Generalized Fourier-Mukai Transform in Heterotic String Theory/F-theory

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November 2, 2019

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Introduction

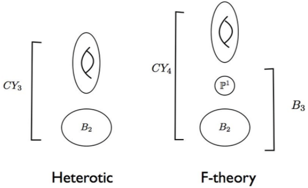
To study the Heterotic string compactifications we should describe gauge fields (vector bundles) over elliptically fibered Calabi-Yau manifolds.

- Most of the toric and CICY Calabi-Yau threefolds are elliptically fibered.

Huang, Taylor, 2019

Anderson, Gao, Gray, Lee, 2017

- The basic duality is between Heterotic on T^2 and F-theory on an elliptically fibered $K3$.



Spectral cover construction on elliptically fibered Calabi-Yau:

Let $\pi : X \longrightarrow B$ be a Weierstrass elliptic fibration, and V be a stable, degree zero, locally free sheaf (vector bundle) on X,

- The restriction of V on a generic (smooth) fiber of π is S-equivalent to Atiyah, 57

$$V|_{E_b} \sim \bigoplus_{i=1}^{n} \mathcal{O}_{E_b}(p_i - \sigma). \qquad (1)$$

  p_i's define the spectral cover S. FMW, 97

- V can be reconstructed as

$$V = \pi_{1*}(\pi_2^* \mathcal{L} \otimes \mathcal{P}^*), \qquad (2)$$

$$\pi_{1,2} : X \times_B X \longrightarrow X, \qquad \mathcal{P} = \mathcal{I}_\Delta \otimes \pi_1^* \mathcal{O}_X(\sigma) \otimes \pi_2^* \mathcal{O}_X(\sigma) \otimes K_B^*.$$

- Formally, one defines a functor,

$$\Phi : D^b(X) \longrightarrow D^b(X), \qquad (3)$$

$$\Phi(V) = R\pi_{2*}(\pi_1^* V \otimes \mathcal{P}). \qquad (4)$$

Spectral cover cont'd

- Since V is (semi)stable, degree zero,

$$\Phi(V) = i_{S*}\mathcal{L}[-1]. \tag{5}$$

- By comparing $Ch(\Phi(V))$ and $Ch(i_{S*}\mathcal{L})$, and making some assumptions about $Pic(S)$, it is possible to derive topological constraints on V.

Anderson, Gao, MK, 2019

FMW, 97

Andreas, Curio, Ruiperez, Yau, 2000

Vector bundle moduli space

- Fourier-Mukai gives a one-to-one correspondence, BBRH, 2008

$$V \leftrightarrow (\mathcal{L}, S), \tag{6}$$

  and preserves semistability.

- FM transform can be used to derive constraints for small instanton transitions. Specially in 4D, M5 branes that wrap the fiber or a curve in base B, can get absorbed or emitted by the end of the world branes. Ovrut, Pantev, Park, 2000

Heterotic/F-theory duality

- Using the data $(\mathcal{L}, S, X)$ we can find the F-theory dual, in particular in 4D, FMW, 97

$$Def(S) \longleftrightarrow Def(X_F), \tag{7}$$
$$H^{1,1}(S) \longleftrightarrow H^{2,2}(X_F), \tag{8}$$
$$J(S) \longleftrightarrow IJ(X_F). \tag{9}$$

- In stable degeneration limit, $(\mathcal{L}, S)$ maps to the spectral data $(\mathcal{L}_F, S_F)$ embedded in $P1$ fibration over singular locus corresponding to the Higgs bundle inside the $7-branes$. Donagi, Wijnholt, 2008

Beasley, Heckman, Vafa, 2008

Limitations

The calculations in FMW are based on many assumptions, that limit their validity. S must smooth and generic, divisors on S are induced from the ambient space, and X is given Weierstrass model. However,

- If S is reducible, Anderson, Gao, MK, 2019

$$S = S_1 \cup_\Sigma S_2, \tag{10}$$

  and $\mathcal{L}$ a general coherent sheaf on (of rank 1) S , $Ch(V)$ will be inconsistent with what has been derived in FMW. Deformations are obstructed.

