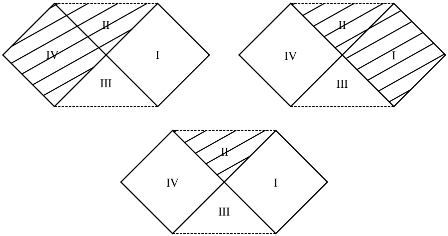
A Dual Approach to Defining Black Holes

James Wheeler

Duke University
Southeastern Regional Mathematical String Theory Meeting

Saturday, October 8, 2022



Outline

What is a Black Hole?

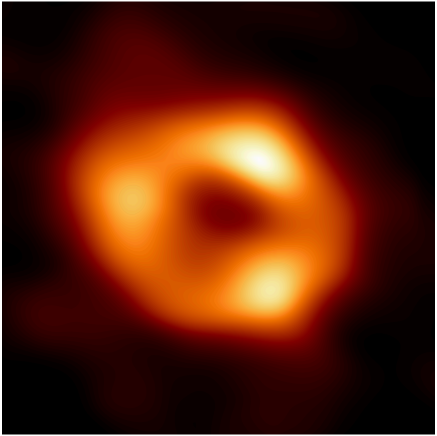


Image of Sgr A* by EHT Collaboration

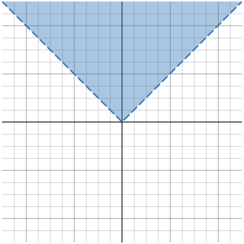
A region of spacetime in which gravity is so strong that light cannot escape.

What is a Black Hole?

A few such regions in Minkowski space...

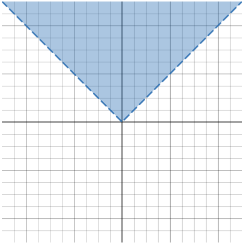
What is a Black Hole?

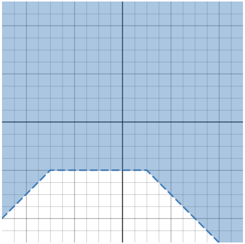
A few such regions in Minkowski space...



What is a Black Hole?

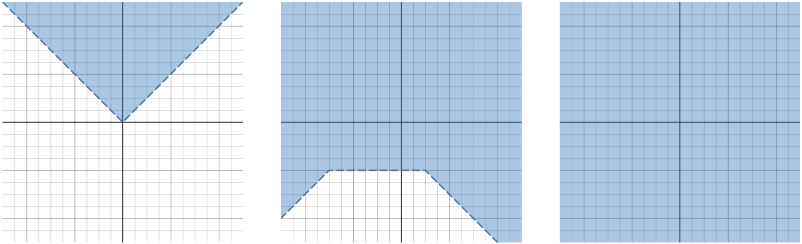
A few such regions in Minkowski space...





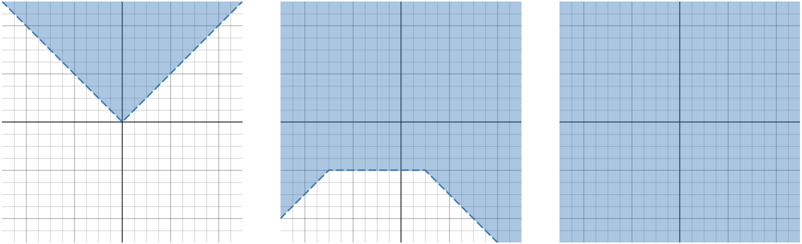
What is a Black Hole?

A few such regions in Minkowski space...



What is a Black Hole?

A few such regions in Minkowski space...



Something more is needed. A black hole must also be "small" in some appropriate sense.

The Standard Approach

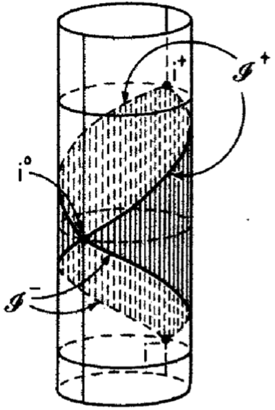
Require spacetime to be *asymptotically flat*.

