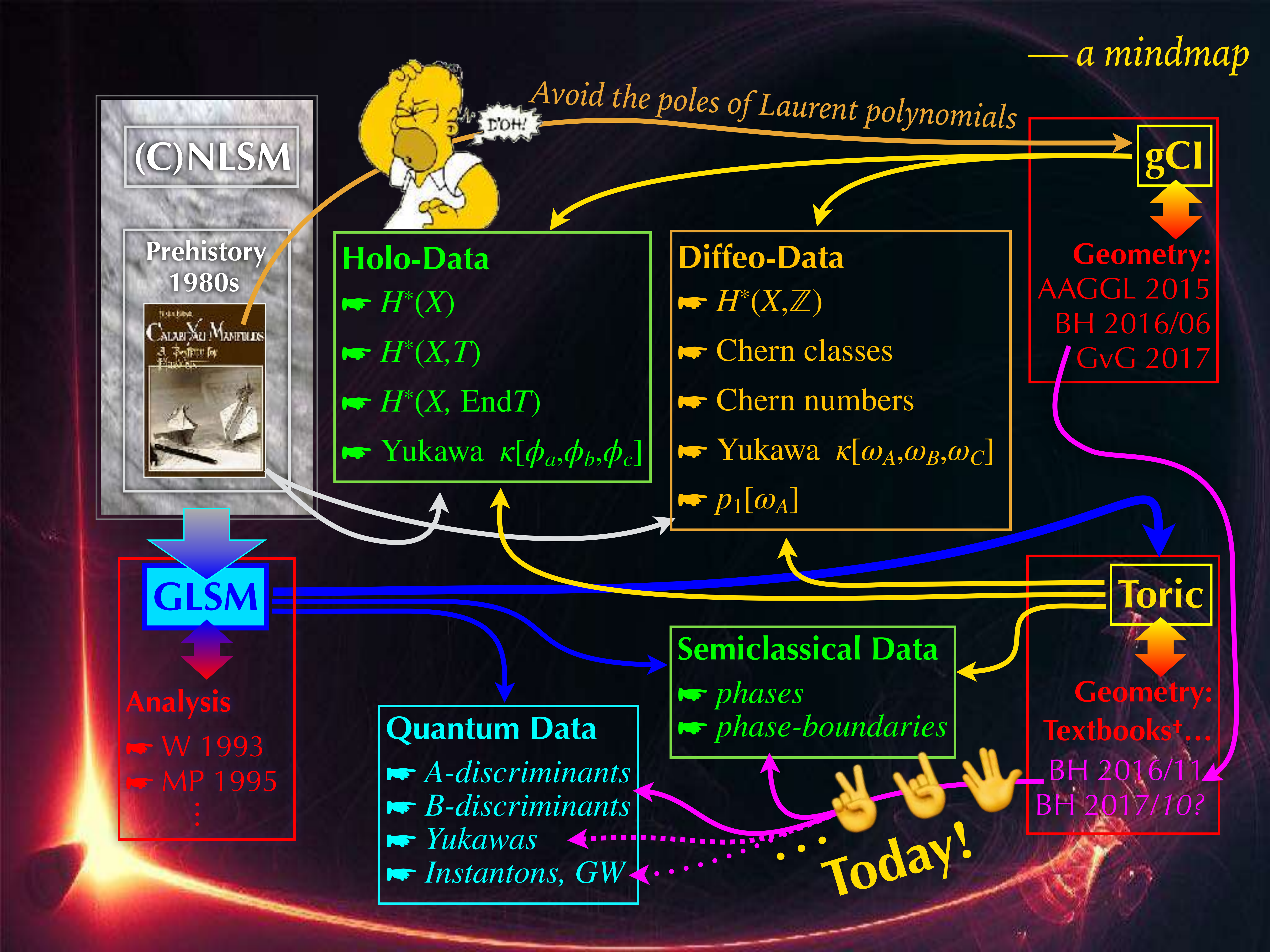

— a mindmap
Avoid the poles of Laurent polynomials
D'OH!
(C)NLSM
Prehistory
1980s
gCI
Geometry:
AAGGL 2015
BH 2016/06
GvG 2017
Holo-Data
☛ $H^*(X)$
☛ $H^*(X,T)$
☛ $H^*(X, \mathrm{End}T)$
☛ Yukawa $\kappa[\phi_a,\phi_b,\phi_c]$
Diffeo-Data
☛ $H^*(X,\mathbb{Z})$
☛ Chern classes
☛ Chern numbers
☛ Yukawa $\kappa[\omega_A,\omega_B,\omega_C]$
☛ $p_1[\omega_A]$
GLSM
Analysis
☛ W 1993
☛ MP 1995
⋮
Toric
Geometry:
Textbooks†…
BH 2016/11
BH 2017/10?
Semiclassical Data
☛ phases
☛ phase-boundaries
Quantum Data
☛ A-discriminants
☛ B-discriminants
☛ Yukawas
☛ Instantons, GW
… Today!