- In GLSM's V is defined by a monad, Anderson, Gao, MK, 2019

$$0 \longrightarrow V \longrightarrow \mathcal{H} \longrightarrow \mathcal{N} \longrightarrow 0, \tag{11}$$

  Generally, the spectral sheaf of such bundles are supported over non-reduced schemes. So FMW's analysis is not sufficient for many Heterotic Pheno models.

- By deforming S, NS(S) can jump. Curio, 2012

Elliptically fibered Calabi-Yau with $SU(n)$ exceptional divisor

- X doesn't need to be Weierstrass model. Assume,

$$E_1, E_2, \ldots, E_{n-1} \tag{12}$$

  are exceptional divisors corresponding to the $SU(n)$ Cartan.
- Since X has a (holomorphic section) section σ, the "Poincare sheaf" defined as,

  Caldararu, 98

$$\mathcal{P} = \mathcal{I}_\Delta \otimes \pi_1^* \mathcal{O}_X(\sigma) \otimes \pi_2^* \mathcal{O}_X(\sigma), \tag{13}$$

- The functor Φ defined as before is still Fourier-Mukai. And can be used to find $Ch(V)$. In particular, E_i can induce (-2)-curves inside S (e), and they contribute as (For $SU(2)$ singularity and $B = \mathbb{P}^2$)

$$c_1(\mathcal{L}) \to c_1(\mathcal{L}) + \beta e, \tag{14}$$

$$c_2(V) \to c_2(V) + \beta^2 f - \beta D \cdot E, \tag{15}$$

$$c_3(V) \to c_3(V). \tag{16}$$

Anderson, Gao, MK, 2019

$SU(n)$ cont's

- To find F-theory dual, we need to go to a limit where $vol(f) >> vol(E_i \cdot f)$, i.e. practically we need to blow down E_i,

$$\begin{array}{ccc} X & \xrightarrow{\rho} & X_{sing} \\ & \searrow & \downarrow \\ & & X_F \end{array} \tag{17}$$

But there is no FM functor between $D^b(X)$ and $D^b(X_{sing})$, so it's not correct to define the spectral data $(\mathcal{L}, S)$ in the singular limit.

Small instanton transitions

- There are new possibilities for small instanton transitions, for example if

$$[c_1] = D \cdot (D - E), \tag{18}$$
$$0 \longrightarrow \tilde{V} \longrightarrow V \longrightarrow i_{c_1 *}\mathcal{O}_{c_1}(-1) \longrightarrow 0 \tag{19}$$

Corresponds to absorption of an $M5$ brane wraps on $[c_1]$.

Anderson, Gao, MK, 2019

- Contrary to older analysis, neither gauge or chirality of the effective theory changes.

Elliptically fibered Calabi-Yau with non trivial MW group

- Assume $\pi : X \longrightarrow B$ is an elliptically fibered Calabi-Yau, with $rk(MW) \neq 0$. Let σ to be zero section and S_i be the generators.
- If σ is holomorphic, the Poincare sheaf that defined as before will be well defined and Φ will be Fourier-Mukai.
- We get new formulas for $Ch(V)$, no new contribution in chirality of EFT.
- If σ is rational then the Poincare sheaf is not well defined. We didn't find a canonical way to deal with this problem.

Possible contribution in EFT's chirality

- If we can X admits a birational transformation (a flop in particular),

$$\begin{array}{ccc} & \tilde{X} & \\ {}^{p}\swarrow & & \searrow^{q} \\ X & & X' \end{array} \tag{20}$$

Then $Rq_* Lp^* : D^b(X) \longrightarrow D^b(X')$ is an equivalence. Bondal, Orlov, 95

- If σ becomes a holomorphic section after the flop, we get can define $\mathcal{P}$ and Φ as before for in X' rather than X.
- We hoped to see extra contribution to the EFT chirality from the (-1)-curves inside σ (in X). We can show (in case S is generic and smooth), Anderson, Gao, MK

$$R\pi_* V = R\pi'_*(\mathcal{L}^\bullet \otimes \mathcal{O}_{\sigma'}) \tag{21}$$

From this it's clear that if $\mathcal{L}_0 \neq 0$ we have extra contributions to the index of V, and therefore the chirality in the effective theory.