The Standard Approach

Require spacetime to be *asymptotically flat*.

asymptotically empty and simple

Image adapted from Figure 11.1 of Wald's *General Relativity*



The Standard Approach

Require spacetime to be *asymptotically flat*.

asymptotically empty and simple $(\widetilde{M}, \tilde{g}) \stackrel{\phi}{\hookrightarrow} (\widehat{M}, \hat{g})$

The Standard Approach

Require spacetime to be *asymptotically flat*.

$$\begin{array}{ccccc} \textit{asymptotically empty and simple} & (\widetilde{M}, \tilde{g}) & \overset{\phi}{\hookrightarrow} & (\widehat{M}, \hat{g}) \\ \downarrow & \uparrow \psi & & \\ \textit{weakly asymptotically empty and simple} & (U, g) & \subset & (M, g) \end{array}$$

The Standard Approach

Require spacetime to be *asymptotically flat*.

$$
\begin{array}{cccc}
\textit{asymptotically empty and simple} & (\widetilde{M}, \tilde{g}) & \overset{\phi}{\hookrightarrow} & (\widehat{M}, \hat{g}) \\
\downarrow & \uparrow \psi & & \\
\textit{weakly asymptotically empty and simple} & (U, g) & \subset & (M, g) \\
\downarrow & & & \\
\textit{future asymptotically predictable} & \mathscr{J}^+ & \subset & \overline{D^+(\mathscr{S})}
\end{array}
$$

The Standard Approach

Require spacetime to be *asymptotically flat*.

$$
\begin{array}{lccc}
\textit{asymptotically empty and simple} & (\widetilde{M}, \tilde{g}) & \overset{\phi}{\hookrightarrow} & (\widehat{M}, \hat{g}) \\
\downarrow & \uparrow \psi & & \\
\textit{weakly asymptotically empty and simple} & (U, g) & \subset & (M, g) \\
\downarrow & & & \\
\textit{future asymptotically predictable} & \mathscr{I}^+ & \subset & \overline{D^+(\mathscr{S})}
\end{array}
$$

$\mathscr{B}_c \subset M$ is then the complement of

$$J^{-1}(\mathscr{I}^+) := J^- \left((\phi \circ \psi)^{-1} \left(\widehat{J}^{-1}(\mathscr{I}^+) \right) \right)$$

Extant Generalizations

Largely centered around trapped surfaces.

Extant Generalizations

Largely centered around trapped surfaces.

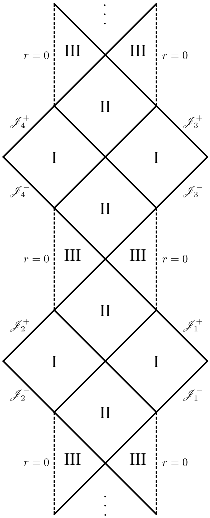
Fruitful for numerical and thermodynamic concerns.

Extant Generalizations

Largely centered around trapped surfaces.

Fruitful for numerical and thermodynamic concerns.

Arguably fall short of capturing the black hole concept.

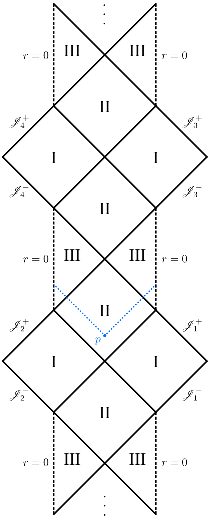


Extant Generalizations

Largely centered around trapped surfaces.

Fruitful for numerical and thermodynamic concerns.

Arguably fall short of capturing the black hole concept.

