Discrete anomaly in QCD

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Based on (1710.08923 w/ Misumi, Sakai & 1807.07666)

Motivation Confinement v.s. Chiral sym. breaking

helicity $\sigma \cdot p$ ~ chirality

Assume massless quarks are confined inside color-singlet hadron

⇓

To be bounded,

$\vec{p} \Rightarrow -\vec{p}$ at bdry

$\rightsquigarrow \quad \sigma \cdot \vec{p} \Rightarrow -\sigma \cdot \vec{p}$

∴ At bdry, chirality flipping process must happen.

$\rightsquigarrow$ Confinement ⇒ Chiral SSB

$\boxed{\text{Q. Can we make (some part of) this precise?}}$

Tool 't Hooft anomaly matching

't Hooft anomaly

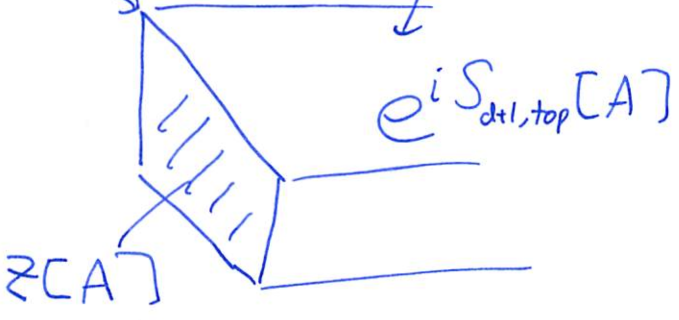
d-dim QFT with global sym. G

A : G-background gauge field.

$$Z[A+\underbrace{d\theta}_{\text{G-gauge trans.}}] = \underbrace{e^{i\mathcal{A}[\theta, A]}}_{\text{anomalous violation phase in a controllable fashion}} Z[A]$$

⇒ 't Hooft anomaly!

Anomaly matching : $\mathcal{A}_{UV} = \mathcal{A}_{IR}$!



Combined system

$$Z[A]\, e^{iS_{d+1,top}[A]}$$

is gauge-inv.

$$\xRightarrow{\text{RG-flow}} Z_{IR}[A]\, e^{iS_{d+1,top}[A]}$$

Warm-up $(1+1)d$ $\mathbb{CP}^{N-1}$ σ-model

$\mathbb{CP}^1$ $\quad S = \int \frac{1}{2g^2} |(d + i\underbrace{a}_{U(1)\text{ gauge field.}})\vec{z}|^2 + i\frac{\theta}{2\pi}\int da$

$$\vec{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \in \mathbb{C}^2 \qquad |\vec{z}|^2 = 1.$$

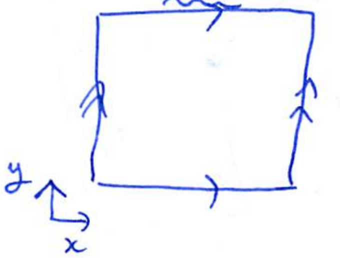
$\theta \sim \theta + 2\pi$. $\quad$ Global sym. $\frac{SU(2)}{\mathbb{Z}_2} \simeq SO(3)$.

θ-periodicity is broken by background gauge field of $SO(3)$

(See Ho-Tat's talk)

Twisted partition func. on T^2.

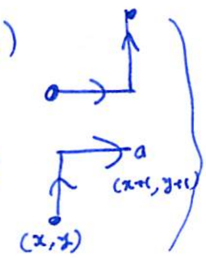
$z(x,y+1) = i\sigma_z(x,y)$



$z(x+1,y) = \underbrace{i\sigma_x}_{\in SU(2)} z(x,y)$

Something is wrong...

$$\left(\begin{aligned} z(x+1,y+1) &= i\sigma_x\, i\sigma_z\, z(x,y) \\ &= \underbrace{i\sigma_z\, i\sigma_x}_{\shortparallel} z(x,y) \end{aligned} \right)$$

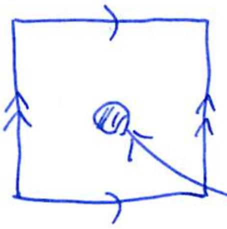


$(x+1,y+1)$ (x,y)

$(-1)\, i\sigma_x\, i\sigma_z$.