Non-Convex Mirror-Models

Prehistory

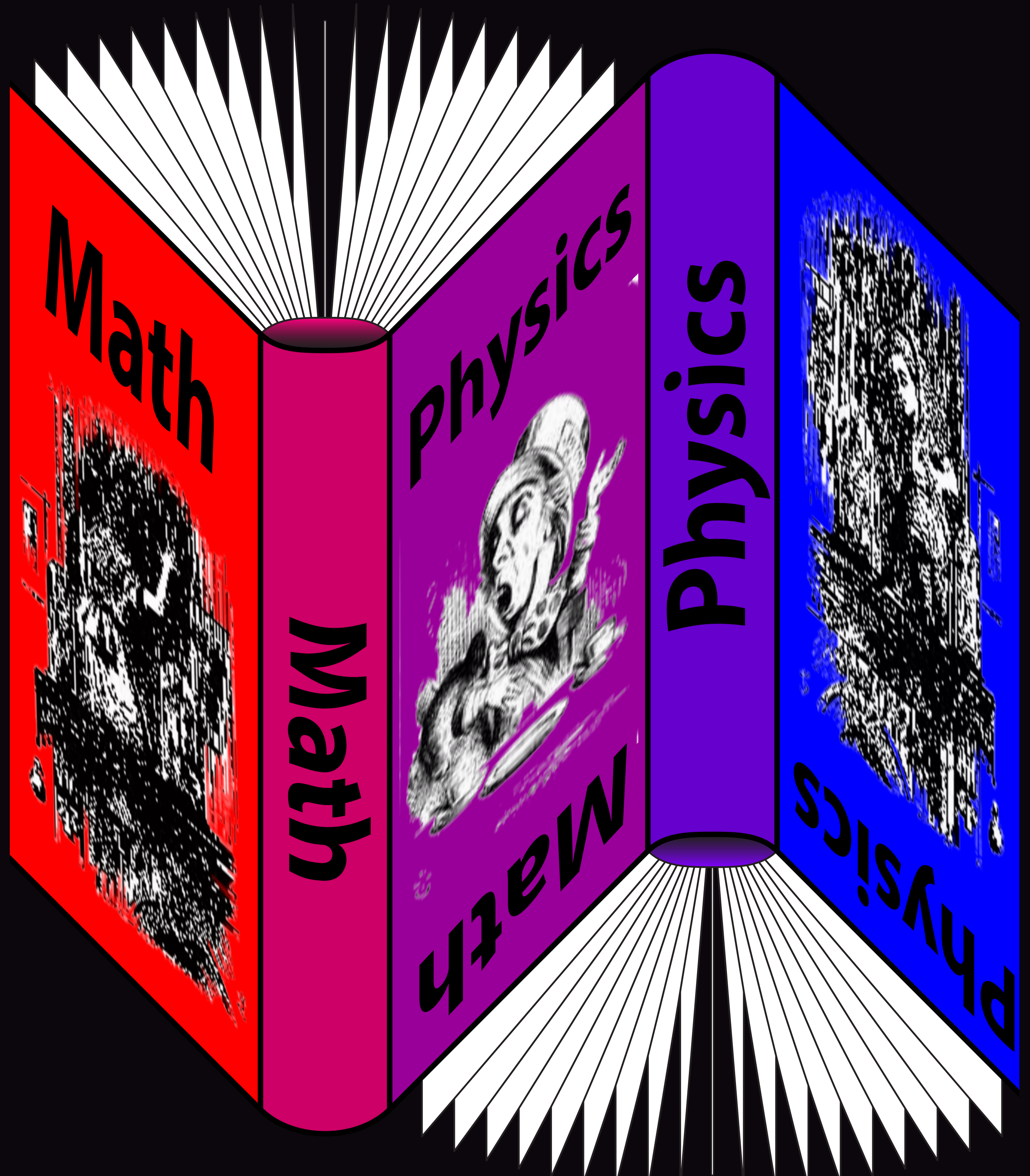
The Big Picture

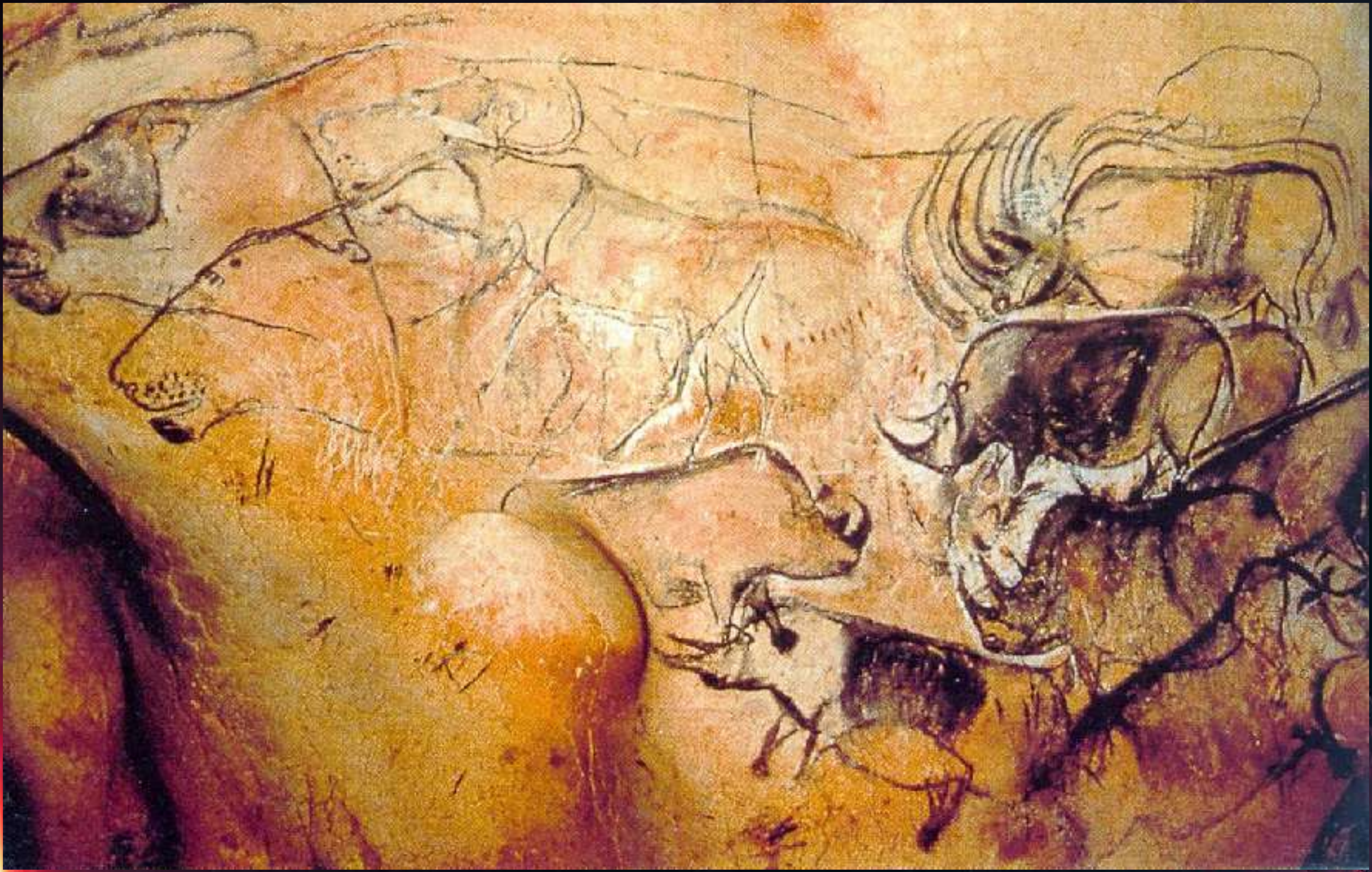
Laurent GLSModels

Phases & Discriminants

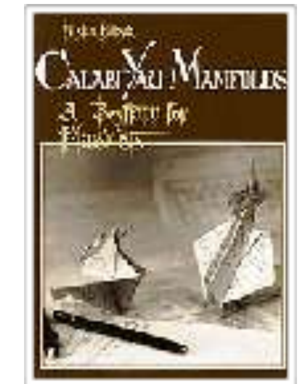
…and in the Mirror

"It doesn't matter what it's called,
…if… it has substance."
S.-T. Yau





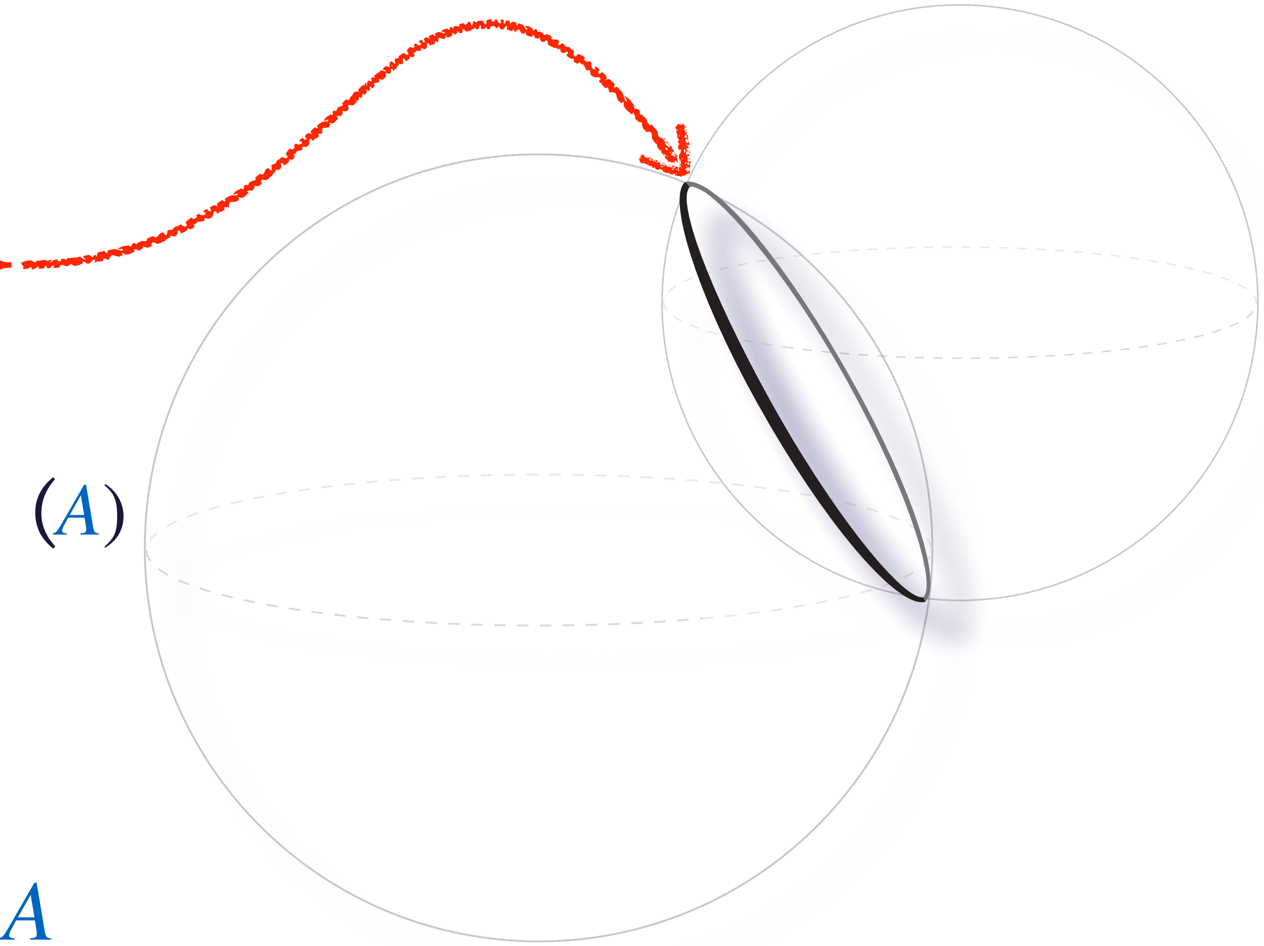
Pre-History
(Where are We Coming From?)



Pre-History

Classical Constructions

- Complete Intersections
  - Ex.: $(x-x_1)^2+(y-y_1)^2+(z-z_1)^2 = R_1{}^2$
    $(x-x_2)^2+(y-y_2)^2+(z-z_2)^2 = R_2{}^2$ }
    - Algebraic (constraint) equations
    - …in a well-understood “ambient” (A)



- Work over complex numbers
  - …& incl. “infinity” (e.g., $\mathbb{CP}^n$’s)
- For *hypersurfaces*: $X=\{p(x) = 0\} \subset A$
  - Functions: $[f(x)]_X = [f(x) \simeq f(x) + \lambda{\cdot}p(x)]_A$
  - Differentials: $[\mathrm{d}x]_X = [\mathrm{d}x \simeq \mathrm{d}x + \lambda{\cdot}\mathrm{d}p(x)]_A$
  - Homogeneity: $\mathbb{CP}^n = U(n+1)/[U(1)\times U(n)]$
    - Differential r-forms on $\mathbb{CP}^n$ are all $U(n+1)$-tensors

Just like gauge transformations

…with tensors

The Big Picture
(What are We Doing?)

Big Picture

Superstrings = *Framework* for Models

- Gauged Linear Sigma Model (GLSM) — on the world-sheet
  - Several "matter" fields + several "gauge" fields
    - Several coordinate functions – equivalence relations

$$A_\mu \simeq A_\mu + (\nabla_\mu \lambda)$$

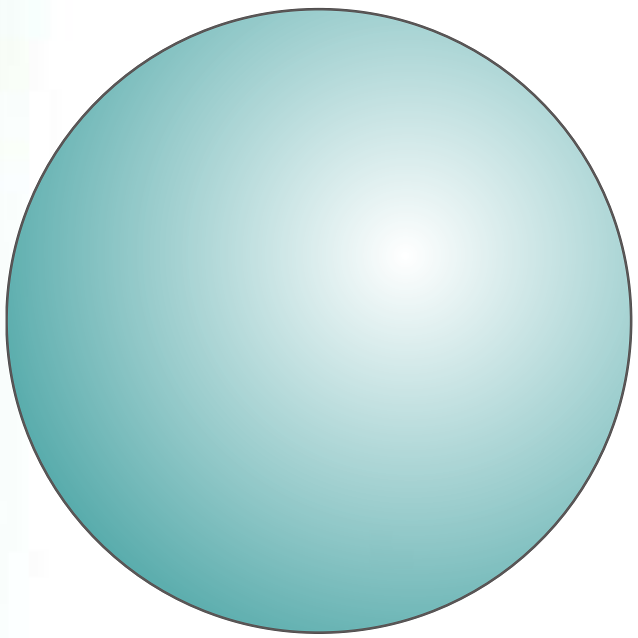
  - "Kinetic" part ($\|[\partial + q_X A]X\|^2$): KE + gauge-matter coupling
  - "Potential" part ($W(X)$): PE (gauge-invariant), "*F*-terms"
  - "Gauge" part ($\|\partial\wedge A\|^2 + \tau\cdot(\partial\wedge A)$): "*D*-terms" & "*F.-I.* terms"
- World-sheet matter & gauge symmetries are both complex
  - E.g.: $(x_1, x_2, x_3) \simeq (\lambda^{q_1} x_1, \lambda^{q_2} x_2, \lambda^{q_3} x_3)$, $\lambda \in \mathbb{C}^*$: $\mathbb{P}^2_{(q_1:q_2:q_3)}$
    - ...makes sense if the fixed-point set is excised (forbidden) from $(x_1, x_2, x_3) \in \mathbb{C}^3$
    - ...or considered as an alternate (separate) location.
- Gauge symmetry "stratifies" the X-field-space
  - & $|vacuum\rangle$ determined by $\min[W(X)]$: hypersurface

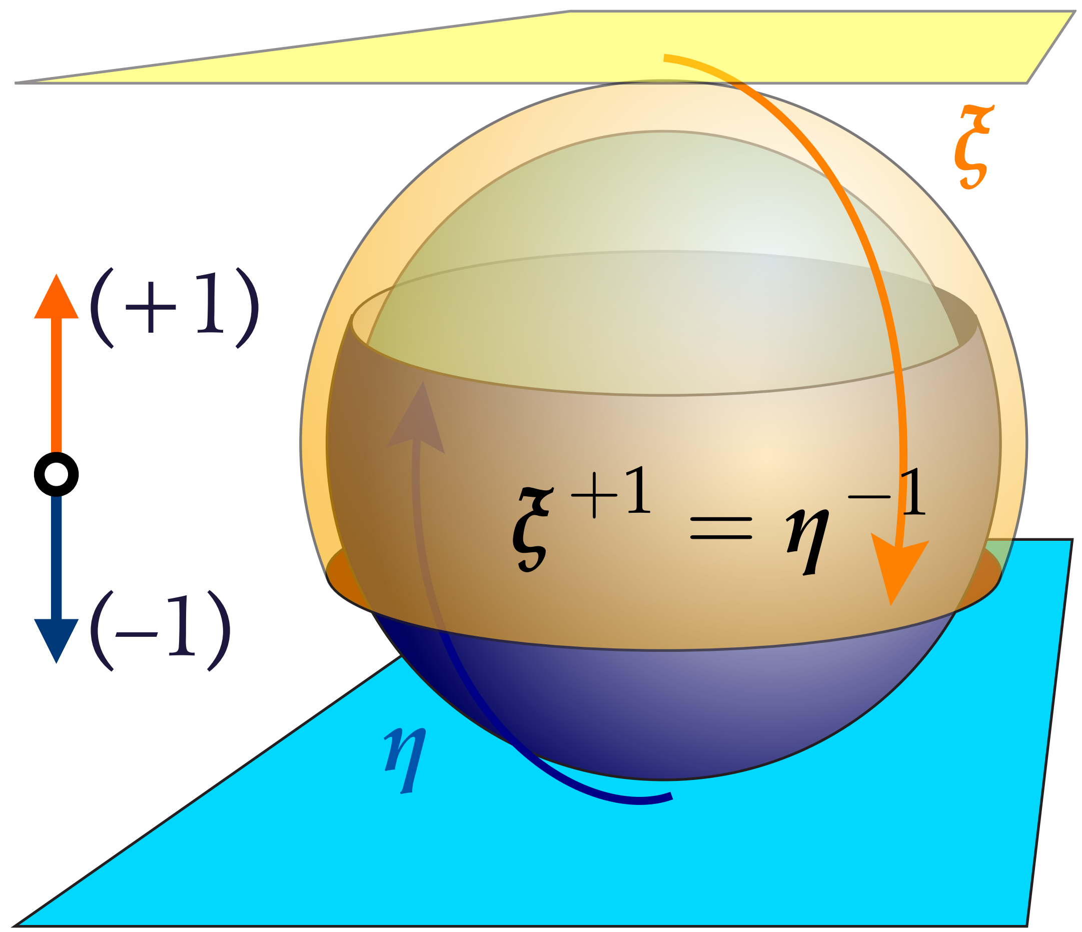
} ⇒ ***spacetime***

Big Picture

Toric Geometry

- Consider $S^2 \simeq \mathbb{P}^1$:





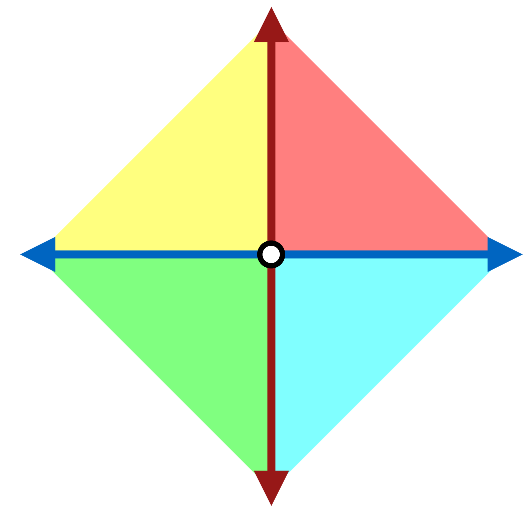
- Need at least two (complex) coordinates:
  - Match (the exponents) near the equator: $(+1)_N = (-1)_S$
  - Symmetry: $\xi \to \lambda^{+1}\xi$ and $\eta \to \lambda^{-1}\eta$, with $\lambda \in \mathbb{C}^* = (\mathbb{C} \smallsetminus \{0\})$
  - Explicitly: $\lambda = e^{i(\alpha+i\beta)} = e^{-\beta} \cdot e^{i\alpha}$ = (real) rescaling · phase-change

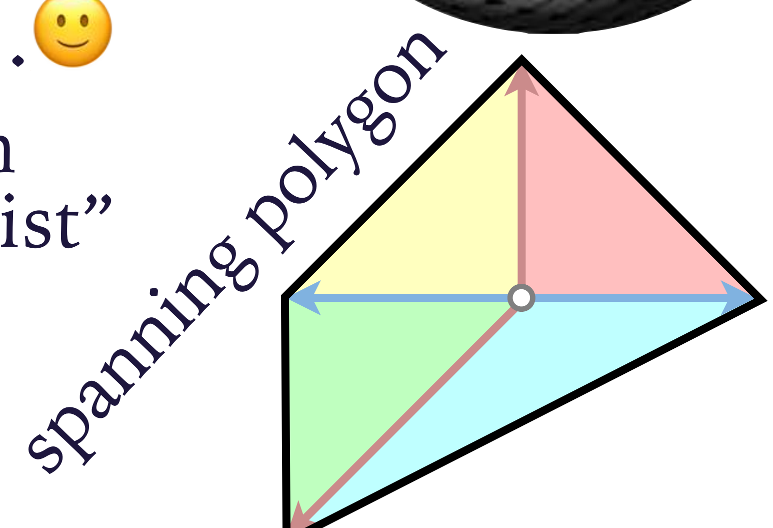
“thickened” S^1 *usual gauge transformation*

Big Picture

Toric Geometry

- More complicated examples: $S^2 \times S^2$
  - An entire 2nd sphere at every point of 1st
  - Orthogonal ↔ linearly independent
  - Top-dim cones ↔ coord. patches
  - 2-dim (enveloping) polytope ↔ (ℂ) 2-dim. geometry
- More complicated yet: “twisted” product
  - Twisted torus $S^1 ⋇ S^1$ (S^1 “twists” about S^1) (≃ crystal w/oblique lattice 🙂).
- Now ×ℂ: Hirzebruch (ℂ) surface, $\mathfrak{F}_1$. 🙂
  - “Slanting” (0,–1) → (–m,–1) the bottom vertex (& two cones) encodes the “twist”
  - … $\mathfrak{F}_m = m$-twisted $\mathbb{P}^1$-bundle over $\mathbb{P}^1$.
  - …and so on: 4 textbooks worth!





Toric Geometry

Polytope Encoding

- The polytope encodes the space
- …but also its symmetries:
  - Assign each vertex a (Cox) coordinate
  - Read off cancelling relations

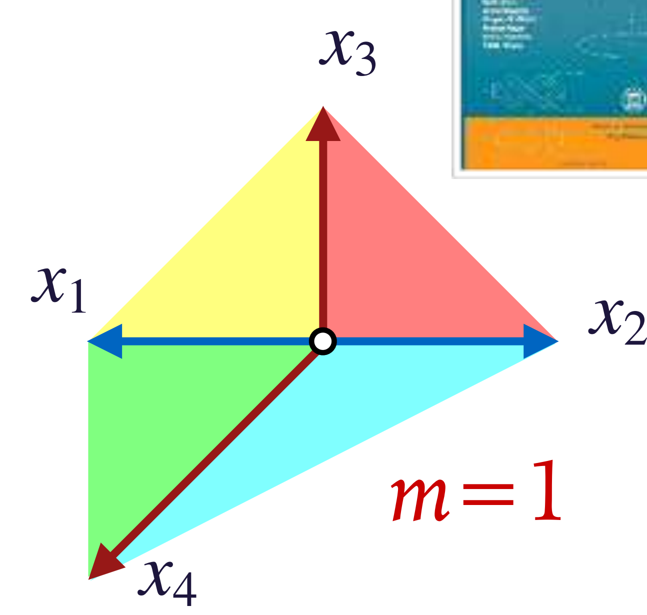
$$1\,\vec{v}_{x_1} + 1\,\vec{v}_{x_2} + 0\,\vec{v}_{x_3} + 0\,\vec{v}_{x_4} = 0$$

$$(x_1, x_2, x_3, x_4) \simeq (\lambda^1 x_1, \lambda^1 x_2, \lambda^0 x_3, \lambda^0 x_4)$$

$$0\,\vec{v}_{x_1} + m\,\vec{v}_{x_2} + 1\,\vec{v}_{x_3} + 1\,\vec{v}_{x_4} = 0$$

$$(x_1, x_2, x_3, x_4) \simeq (\lambda^0 x_1, \lambda^m x_2, \lambda^1 x_3, \lambda^1 x_4)$$

- Defines two independent (gauge) symmetries
  - a GLSM w/gauge-invariant Lagrangian
  - and | *ground state* ⟩ where KE = 0 = PE
  - & (quantum) Hilbert space on it



Laurent GLSModels
(and their Toric Geometry)

A Generalized Construction of
Calabi-Yau Models and Mirror Symmetry
arXiv:1611.10300

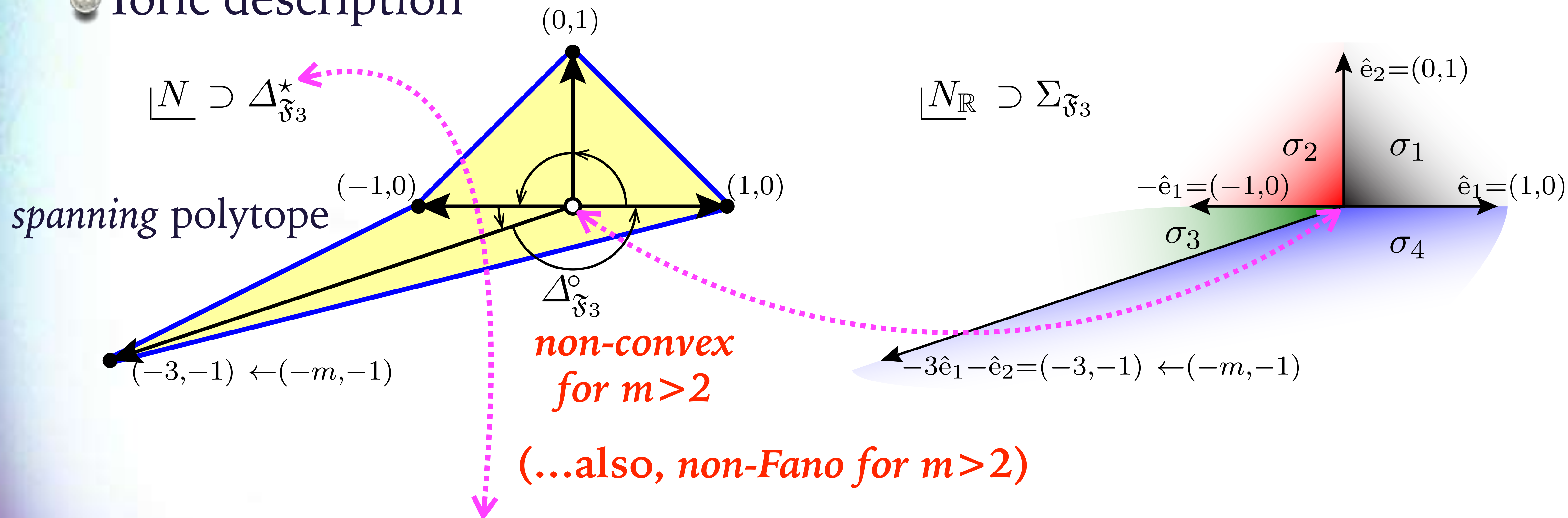
Laurent GLSMs

& Non-Convex Mirrors

—*Proof-of-Concept*—

arXiv:1611.10300

- 2-torus in the Hirzebruch surface $\mathfrak{F}_m$:
  - "Anticanonical" (Calabi-Yau, Ricci-flat) hypersurface in $\mathfrak{F}_m$
  - Toric description



The star-triangulation of the *spanning* polytope defines the fan of the underlying toric variety