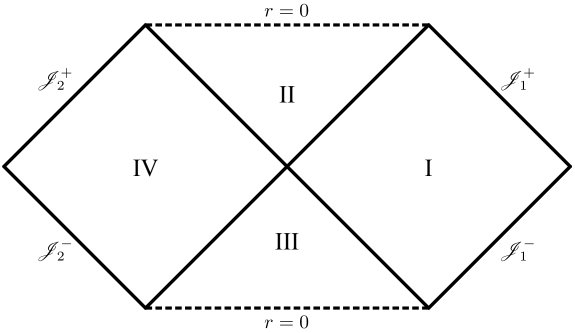


A New Approach

We would like to capture “smallness” without referencing $\mathscr{I}^+$.

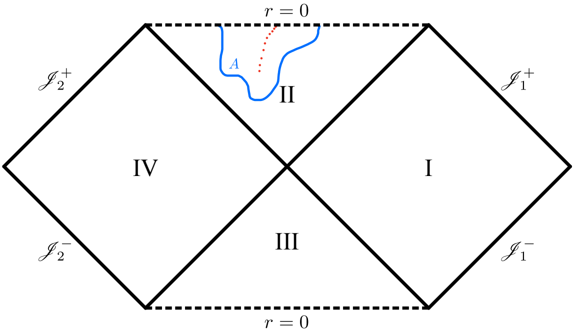
A New Approach

We would like to capture "smallness" without referencing $\mathscr{I}^+$.



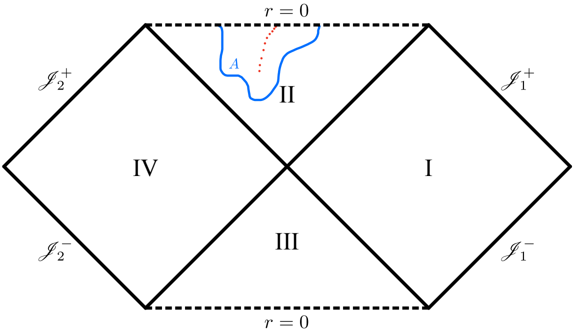
A New Approach

We would like to capture "smallness" without referencing $\mathscr{I}^+$.



A New Approach

We would like to capture "smallness" without referencing $\mathscr{I}^+$.



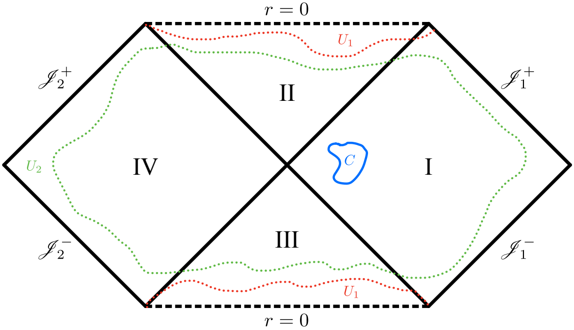
Key observation: compactness **only** fails due to singular limits.

Salvaging Compactness

Suppose we are gifted the family $\mathscr{U}$ of *singular neighborhoods*.

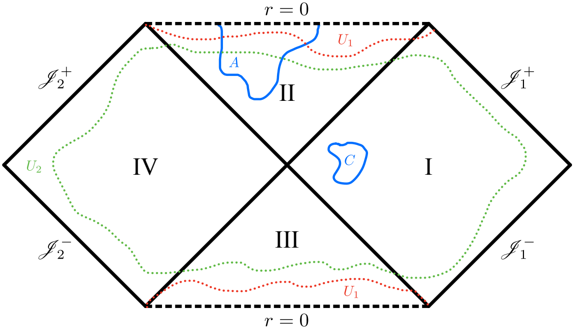
Salvaging Compactness

Suppose we are gifted the family $\mathscr{U}$ of *singular neighborhoods*.



Salvaging Compactness

Suppose we are gifted the family $\mathscr{U}$ of *singular neighborhoods*.



We now define a closed set $A \subset M$ to be *singularly compact* if $A \backslash U$ is compact for every $U \in \mathscr{U}$.

