Triality of Two-dimensional (0,2) Theories

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J.Guo,B.Jia,E.Sharpe,arXiv:1501.00987

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Outline

2d N = (0,2) Gauge Theories

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- Fermi Multiplet Ψ: $\overline{\mathcal{D}}_+\Psi = \sqrt{2}E(\Phi)$
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- Vector Multiplet: $V = v - 2i\theta^+\lambda_- - 2i\overline{\theta}^+\overline{\lambda}_- + 2\theta^+\overline{\theta}^+D$

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- Potential for the scalars:

$$V = \frac{e^2}{2} D^a D^a + \sum_a |E_i(\phi)|^2 + \sum_a |J_i(\phi)|^2$$

Triality

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	Φ	Ψ	P	Γ	Ω
U(N_c)	$\square$	$\bar{\square}$	$\bar{\square}$	$\mathbf{1}$	det
SU(N_1)	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\bar{\square}$	$\mathbf{1}$
SU(N_2)	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
SU(N_3)	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
SU(2)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$

$$N_c = (N_1 + N_2 - N_3)/2$$

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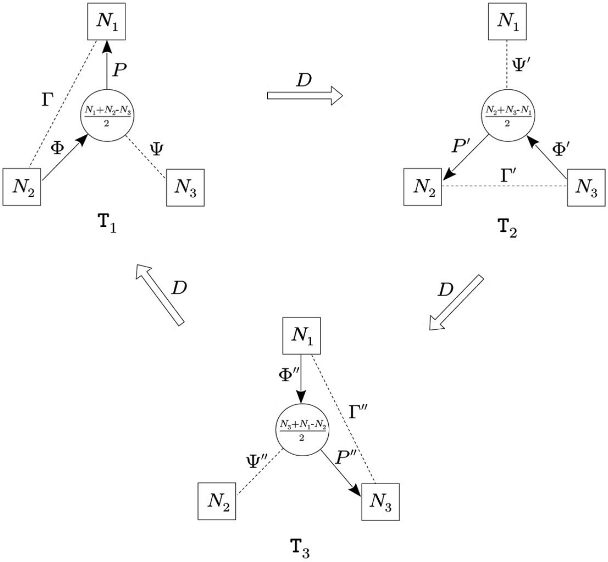
- Triality under permutation of N_1, N_2, N_3

Quiver Diagram



Checks

- Non-abelian Flavor Anomalies:

$$k_{SU(N_1)} = -\frac{1}{4}(-N_1 + N_2 + N_3)$$
$$k_{SU(N_2)} = -\frac{1}{4}(+N_1 - N_2 + N_3)$$
$$k_{SU(N_3)} = -\frac{1}{4}(+N_1 + N_2 - N_3)$$

Invariant under cyclic permutations of N_1, N_2, N_3

Checks

- Central Charge:

$$c_R = \frac{3}{4}\frac{(-N_1+N_2+N_3)(N_1-N_2+N_3)(N_1+N_2-N_3)}{N_1+N_2+N_3}$$

$$c_L = c_R - \frac{1}{4}(N_1^2+N_2^2+N_3^2-2N_1N_2-2N_2N_3-2N_3N_1)+2$$

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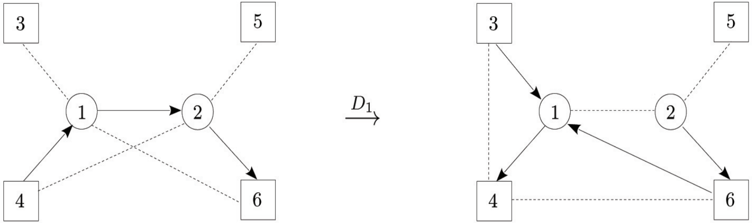
- Elliptic Genus is also invariant under the permutations.

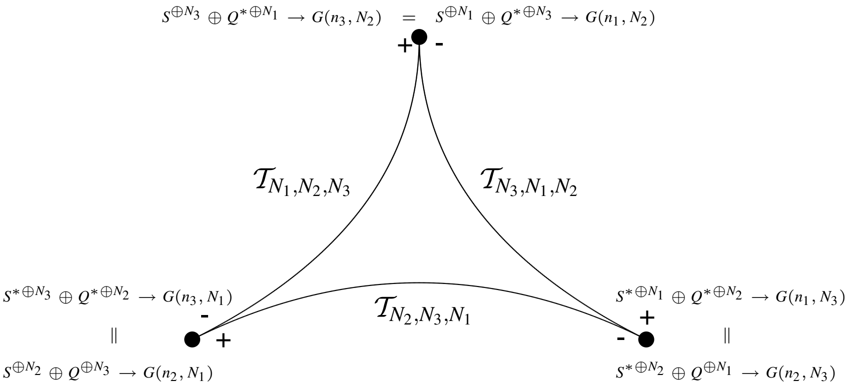
General Quiver Gauge Theories

The triality transformation can be applied to any gauge node in a general quiver gauge theory



[Gadde,Gukov,Putrov 1310.0818]