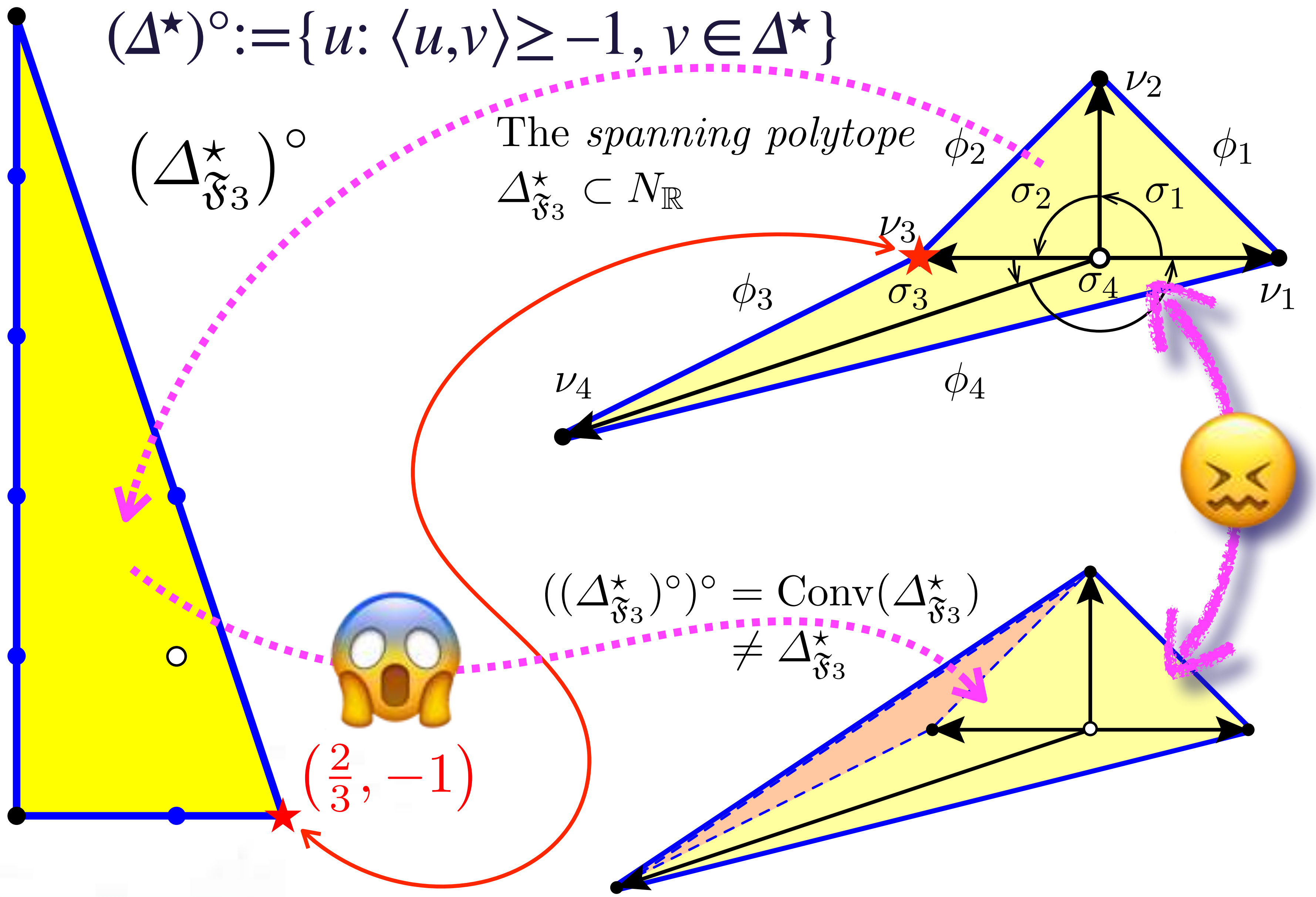
Laurent GLSMs

& Non-Convex Mirrors

—*Proof-of-Concept*— arXiv:1611.10300

- The *Newton* polytope (polar of *spanning* polytope):
  - The "standard" polar polytope is non-integral
  - The "standard" polar of the polar is not the spanning polytope that we started with
  - Is no good for mirror symmetry

$(\Delta^\star)^\circ := \{u\colon \langle u,v\rangle \geq -1,\ v\in\Delta^\star\}$

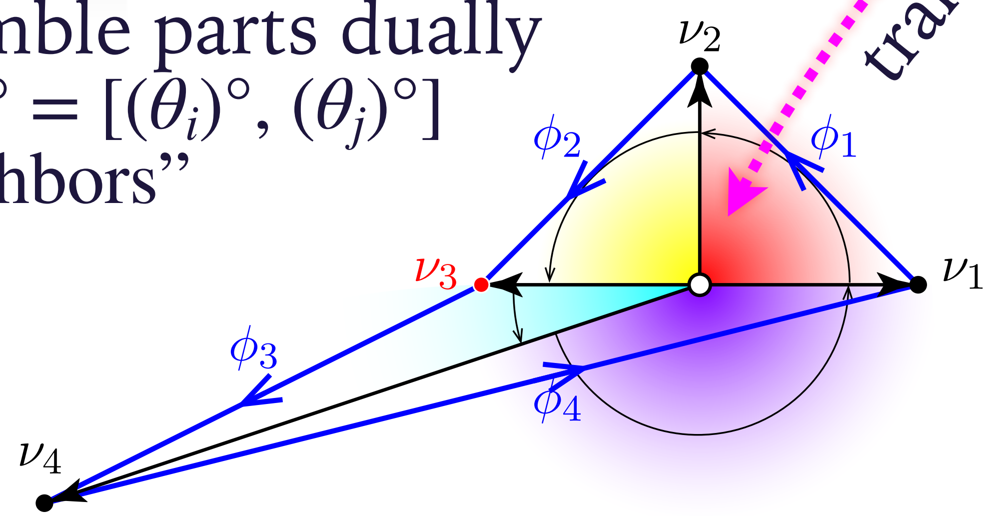


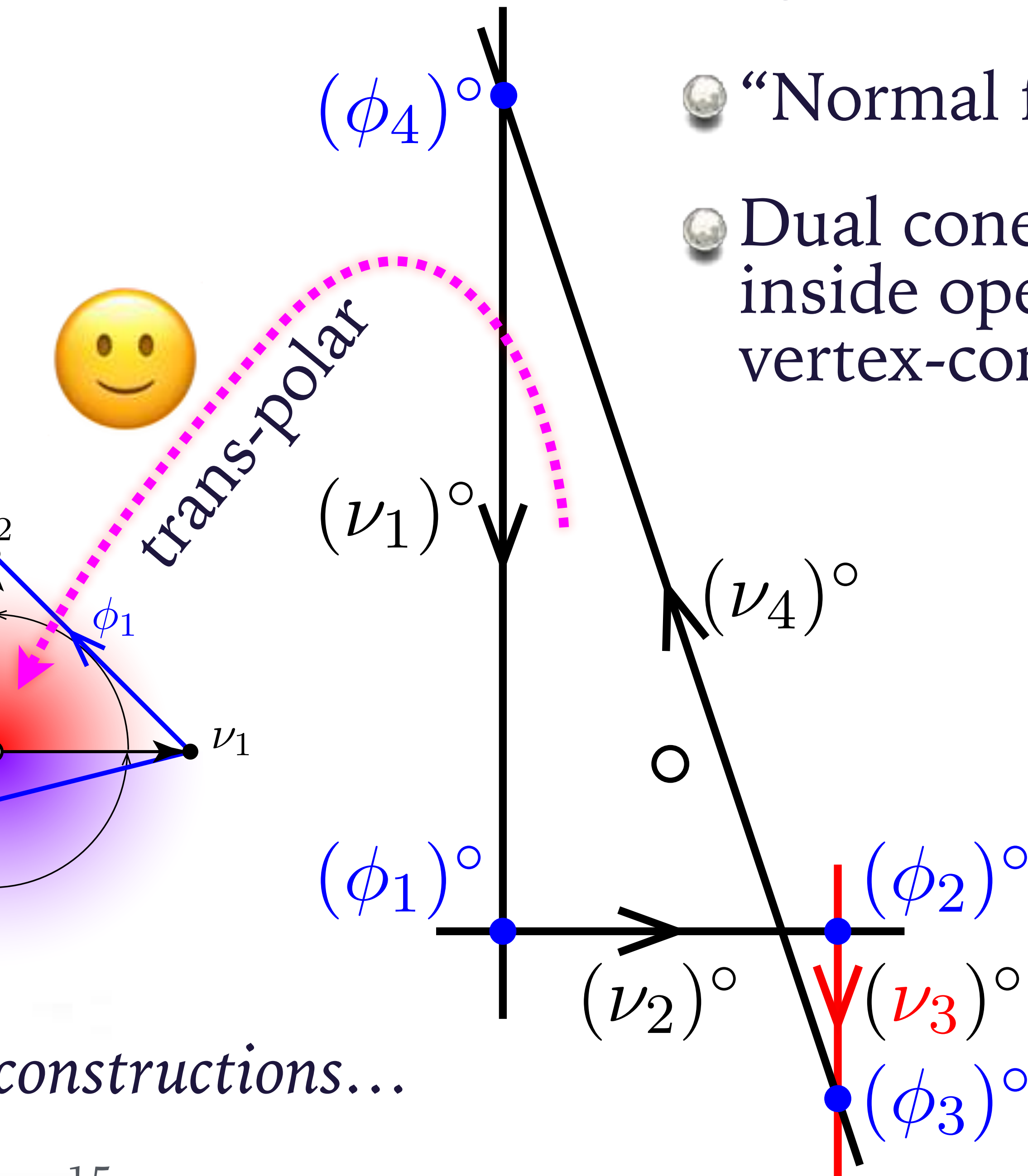
Laurent GLSMs

& Non-Convex Mirrors

—Proof-of-Concept— arXiv:1611.10300

- The *oriented Newton* polytope (trans-polar of *spanning* polytope):
- **Construction** (trans-polar)
  - Decompose $\Delta^\star$ into convex faces θ_i;
  - Find the (standard) polar $(\theta_i)^\circ$ for each (convex) face
  - (Re)assemble parts dually to $(\theta_i \cap \theta_j)^\circ = [(\theta_i)^\circ, (\theta_j)^\circ]$ with "neighbors"





- "Normal fan"
- Dual cones $\mapsto$ inside opening vertex-cones

- *Agrees with standard (if obscure?) constructions…*

Laurent GLSMs

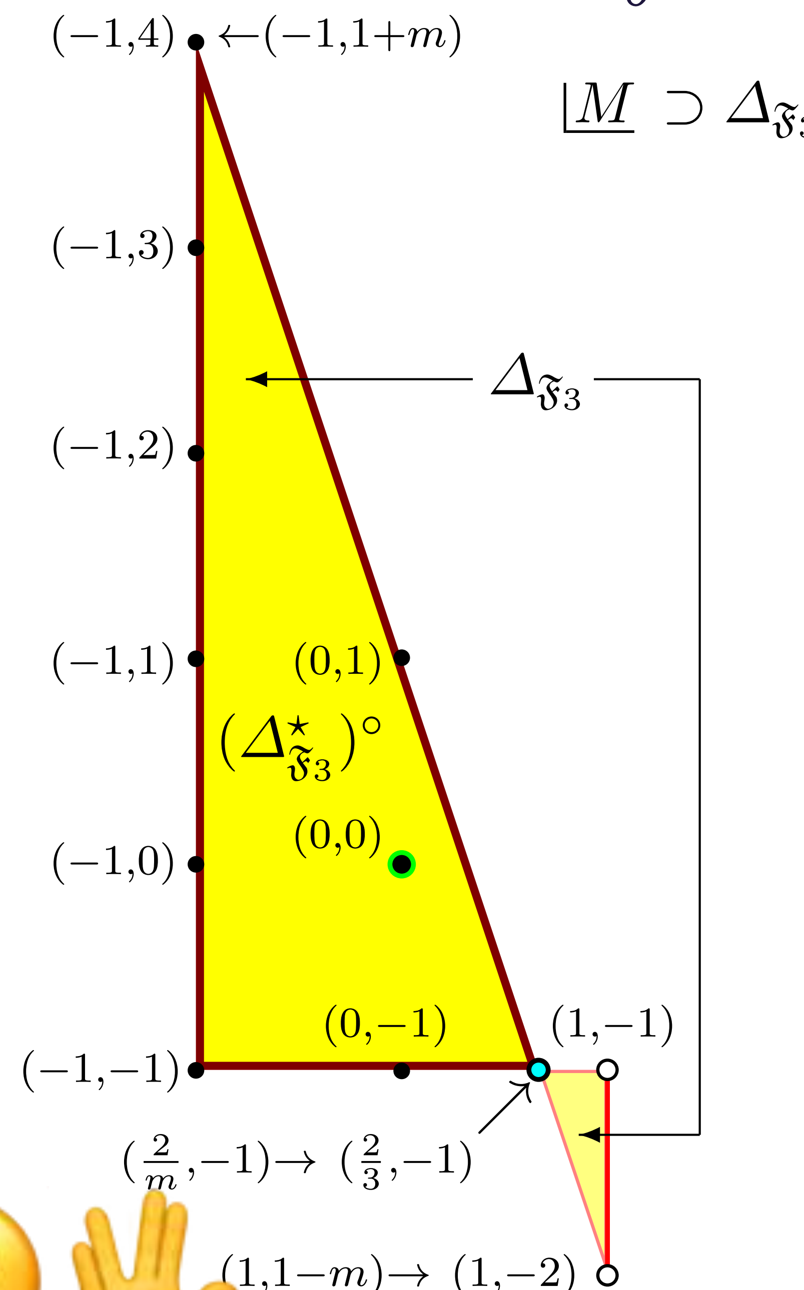
& Non-Convex Mirrors

—*Proof-of-Concept*— arXiv:1611.10300

- The *oriented Newton* polytope:
  - specifies allowed monomials
  - The so-defined 2-tori are all ***singular*** @ (0,0,1)
  - …as each monomial has at least an x_1 factor, so $f(x) = x_1 \cdot g(x)$
  - The extension corresponds to Laurent monomials:

$$(1,-1) \mapsto \frac{x_2^2}{x_4}, \quad (1,-2) \mapsto \frac{x_2^2}{x_3} \Bigg\} \text{ make the 2-tori } \Delta\text{-regular.}$$