A Dual Perspective

Definition

Let $\mathscr{F}$ be the family of singularly compact future sets. The *Black Region* *$\mathscr{B}$ is given by*

$$\mathscr{B} := \bigcup_{A \in \mathscr{F}} A.$$

A Dual Perspective

Definition

Let $\mathscr{F}$ be the family of singularly compact future sets. The Black Region $\mathscr{B}$ is given by

$$\mathscr{B} := \bigcup_{A \in \mathscr{F}} A.$$

Rather than the points from which one **can't** reach infinity, we identify $\mathscr{B}$ as the points from which one **must** approach a singularity.

A Dual Perspective

Definition

Let $\mathscr{F}$ be the family of singularly compact future sets. The Black Region $\mathscr{B}$ is given by

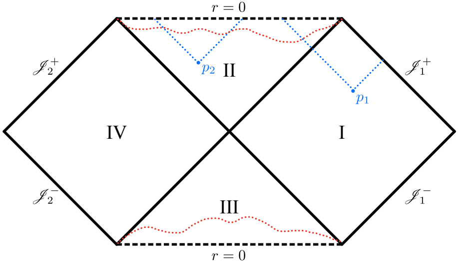
$$\mathscr{B} := \bigcup_{A \in \mathscr{F}} A.$$

Rather than the points from which one **can't** reach infinity, we identify $\mathscr{B}$ as the points from which one **must** approach a singularity.