$\Downarrow$ To do this correctly,

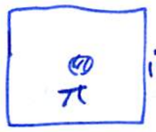
$i\sigma_z$-twist



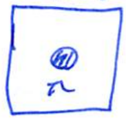
$i\sigma_x$-twist

$U(1)$ Aharonov-Bohm flux with $\int da = \pi \pmod{2\pi}$.

$$\leadsto Z_{\theta+2\pi}\left(\right) = \underbrace{(-1)}_{\uparrow} Z_\theta\left(\right)$$



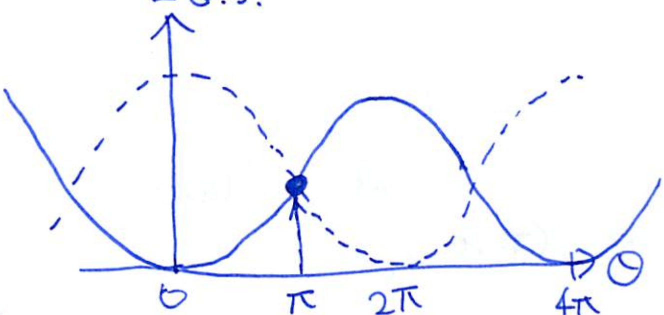
$i\sigma_z$ π $i\sigma_x$



$i\sigma_z$ π $i\sigma_x$

Anomalous phase for θ-periodicity by 2π

$E_{G.S.}$

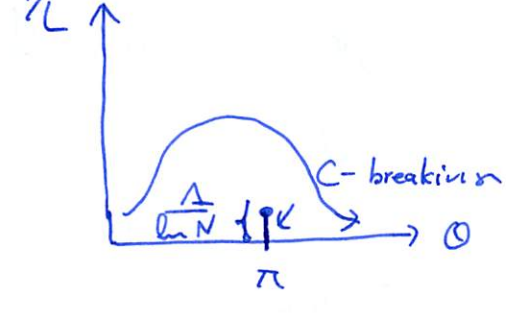


Now, we consider the twisted compactification:

2d $\mathbb{CP}^{N-1}$ model $\rightarrow$ Twisted $\mathbb{CP}^{N-1}$ QM $(\mathbb{R}\times \underset{\substack{\text{very small} \\ L}}{\underline{S^1}})$

(cf) Thermal case If we take $\vec{z}(x, \tau+L) = \vec{z}(x)$.

Phase diagram
(Affleck)

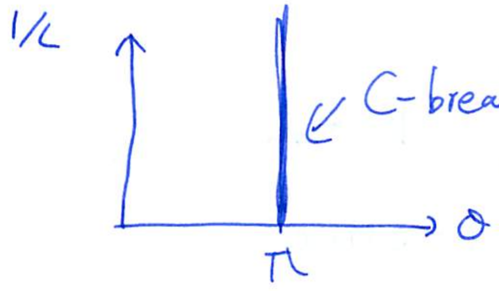


as finite-$T\left(=\frac{1}{L}\right)$ phases,
$(\theta=0)$ phase $=$ $(\theta=2\pi)$ phase.

Although $\theta=0$ and 2π are separated at $T=0$.

Twisted compactification B.C. $\vec{z}(x,\tau+L) = \underbrace{\begin{pmatrix} 1 & & \\ & \omega & \\ & & \ddots \\ & & & \omega^{N-1}\end{pmatrix}}_{C} z(x,\tau)$ $(\omega^N=1)$

Phase diagram

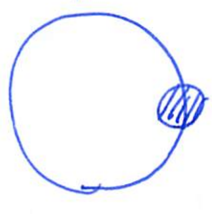


C-breaking at any $1/L$!!

Polyakov loop $P = e^{i\oint_{S^1} a} \in U(1)$

Effective potential

Thermal

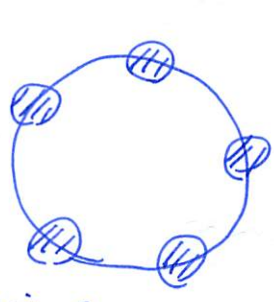


Unique minimum
$P \to e^{i\alpha}P$ is <u>not</u> sym.

$\Downarrow$

$\langle P\rangle = 0 \Rightarrow$ C-breaking at $\theta=\pi$

Twisted



N minima
$\boxed{P \to e^{\frac{2\pi i}{N}}P \text{ is sym.}}$

$\Downarrow$

$\mathbb{Z}_N$-sym. $\begin{cases} \vec{z}_{(x,\tau+L)} = \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & & 1 \\ 1 & & & 0\end{pmatrix}\vec{z}(x,\tau) \\ a_0 \to a_0 + \frac{2\pi}{NL}\end{cases}$

Interesting thing

$Z_{2d\,\mathbb{CP}^{N-1},\theta+2\pi}\left[\begin{smallmatrix}SO(3)\\ \text{backgrd}\end{smallmatrix}\right] = (\pm 1)e^{\frac{2\pi i}{N}} Z_{2d\,\mathbb{CP}^{N-1},\theta}\left[\begin{smallmatrix}SO(3)\\ \text{backgrd}\end{smallmatrix}\right]$