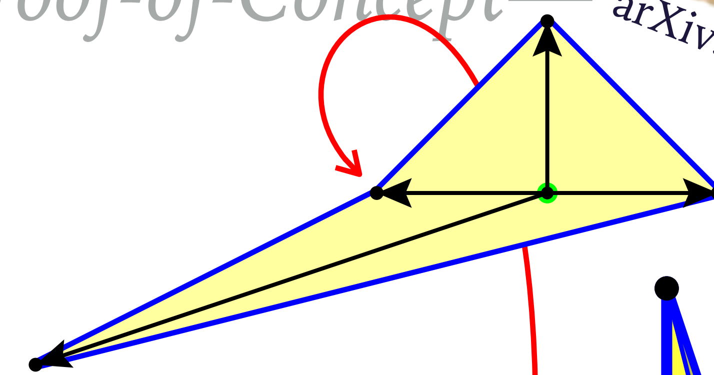
	$x_1^2 x_3^5$
	$x_1^2 x_3^4 x_4$
	$x_1^2 x_3^3 x_4^2$
$x_1 x_2 x_3^2$	$x_1^2 x_3^2 x_4^3$
$x_1 x_2 x_3 x_4$	$x_1^2 x_3 x_4^4$
$x_1 x_2 x_4^2$	$x_1^2 x_4^5$



Laurent GLSMs

& Non-Convex Mirrors

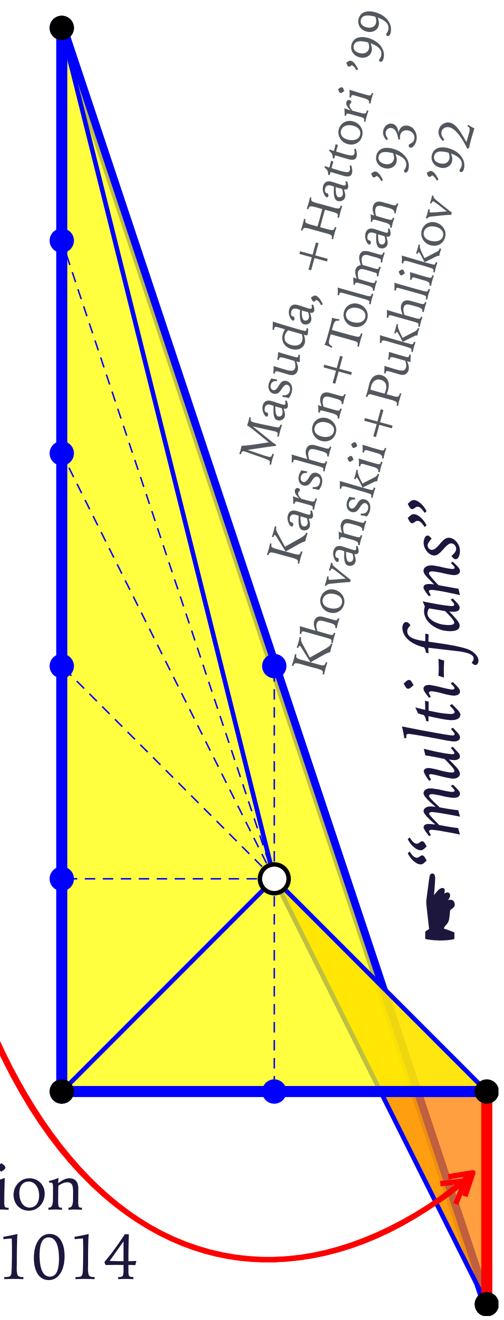
—*Proof-of-Concept*— arXiv:1611.10300



- The *oriented Newton* polytope:
  - is star-triangulable → a toric variety
  - differs from its convex hull by "flip-folded" simplices
- Associating coordinates to corners:
  - *SP*: $x_1=(-1,0)$, $x_2=(1,0)$, $x_3=(0,1)$, $x_4=(-3,-1)$
  - *NP*: $y_1=(-1,4)$, $y_2=(-1,-1)$, $y_3=(1,-1)$, $y_4=(1,-2)$
  - Expressing each as a monomial in the others:

$$NP:\ x_1^2\,x_3^5 \oplus x_1^2\,x_4^5 \oplus \frac{x_2^2}{x_4} \oplus \frac{x_2^2}{x_3} \quad vs. \quad SP:\ y_1^2\,y_2^2 \oplus y_3^2\,y_4^2 \oplus \frac{y_1^5}{y_4} \oplus \frac{y_2^5}{y_3}$$

$$\mathbb{P}^2_{(1:1:3)}[5]\ \begin{bmatrix} 2 & 0 & 5 & 0 \\ 2 & 0 & 0 & 5 \\ 0 & 2 & 0 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xleftrightarrow{\text{BHK}} \begin{bmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 5 & 0 & 0 & -1 \\ 0 & 5 & -1 & 0 \end{bmatrix}\ \mathbb{P}^2_{(3:2:5)}[10]$$



Mirror Construction
arXiv:hep-th/9201014

Laurent GLSMs

& Non-Convex Mirrors

—*Proof-of-Concept*— arXiv:1611.10300

- K3 in Hirzebruch 3-folds, "cornerstone" mirrors:

$$a_1\, x_4^8 + a_2\, x_3^8 + a_3\, \frac{x_1^3}{x_3} + a_5\, \frac{x_2^3}{x_3} : \quad \exp\left\{ 2i\pi \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ \frac{1}{24} & \frac{1}{24} & \frac{1}{8} & 0 \\ \hline \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & \frac{1}{8} \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : \quad \begin{cases} G = \mathbb{Z}_3 \times \mathbb{Z}_{24}, \\ Q = \mathbb{Z}_8. \end{cases}$$

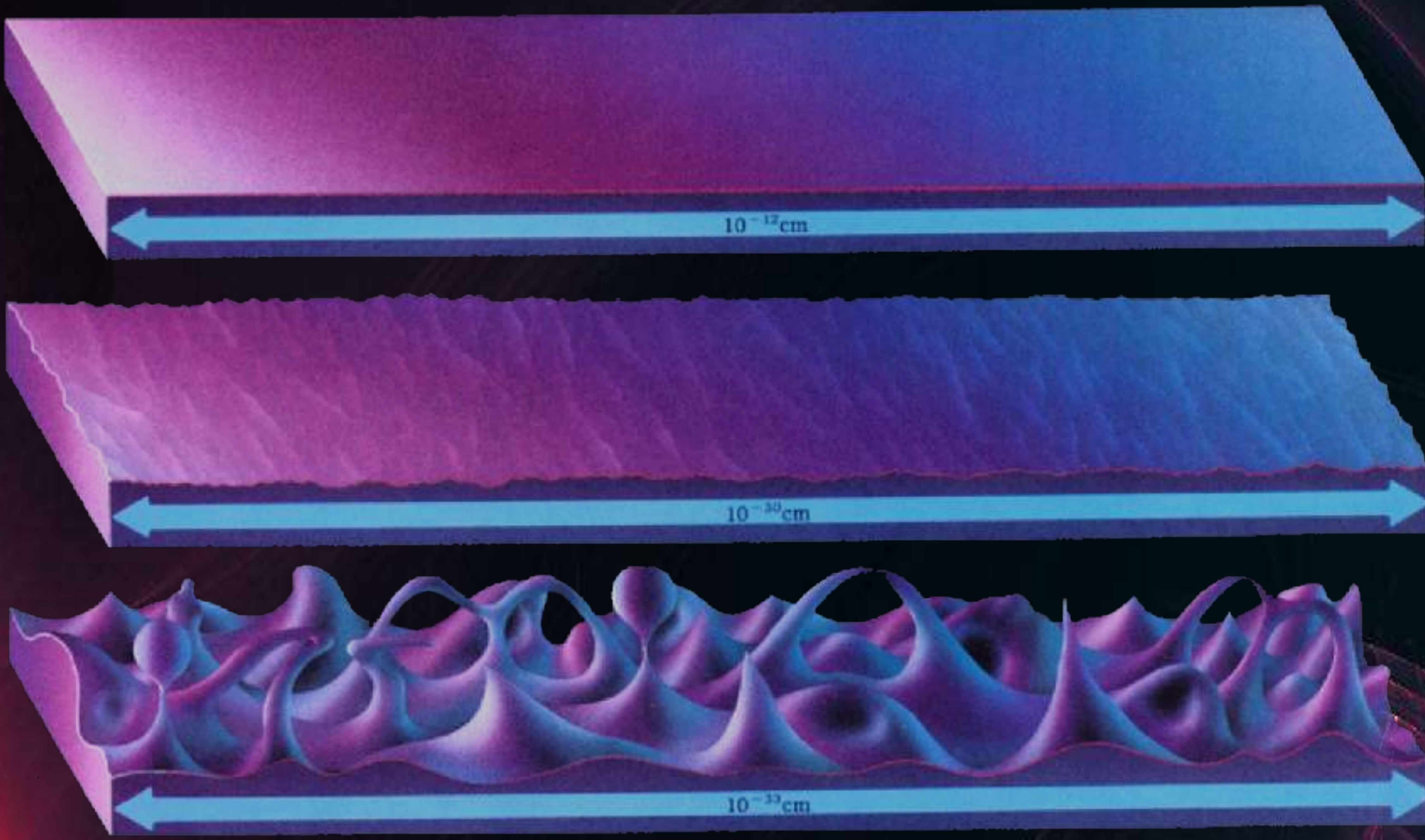
$$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 \\ 3 & 0 & -1 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix} \qquad \mathbb{P}^3_{(3:3:1:1)}[8] \qquad \mathbb{P}^3_{(3:5:8:8)}[24]/\mathbb{Z}_3$$

BHK

$$b_1\, y_3^3 + b_2\, y_5^3 + b_3\, \frac{y_2^8}{y_3\, y_5} + b_4\, y_1^8 : \quad \exp\left\{ 2i\pi \begin{bmatrix} \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ \hline \frac{3}{24} & \frac{5}{24} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \right\} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_5 \end{bmatrix} : \quad \begin{cases} G^{\nabla} = \mathbb{Z}_8 \\ Q^{\nabla} = \mathbb{Z}_{24} \times \mathbb{Z}_3 \end{cases}$$

$$\frac{|G|}{|Q|} = \frac{3{\cdot}24}{8} = 9 = \frac{d(\Delta_{\mathcal{F}_3})}{d(\Delta^{\star}_{\mathcal{F}_3})} = \frac{54}{6}$$

- The Hilbert space & interactions restricted by the symmetries
  - Analysis: classical, semi-classical, quantum corrections…
  - …in spite of the manifest singularity in the (super)potential



Discriminants
(How Small Can We Go?)