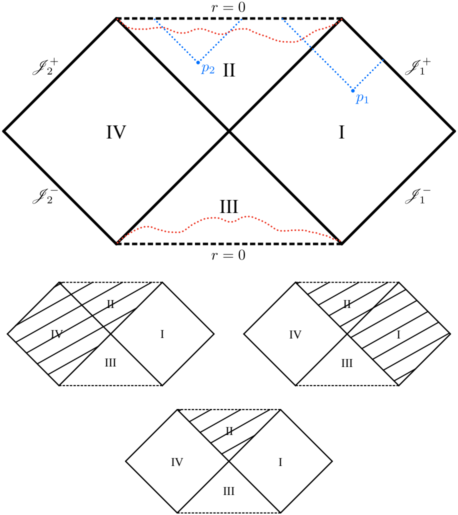
Lemma

$p \in \mathscr{B} \iff \overline{J^+(p)}$ *is singularly compact.*

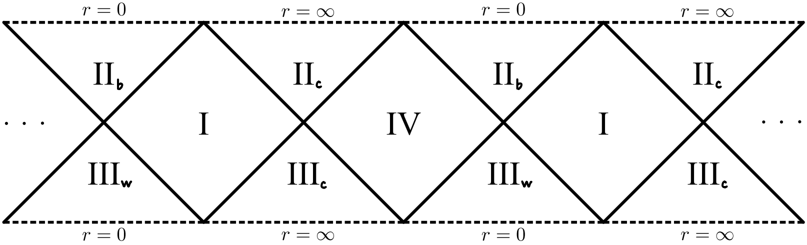
Examples: Schwarzschild



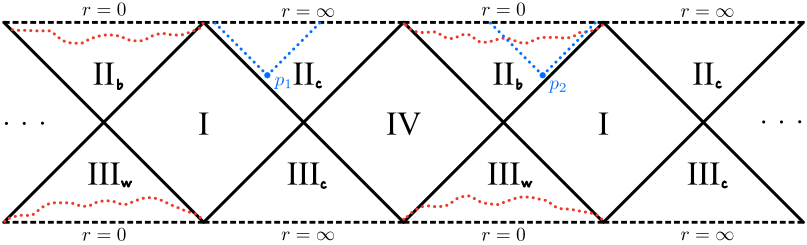
Examples: Schwarzschild



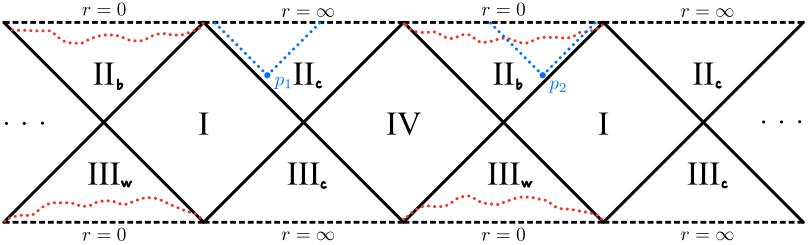
Examples: deSitter Schwarzschild

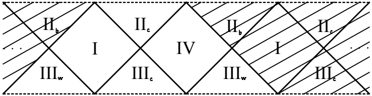


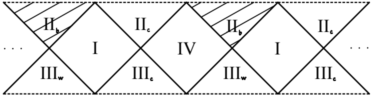
Examples: deSitter Schwarzschild



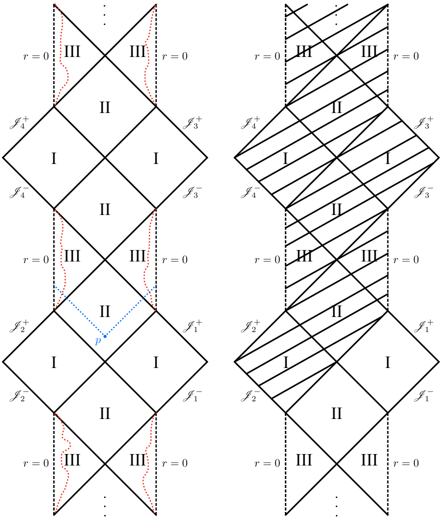
Examples: deSitter Schwarzschild



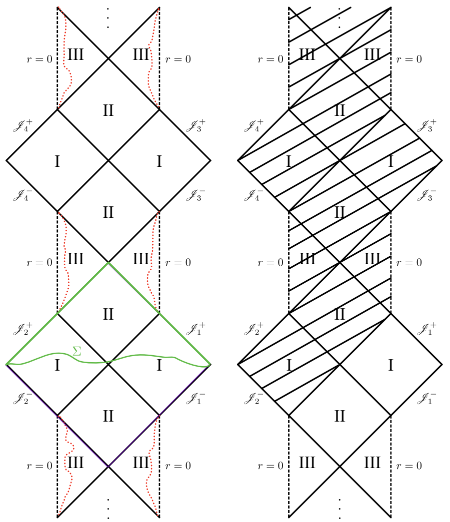




Examples: Kerr



Examples: Kerr



Strong Cosmic Censorship indicates this should not be an issue.

Locating Singularities

Must confront the identification of $\mathscr{U}$.

Locating Singularities

Must confront the identification of $\mathscr{U}$.

What do we mean by a physical "singularity"?

Locating Singularities

Must confront the identification of $\mathscr{U}$.

What do we mean by a physical “singularity”?

How can we characterize being “close” to a singularity?

Boundary Constructions

Idea: construct topological space $\overline{M}$ comprised of M together with "boundary points".

Boundary Constructions

Idea: construct topological space $\overline{M}$ comprised of M together with "boundary points".

Several approaches in '70s:

- c-boundary
- g-boundary
- b-boundary

Boundary Constructions

Idea: construct topological space $\overline{M}$ comprised of M together with "boundary points".

Several approaches in '70s:

- c-boundary
- g-boundary
- b-boundary

Topological defects (Geroch, Can-Bin, & Wald - 1982).

Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

Abstract Boundary

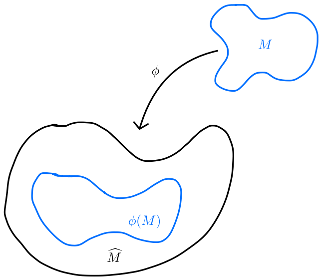
Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

Motivation: generalize coordinate chart intuition.

Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

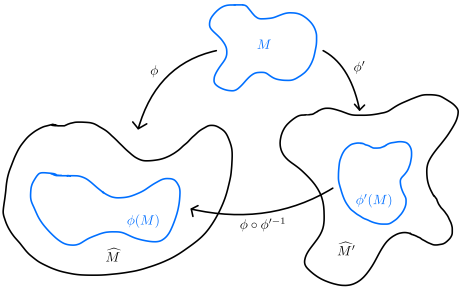
Motivation: generalize coordinate chart intuition.



Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

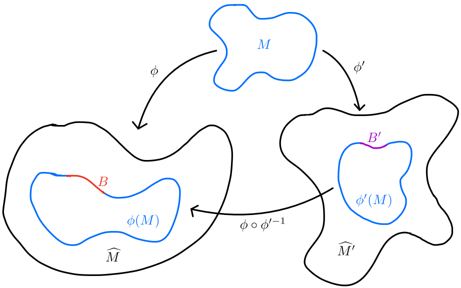
Motivation: generalize coordinate chart intuition.



Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

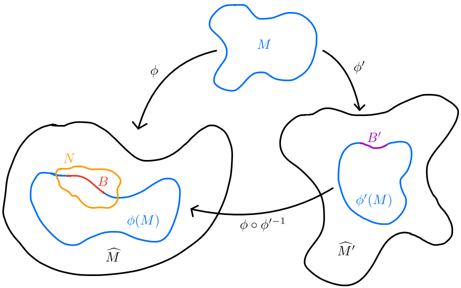
Motivation: generalize coordinate chart intuition.



Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

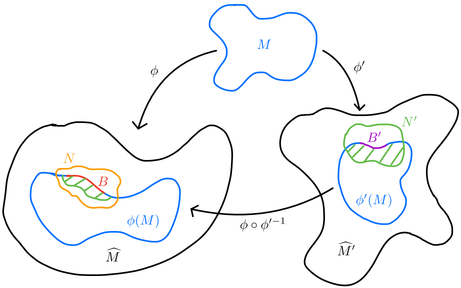
Motivation: generalize coordinate chart intuition.



Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

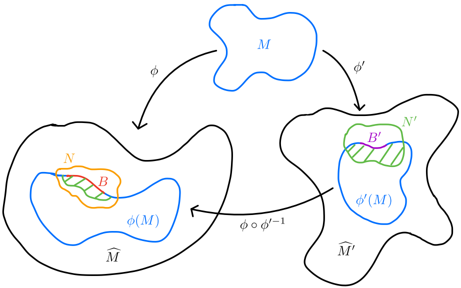
Motivation: generalize coordinate chart intuition.



Abstract Boundary

Proposed in 1994 (Scott and Szekeres); topology in 2014 (Barry and Scott).

Motivation: generalize coordinate chart intuition.



Here, B *covers* B', written $B \triangleright B'$.

Abstract Boundary

Equivalence relation: $B \sim B' \iff B \triangleright B'$ and $B' \triangleright B$.

Abstract Boundary

Equivalence relation: $B \sim B' \iff B \triangleright B'$ and $B' \triangleright B$.

The *abstract boundary* is then

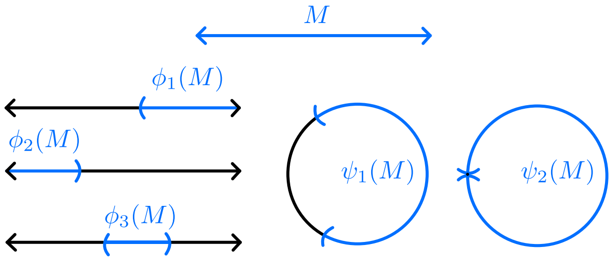
$$\mathcal{B}(M) := \left\{ [p] \;\middle|\; p \in \partial(\phi(M)) \text{ for some } \phi : M \to \widehat{M} \right\}$$

Abstract Boundary

Equivalence relation: $B \sim B' \iff B \triangleright B'$ and $B' \triangleright B$.