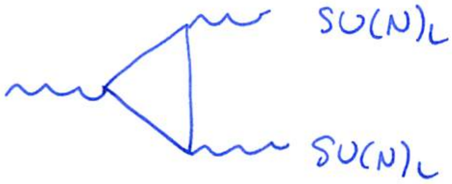
$\overset{\text{equiv.}}{\Longleftrightarrow}$ $Z_{1d\text{ twisted }\mathbb{CP}^{N-1},\theta+2\pi}[\mathbb{Z}_N \text{ background}] = e^{\frac{2\pi i}{N}} Z_{1d\text{ twisted},\theta}\left[\mathbb{Z}_N \text{ backgrd}\right]$.

QCD

Consider $N = N_c = N_f$. Massless quarks.

$U(1) - SU(N)_L - SU(N)_L$ triangle anomaly

$\underset{\text{baryon current}}{\underbrace{\partial_\mu J^\mu_B}}$



$$\partial_\mu J^\mu_B \sim \mathrm{tr}(F_L^2).$$

($\Rightarrow$ Anomalous baryon # violation by E-W. instanton)

At low energy,

$$J^\mu_{\text{Skyrmion}} \sim \frac{1}{24\pi^2} \mathrm{tr}\left((U^\dagger dU)^3\right).$$

$$d\, J_{\text{skyrmion}}\left[\begin{matrix}\text{Background}\\ A_L\end{matrix}\right] \sim \mathrm{tr}(F_L^2).$$

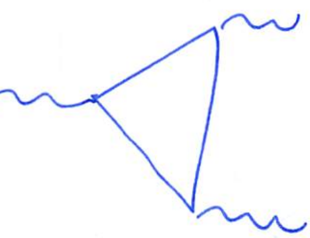
To obtain discrete anomally (in an "illegal" way),

consider

$$\underline{\frac{SU(N)_V \times U(1)}{\mathbb{Z}_N \times \mathbb{Z}_N} \times (\mathbb{Z}_{2N})_A} \subset \frac{SU(N)_L \times SU(N)_R \times U(1)}{\mathbb{Z}_N \times \mathbb{Z}_N}.$$

$\uparrow$ Consider the background gauge field for this

$\partial_\mu J^\mu_B$



$SU(N)_L \Rightarrow \dfrac{SU(N)_V}{\mathbb{Z}_N}$

$SU(N)_L \Rightarrow (\mathbb{Z}_{2N})_A$

$$dJ_B = \frac{N_f}{(2\pi)^2} \underline{\underline{\mathrm{tr}(F_L)}} \wedge dA_\chi$$

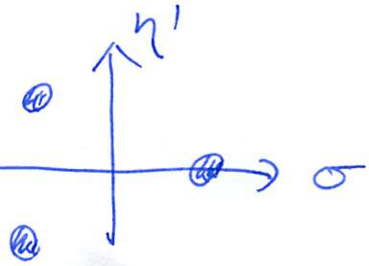
$$\parallel$$

$$B_f$$

In 4d, we have shown that new anomaly constraint exists for chiral SSB

- $SU(N)_L \times SU(N)_R \to SU(N)_V \qquad \langle \bar{\Psi}_L \Psi_R \rangle \neq 0$

- ~~$SU(N)_L \times SU(N)_R \to SU(N)_V \times (\mathbb{Z}_{2N})_A$~~ $\left\{\begin{matrix}\langle \bar{\Psi}_L \Psi_R \rangle = 0. \\ \langle \bar{\Psi}_L T^a \Psi_R \bar{\Psi}_R T^a \Psi_L \rangle \neq 0\end{matrix}\right.$



$\uparrow$ This is ruled out, although these two locally look the same!

Now, consider <u>conf. vs. Chiral SSB</u>

As in $\mathbb{CP}^{N-1}$ case, consider twisted b.c.

$$q(\vec{x}, t+L) = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{N-1} \end{pmatrix} q(\vec{x}, t).$$

$\Rightarrow$ We have $\mathbb{Z}_N$ sym. that acts on Polyakov loop

$$P \longrightarrow e^{\frac{2\pi i}{N}} P \quad (\text{as in pure YM})$$

<u>Claim by anomaly matching</u>

$$\underbrace{\langle P \rangle = 0}_{\substack{\parallel \\ \text{Conf.}}} \Rightarrow \underbrace{\langle \bar{\Psi}_L \Psi_R \rangle \neq 0}_{\substack{\parallel \\ \text{Chiral SSB}}}$$

Conf. (if we def. confinement by Polyakov loop...)

Chiral SSB

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