Phases & Discriminants

The Phase-Space

—Proof-of-Concept— arXiv:1611.10300

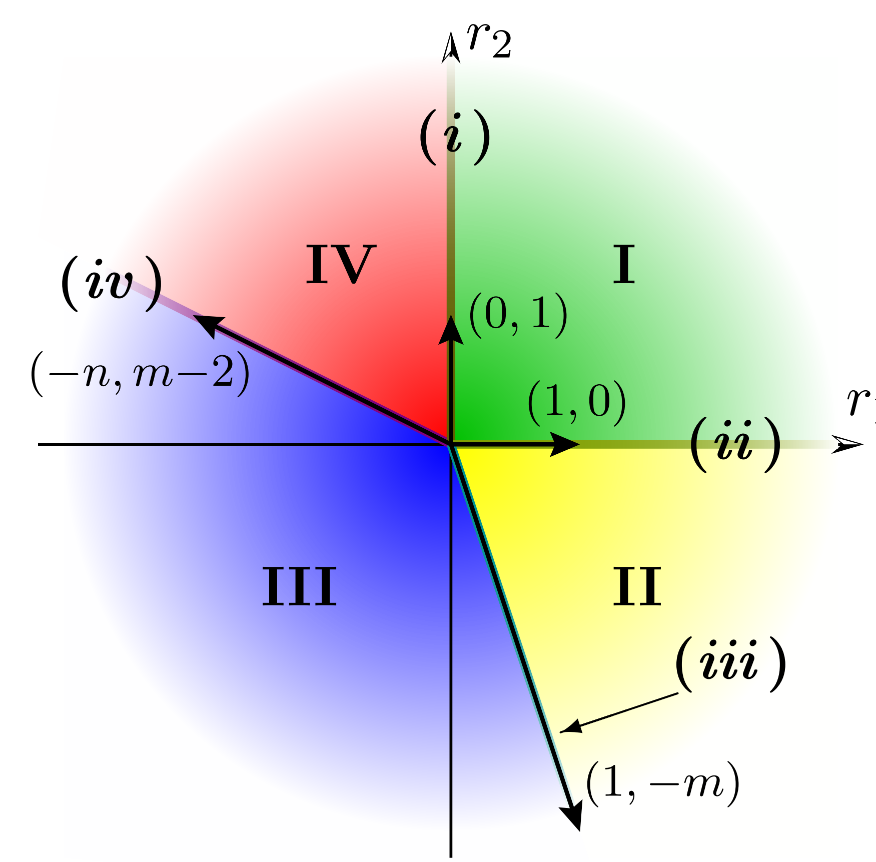
- The (super)potential: $W(X) := X_0 \cdot f(X)$,

$$f(X) := \sum_{j=1}^{2} \Big(\sum_{i=2}^{n} (a_{ij}\, X_i^{\,n}) X_{n+j}^{\,2-m} + a_j\, X_1^{\,n} X_{n+j}^{\,(n-1)m+2} \Big)$$

	X_0	X_1	X_2	$\cdots$	X_n	X_{n+1}	X_{n+2}
Q^1	$-n$	1	1	$\cdots$	1	0	0
Q^2	$m-2$	$-m$	0	$\cdots$	0	1	1

- The possible vevs

	$\lvert x_0\rvert$	$\lvert x_1\rvert$	$\lvert x_2\rvert$	$\cdots$	$\lvert x_n\rvert$	$\lvert x_{n+1}\rvert$	$\lvert x_{n+2}\rvert$
i	0	0	0	$\cdots$	0	$*$	$*$
I	0	$*$	$*$	$\cdots$	$*$	$*$	$*$
ii	0	0	$*$	$\cdots$	$*$	0	0
II	0	$\lvert x_1\rvert = \sqrt{\frac{\sum_j \lvert x_{n+j}\rvert^2 - r_2}{m}} = \sqrt{r_1 - \sum_{i=2}^{n} \lvert x_i\rvert^2} > 0$	$*$	$\cdots$	$*$	$*$	$*$
iii	0	$\sqrt{r_1}$	0	$\cdots$	0	0	0
III	$\sqrt{\frac{mr_1+r_2}{(n-1)m+2}}$	$\sqrt{\frac{(m-2)r_1+nr_2}{(n-1)m+2}}$	0	$\cdots$	0	0	0
iv	$\sqrt{-r_1/n}$	0	0	$\cdots$	0	0	0
IV	$\sqrt{-r_1/n}$	0	0	$\cdots$	0	$*$	$*$

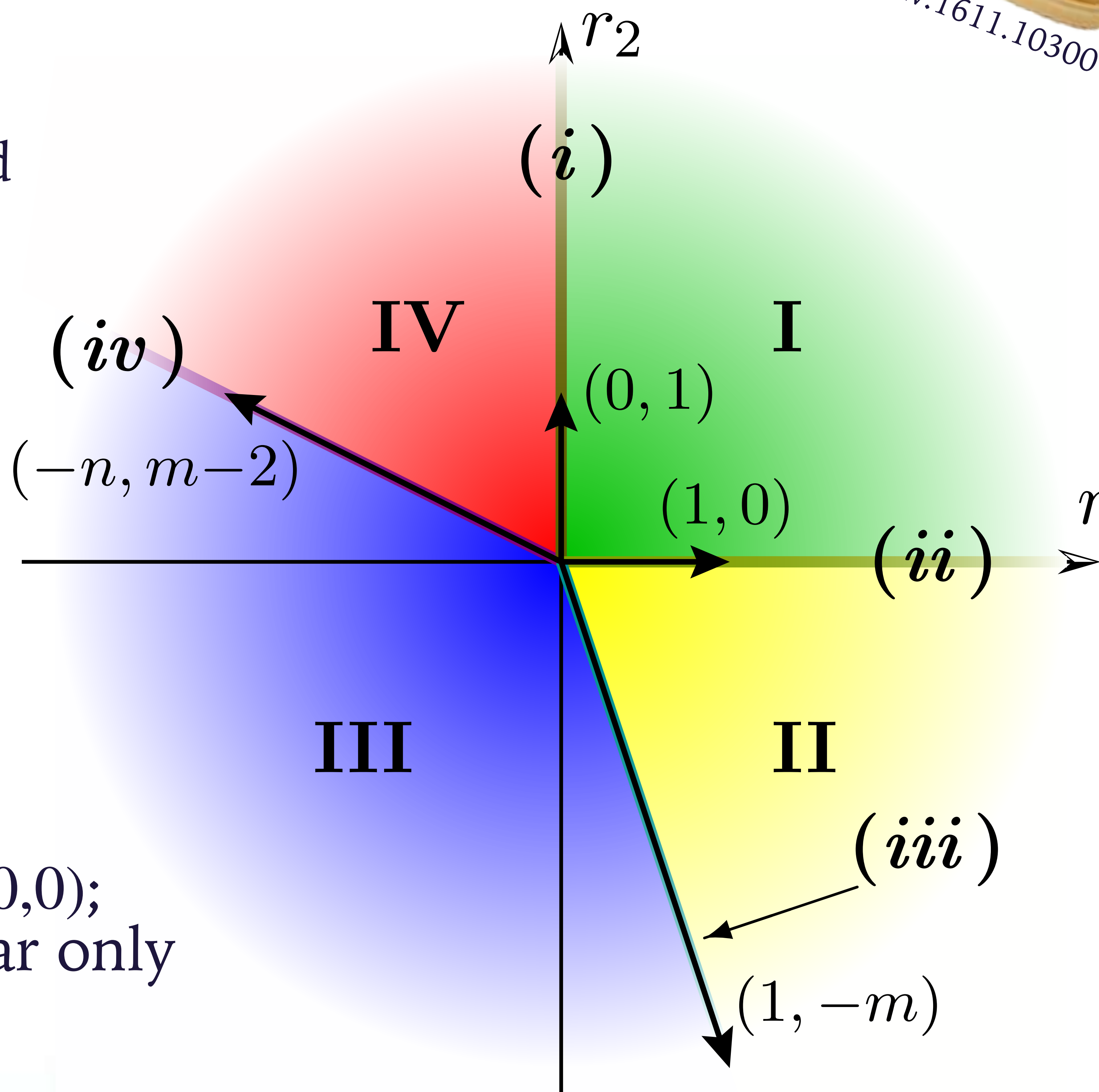


Phases & Discriminants

—Proof-of-Concept— arXiv:1611.10300

The Phase-Space

- Phase-space:
  - I: Calabi-Yau (n–1)-fold hypersurface in F_m
  - II: Calabi-Yau (n–1)-fold hypersurface in F_m (flopped)
  - III: Calabi-Yau $\mathbb{Z}_{(n-1)m+2}$ Landau -Ginzburg orbifold
  - IV: Calabi-Yau hybrid
- The $\langle x_i(r_1,r_2)\rangle$ change continuously 'round (0,0); boundaries are singular only for special values of θ.

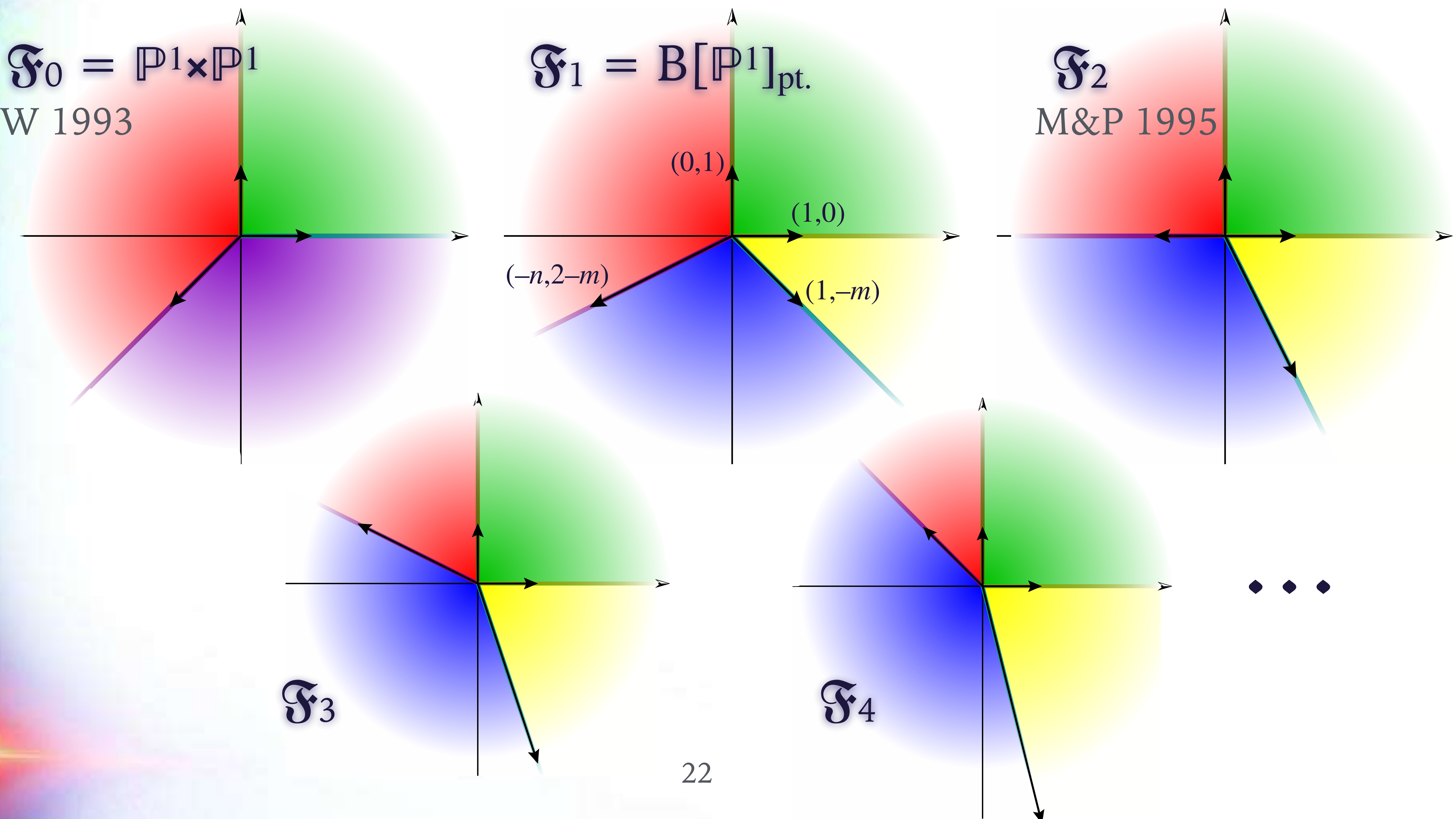


Phases & Discriminants

—*Proof-of-Concept*— arXiv:1611.10300

The Phase-Space

- Varying m in $\mathfrak{F}_m$:



Phases & Discriminants

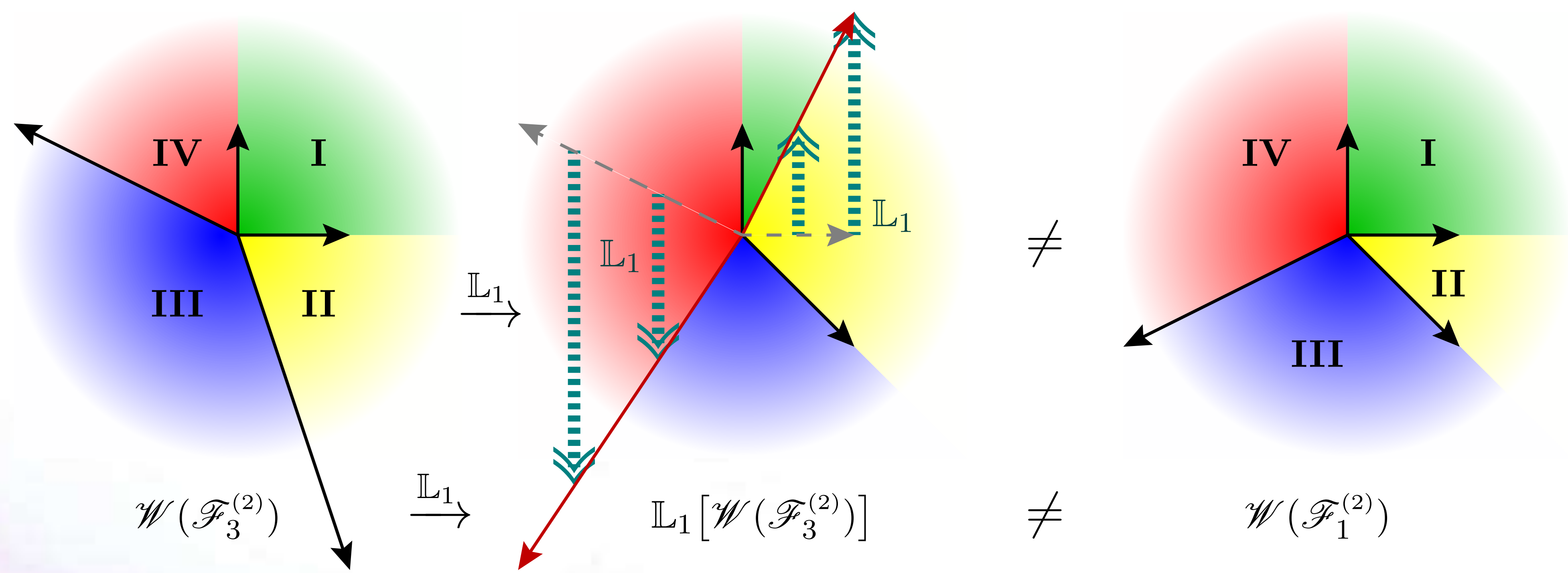
The Phase-Space

—*Proof-of-Concept*— arXiv:1611.10300

- Infinite diversity in the $\mathfrak{F}_m$:
- The $[m \pmod{n}]$ diffeomorphism $\quad \mathbb{L}_k : \mathscr{F}^{(n)}_m[c_1] \to \mathscr{F}^{(n)}_{m+nk}[c_1]$

$$\mathbb{L}_1 : \{ \overbrace{(0,1),\ (1,-m)}^{\mathscr{W}(\mathscr{F}^{(n)}_m[c_1])} \} \xrightarrow{\cdot \left[\begin{smallmatrix}1 & n\\ 0 & 1\end{smallmatrix}\right]} \{ \overbrace{(0,1),\ (1,-(m-n))}^{\mathscr{W}(\mathscr{F}^{(n)}_{m-n}[c_1])} \}$$

$$\mathbb{L}_1 : \{ \overbrace{(1,0),\ (-n,m-2)}^{\mathscr{W}(\mathscr{F}^{(n)}_m[c_1])} \} \xrightarrow{\cdot \left[\begin{smallmatrix}1 & n\\ 0 & 1\end{smallmatrix}\right]} \{(1,n),\ (-n,m-2-n^2)\} \neq \{ \overbrace{(1,0),\ (-n,(m-n)-2)}^{\mathscr{W}(\mathscr{F}^{(n)}_{m-n}[c_1])} \}$$



Phases & Discriminants

—Proof-of-Concept— arXiv:RealSoon

The Discriminant

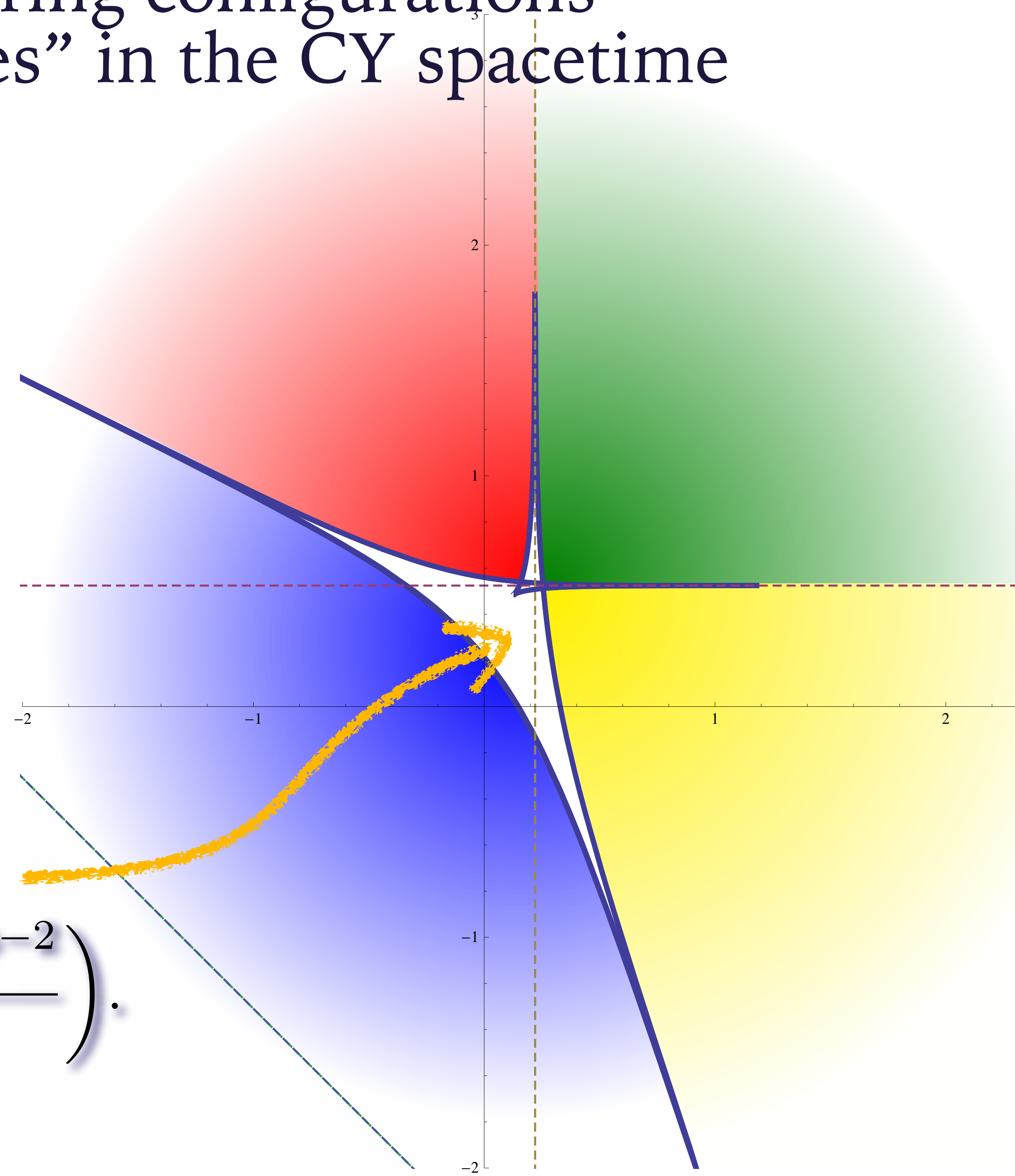
- Now add "instantons": 0-energy string configurations wrapped around "tunnels" & "holes" in the CY spacetime
  - Near $(r_1,r_2) \sim (0,0)$, classical analysis of the Kähler (metric) phase-space fails [M&P: arXiv:hep-th/9412236]
- With

	X_0	X_1	X_2	$\cdots$	X_n	X_{n+1}	X_{n+2}
Q^1	$-n$	1	1	$\cdots$	1	0	0
Q^2	$m-2$	$-m$	0	$\cdots$	0	1	1

- the instanton resummation gives:

$$r_1 + \frac{\hat{\theta}_1}{2\pi i} = -\frac{1}{2\pi}\log\left(\frac{\sigma_1^{n-1}(\sigma_1 - m\,\sigma_2)}{[(m-2)\sigma_2 - n\sigma_1]^n}\right),$$

$$r_2 + \frac{\hat{\theta}_2}{2\pi i} = -\frac{1}{2\pi}\log\left(\frac{\sigma_2^2\,[(m-2)\sigma_2 - n\sigma_1]^{m-2}}{(\sigma_1 - m\,\sigma_2)^m}\right).$$



…and in the Mirror
(Yes, the BHK-mirrors)

Phases & Discriminants

The Discriminant

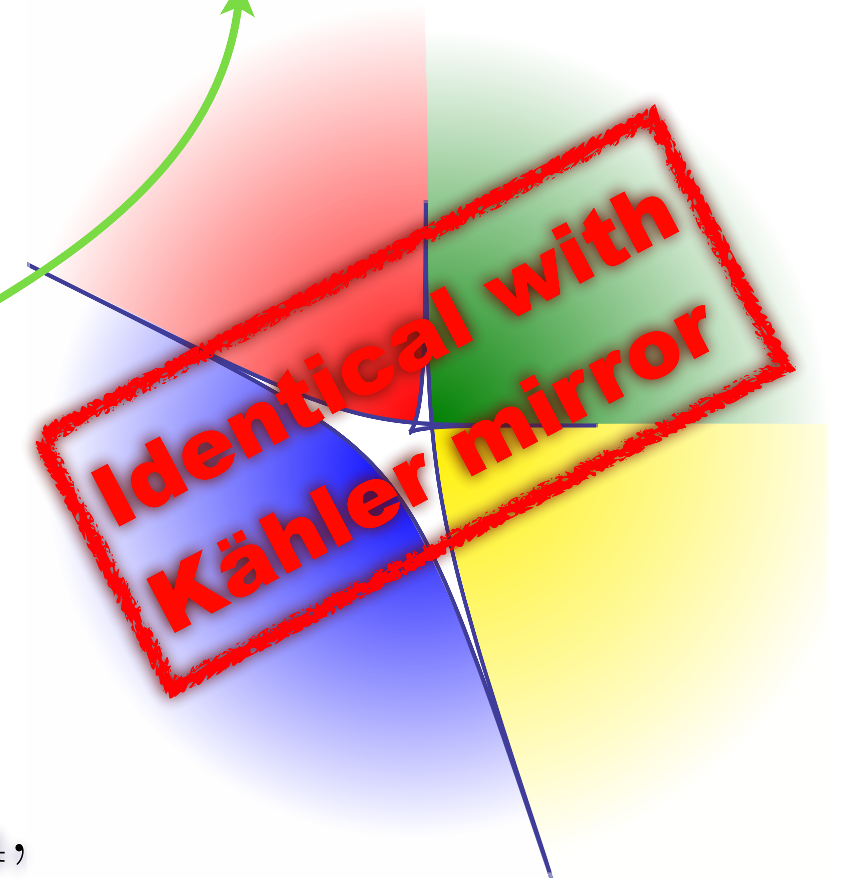
—*Proof-of-Concept*— arXiv:RealSoon

- Now compare with the complex structure of the BHK-mirror
  - Restricted to the "cornerstone" def. poly

$$f(x) = a_0 \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_0 \rangle + 1} + \sum_{\mu_I \in \Delta} a_{\mu_I} \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_I \rangle + 1}$$