The *abstract boundary* is then

$$\mathcal{B}(M) := \left\{ [p] \;\middle|\; p \in \partial(\phi(M)) \text{ for some } \phi : M \to \widehat{M} \right\}$$



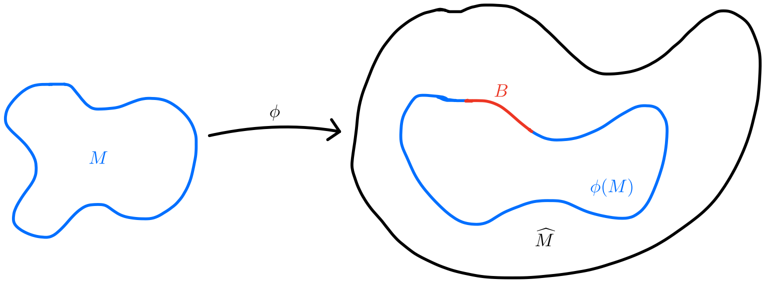
$M = \mathbb{R}$ example. $\mathcal{B}(M)$ contains three points.

Singular Neighborhoods

How should one characterize a neighborhood in M of an abstract boundary set $[B]$?

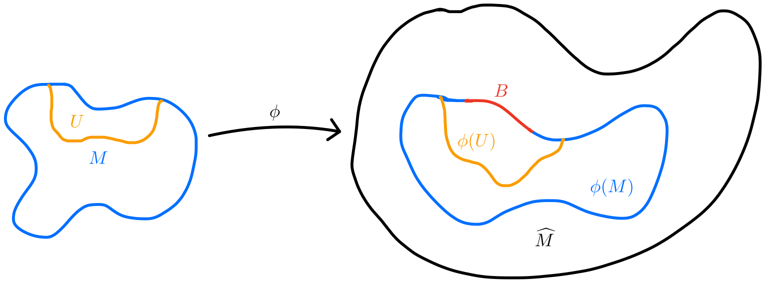
Singular Neighborhoods

How should one characterize a neighborhood in M of an abstract boundary set $[B]$?



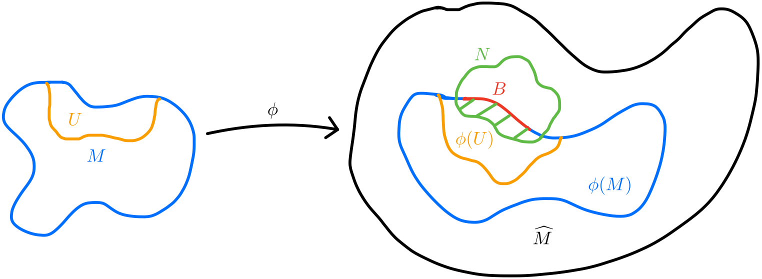
Singular Neighborhoods

How should one characterize a neighborhood in M of an abstract boundary set $[B]$?



Singular Neighborhoods

How should one characterize a neighborhood in M of an abstract boundary set $[B]$?



Here, B is *strongly attached* to U.

Singular Neighborhoods

This yields a natural topology on $\overline{M} := M \cup \mathcal{B}(M)$, with basis

$$\{U \cup \mathcal{B}_U \mid U \subset M \text{ open}\}$$

Singular Neighborhoods

This yields a natural topology on $\overline{M} := M \cup \mathcal{B}(M)$, with basis

$$\{U \cup \mathcal{B}_U \mid U \subset M \text{ open}\}$$

The metric may now be invoked to classify certain abstract boundary points as singularities.

Singular Neighborhoods

This yields a natural topology on $\overline{M} := M \cup \mathcal{B}(M)$, with basis

$$\{U \cup \mathcal{B}_U \mid U \subset M \text{ open}\}$$

The metric may now be invoked to classify certain abstract boundary points as singularities.

The *singular neighborhoods* of M may finally be identified as open sets in $\overline{M}$ containing all singularities.

Characterizing $\mathscr{B}$

With our framework fully specified, we may prove results towards the structure of $\mathscr{B}$.

Characterizing $\mathscr{B}$

With our framework fully specified, we may prove results towards the structure of $\mathscr{B}$.

Theorem

Let (M, g) be strongly causal. If $p \in \mathscr{B}$, then every sequence along the end of an inextendible, future-directed causal curve through p has a pure singularity as an accumulation point in $\overline{M}$.

Characterizing $\mathscr{B}$

With our framework fully specified, we may prove results towards the structure of $\mathscr{B}$.

Theorem

Let (M, g) be strongly causal. If $p \in \mathscr{B}$, then every sequence along the end of an inextendible, future-directed causal curve through p has a pure singularity as an accumulation point in $\overline{M}$.

This formalizes the intuition that one must approach a singularity from $\mathscr{B}$.

Characterizing $\mathscr{B}$

Theorem

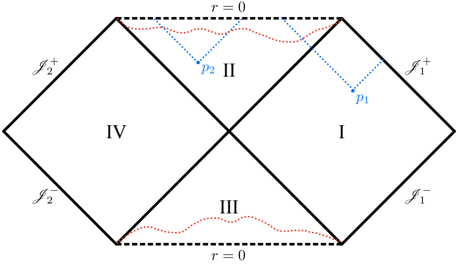
If there exists an envelopment $\phi : M \to \widehat{M}$ *under which* $\overline{\phi(I^+(p))}$ *is compact and every boundary point in* $\widehat{M}$ *attached to* $I^+(p)$ *is a pure singularity, then* $p \in \mathscr{B}$*.*

Characterizing $\mathscr{B}$

Theorem

If there exists an envelopment $\phi : M \to \widehat{M}$ under which $\overline{\phi(I^+(p))}$ is compact and every boundary point in $\widehat{M}$ attached to $I^+(p)$ is a pure singularity, then $p \in \mathscr{B}$.

This formalizes our prior procedure for identifying $\mathscr{B}$ in examples.



Framing Cosmic Censorship

Heuristically, the weak cosmic censorship conjecture states that singularities must be hidden behind black holes.

Framing Cosmic Censorship

Heuristically, the weak cosmic censorship conjecture states that singularities must be hidden behind black holes. In our framework,

Conjecture (Global Weak Cosmic Censorship)

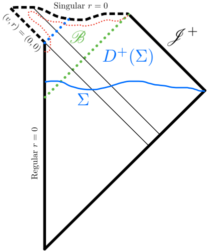
In a generic, maximal, physically admissible spacetime (M, g) *which admits a complete space-like hypersurface* Σ*, there exists a singular neighborhood* $U \in \mathscr{U}$ *such that* $U \cap D^+(\Sigma) \subset \mathscr{B}$*.*

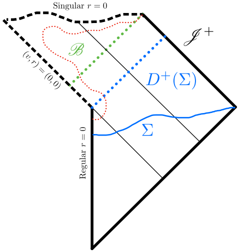
Framing Cosmic Censorship

Heuristically, the weak cosmic censorship conjecture states that singularities must be hidden behind black holes. In our framework,

Conjecture (Global Weak Cosmic Censorship)

In a generic, maximal, physically admissible spacetime (M, g) *which admits a complete space-like hypersurface* Σ*, there exists a singular neighborhood* $U \in \mathscr{U}$ *such that* $U \cap D^+(\Sigma) \subset \mathscr{B}$*.*





Contributions

- Provided a general program for identifying black holes in any spacetime, squarely grounded in their intuitive description.

Contributions

- Provided a general program for identifying black holes in any spacetime, squarely grounded in their intuitive description.
- Demonstrated this program to enjoy intuitive and desirable features.

Contributions

- Provided a general program for identifying black holes in any spacetime, squarely grounded in their intuitive description.
- Demonstrated this program to enjoy intuitive and desirable features.
- Used this framework to provide a more general alternative rigorous formulation of an important open problem in general relativity.

Contributions

- Provided a general program for identifying black holes in any spacetime, squarely grounded in their intuitive description.
- Demonstrated this program to enjoy intuitive and desirable features.
- Used this framework to provide a more general alternative rigorous formulation of an important open problem in general relativity.

Further detail can be found in preprint, *On the Definition of Black Holes: Bridging the Gap Between Black Holes and Singularities (2022)*.