Batyrev

$$g(y) = b_0 \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_0 \rangle + 1} + \sum_{\nu_i \in \Delta^\star} b_{\nu_i} \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_i \rangle + 1}$$



- In particular,

$$g(y) = \sum_{i=0}^{n+2} b_i\, \phi_i(y) \;= b_0\, \phi_0 + b_1\, \phi_1 + b_2\, \phi_2 + b_3\, \phi_3 + b_4\, \phi_4,$$

$$\phi_0 := y_1 \cdots y_4, \quad \phi_1 := y_1^2\, y_2^2, \quad \phi_2 := y_3^2\, y_4^2, \quad \phi_3 := \frac{y_1^{m+2}}{y_3^{m-2}}, \quad \phi_4 := \frac{y_2^{m+2}}{y_4^{m-2}},$$

$$z_1 = -\frac{\beta\,[(m{-}2)\beta + m]}{m{+}2}, \quad z_2 = \frac{(2\beta{+}1)^2}{(m+2)^2\, \beta^m}, \quad \beta := \left[\frac{b_1\, \phi_1}{b_0\, \phi_0} \Big/ {}^{A}\mathscr{J}(g)\right],$$

Phases & Discriminants

—Proof-of-Concept— arXiv:RealSoon

The Discriminant

- So,

$$\mathscr{W}(\mathscr{F}_m^{(n)}):\quad \begin{cases} e^{-2\pi r_1+i\hat\theta_1} = \dfrac{1-m\,\rho}{[(m{-}2)\rho-n]^n}, \\[2ex] e^{-2\pi r_2+i\hat\theta_2} = \dfrac{\rho^2\,[(m{-}2)\rho-n]^{m-2}}{(1-m\,\rho)^m}; \end{cases} \qquad \rho := \frac{\sigma_2}{\sigma_1}$$

- and

$$\mathscr{M}({}^\nabla\!\mathscr{F}_m^{(n)}[c_1]):\quad \begin{cases} z_1 = (-1)^{n-1}\dfrac{\beta\,[(m{-}2)\,\beta+m]^{n-1}}{[(n{-}1)m{+}2]^{n-1}}, \\[2ex] z_2 = \dfrac{(1+n\,\beta)^2}{[(n{-}1)m{+}2]^2\,\beta^m}, \end{cases} \qquad \beta := \left[\frac{b_1\,\phi_1}{b_0\,\phi_0}\Big/{}^A\mathscr{J}(g)\right]$$

$$\gamma := \left[\frac{b_3\,\phi_3}{b_2\,\phi_2}\Big/{}^A\mathscr{J}(g)\right]$$

$$z_1 = \frac{1-m\,\gamma}{[(m{-}2)\gamma-2]^2}, \qquad z_2 = \frac{\gamma^2\,[(m{-}2)\gamma-2]^{m-2}}{(1-m\,\gamma)^m},$$

- are identical?!
  - You bet: $\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} -n & m{-}2 \\ 1 & -m \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$

$$\mathscr{M}({}^\nabla\!\mathscr{F}_m^{(n)}) \supset \mathbb{P}^1 \xrightarrow{\ \approx\ } \mathbb{P}^1 \subset \mathscr{W}(\mathscr{F}_m^{(n)})$$

Horn uniformization — Morrison+Plesser, '93

- Better yet: $(\sigma_1,\sigma_2) \overset{\text{MM}}{\approx} (b_2\phi_2, b_3\phi_3)/{}^A\mathscr{J}$

Phases & Discriminants

The Discriminant

—Proof-of-Concept—

- So: $\mathscr{W}(\mathscr{F}^{(n)}_m[c_1]) \overset{\text{mm}}{\approx} \mathscr{M}({}^\nabla\mathscr{F}^{(n)}_m[c_1])$
- In fact, also: $\mathscr{W}({}^\nabla\mathscr{F}^{(n)}_m[c_1]) \overset{\text{mm}}{\approx} \mathscr{M}(\mathscr{F}^{(n)}_m[c_1])$
  - ✅…when restricted to no (MPCP) blow-ups & "cornerstone" polynomial
- Then, $\dim \mathscr{W}({}^\nabla\mathscr{F}^{(n)}_m[c_1]) = n = \dim \mathscr{M}(\mathscr{F}^{(n)}_m[c_1])$
- Same method:

$$e^{2\pi i\,\tilde{\tau}_\alpha} = \prod_{I=0}^{2n}\Big(\sum_{\beta=1}^{2}\widetilde{Q}^\beta_I\,\tilde{\sigma}_\beta\Big)^{\widetilde{Q}^\alpha_I}$$

$$\tilde{z}_a = \prod_{I=0}^{2n}\big(a_I\,\varphi_I(x)\big)^{\widetilde{Q}^\alpha_I}\Big/{}^A\!\mathscr{J}$$

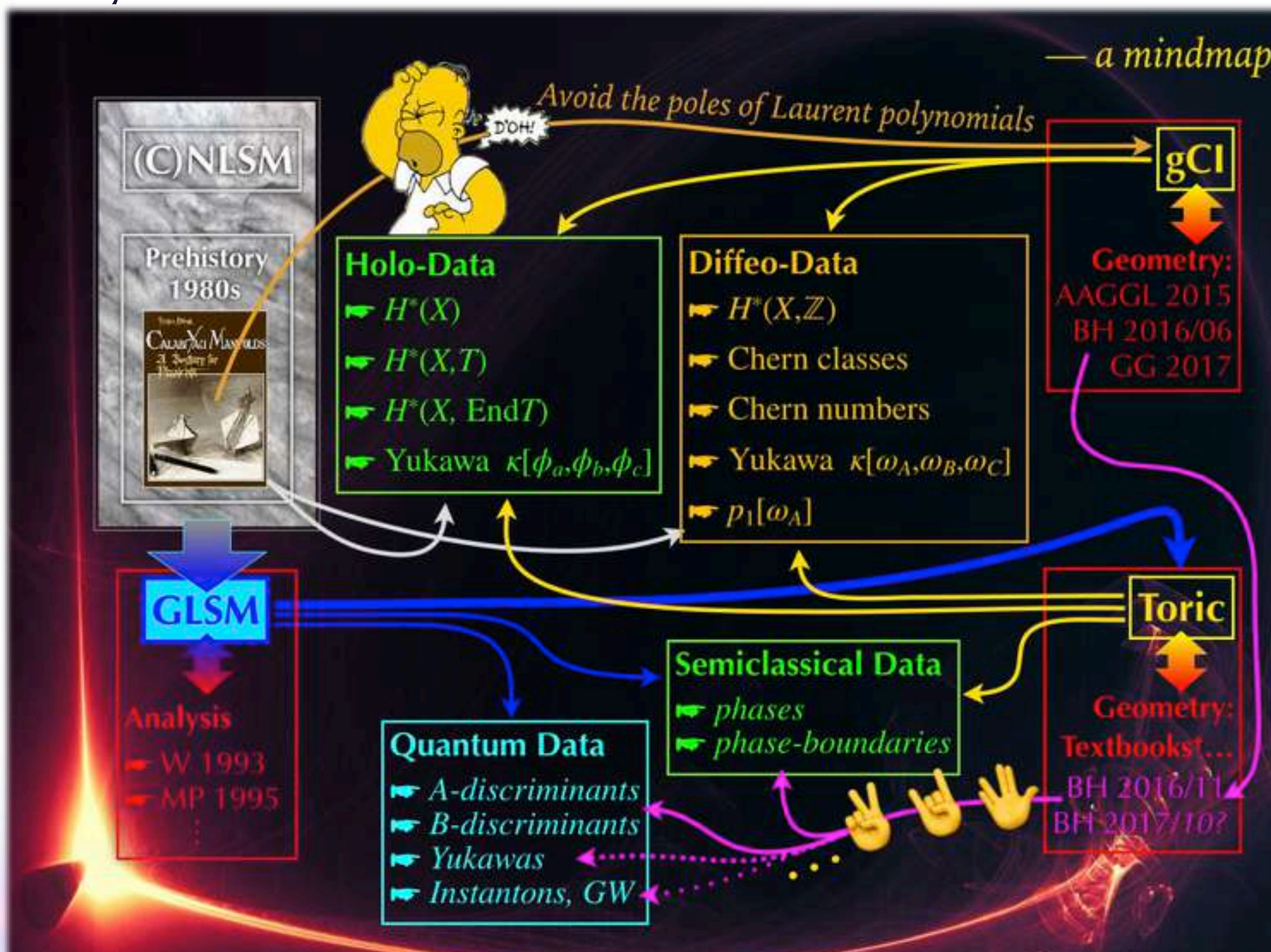
I	$(\sum_\beta \widetilde{Q}^\beta_I\,\tilde{\sigma}_\beta)$	$(a_I\,\varphi_I)/{}^A\mathscr{J}_{(210)}(f)$
0	$-2(m{+}2)(\tilde{\sigma}_1+\tilde{\sigma}_2)$	$-2\big((a_3\,\varphi_3)+(a_4\,\varphi_4)\big)$
1	$m\,\tilde{\sigma}_1+2\,\tilde{\sigma}_2$	$\frac{m\,(a_3\,\varphi_3)+2\,(a_4\,\varphi_4)}{m+2}$
2	$2\,\tilde{\sigma}_1+m\,\tilde{\sigma}_2$	$\frac{2\,(a_3\,\varphi_3)+m\,(a_4\,\varphi_4)}{m+2}$
3	$(m{+}2)\,\tilde{\sigma}_1$	$(a_3\,\varphi_3)$
4	$(m{+}2)\,\tilde{\sigma}_2$	$(a_4\,\varphi_4)$

Laurent GLSMs

Summary

—*Proof-of-Concept*—

arXiv:1611.10300 + more



Laurent GLSMs

Summary

—*Proof-of-Concept*— arXiv:1611.10300 + more

- CY(n–1)-folds in Hirzebruch 4-folds
  - Euler characteristic ✅
  - Chern class, term-by-term ✅
  - Hodge numbers ✅
  - Cornerstone polynomials & mirror ✅
  - Phase-space regions & mirror ✅
  - Phase-space discriminant & mirror ✅
  - The "other way around" ✅ *(limited)*
  - Yukawa couplings ✅
  - World-sheet instantons ✅
  - Gromov-Witten invariants 🔜✅
    - *Will there be anything else?*

$d(\theta^{(k)}) := k!\ \mathrm{Vol}(\theta^{(k)})$ [BH: *signed* by orientation!]

- Oriented polytopes
- Trans-polar$^\triangledown$ constr.
- Newton $\Delta_X := (\Delta^\star{}_X)^\triangledown$
- VEX polytopes

  s.t.: $((\Delta)^\triangledown)^\triangledown = \Delta$

  (B)BHK mirrors
- Star-triangulable

  w/flip-folded faces
- Polytope extension

  ⇔ Laurent monomials

Textbooks to be (re)written, amended

Thank You!

http://physics1.howard.edu/~thubsch/

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