Geometry of the Triality



$$N = \frac{N_1+N_2+N_3}{2}, n_i = N - N_i$$

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- The full spectrum $\mathcal{H} = \bigoplus_{\lambda} \mathcal{H}_L^{\lambda} \bigotimes \mathcal{H}_R^{\lambda}$, λ runs over all integrable representations of $\mathcal{R}$

- The partition function $Z(\tau, \xi_i; \overline{\tau}, \overline{\eta}) = \sum_\lambda \chi_\lambda(\tau, \xi_i) K_\lambda(\overline{\tau}, \overline{\eta})$

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- $$\mathcal{H} = \bigoplus_{\lambda,\lambda'} L_{\lambda,\lambda'} \mathcal{H}^{\lambda}_{L,WZW} \bigotimes \mathcal{H}^{\lambda'}_{R,KS}$$

[Gadde,Gukov,Putrov 1404.5314]

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- The massless chiral states realize the sheaf cohomology groups

$$H^n\left(X, (\wedge^m\mathcal{E})\otimes(\det\mathcal{E})^{-1/2}\otimes K_X^{+1/2}\right)$$

Bott-Borel-Weil Theorem

Let β and γ be some dominant weights of $U(k)$ and $U(n-k)$ respectively, if there is a way to transfer $\alpha = (\beta, \gamma)$ into a dominant weight $\tilde{\alpha}$ of $U(n)$, then the cohomology

$$H^\bullet(G(k,n), K_\beta S^* \otimes K_\gamma Q^*) = K_{\tilde{\alpha}} V^* \delta^{\bullet, l(\alpha)},$$

where V is the fundamental representation of $U(n)$, and $l(\alpha)$ is the number of elementary transformations performed. Otherwise, $H^\bullet(G(k,n), K_\beta S^* \otimes K_\gamma Q^*) = 0$.

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- Example 1: $Sym^2 S^* \otimes Sym^2 Q^*$ over $G(2,4)$
  $(2,0,2,0) \rightarrow (2,1,1,0)$
  $H^m(G(2,4), Sym^2 S^* \otimes Sym^2 Q^*) = 0$ for $m \neq 1$
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- Example 2: $H^\bullet(G(2,4), \wedge^2 S^* \otimes Sym^2 Q^*) = 0$

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- Positive Phase: $S^{\oplus 3} \bigoplus Q^{*\oplus 2} \bigoplus S^{*\oplus 2} \to \mathbb{P}^2$
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- Global Symmetry:
  $SU(2) \times SU(3) \times SU(3) \times SU(2) \times U(1)^3$

States shared between the two phases:

States as representations of $SU(2) \times SU(3) \times SU(3) \times SU(2)$	$r >> 0$ $\wedge^\bullet \mathcal{E}$	$r >> 0$ $H^\bullet(\mathbb{P}^2)$	$r << 0$ $\wedge^\bullet \mathcal{E}$	$r << 0$ $H^\bullet(\mathbb{P}^1)$	$U(1)^3$ charges
(1,1,1,1)	0	0	0	0	(+3, 0, -3)
(1,3,1,2)	1	0	2	1	(+2, -1/2, -3/2)
(1,$\bar{3}$,1,1)	2	1	2	1	(+1, +2, -3)
(2,1,3,1)	2	1	1	0	(+2,0,-2)
⋮					
(2,1,$\bar{3}$,1)	7	1	7	1	(-2,0,+2)
(1,3,1,1)	7	1	6	0	(-1,-2,+3)
(1,$\bar{3}$,1,2)	8	2	6	0	(-2,+1/2,+3/2)
(1,1,1,1)	9	2	8	1	(-3,0,+3)

States not shared between the two phases:

States as representations of	$r >> 0$		$r << 0$		
$SU(2) \times SU(3) \times SU(3) \times SU(2)$	$\wedge^{\bullet}\mathcal{E}$	$H^{\bullet}(\mathbb{P}^2)$	$\wedge^{\bullet}\mathcal{E}$	$H^{\bullet}(\mathbb{P}^1)$	$U(1)^3$ charges
(1,6,1,1)	2	0	-	-	(+1, -1, 0)
(2,8,1,1)	3	0	-	-	(0, 0, 0)
(1,6,1,1)	4	0	-	-	(-1, +1, 0)
(1,6,1,1)	5	2	-	-	(+1, -1,0)
(2,8,1,1)	6	2	-	-	(0,0,0)
(1,6,1,1)	7	2	-	-	(-1, +1, 0)
(4,1,1,1)	-	-	3	0	(0,0,0)
(4,1,1,1)	-	-	5	1	(0,0,0)

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- The third geometry is obtained by switching N_2 and N_3 $\Rightarrow$ same as the negative phase in this example

Second Example

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(1,5,1,1)	2	0	-	-	(0, 0, 0)
(1,5,1,1)	8	1	-	-	(0, 0, 0)
(10,1,1,1)	-	-	4	0	(+2, -2, 0)
($\overline{10}$,1,1,1)	-	-	8	3	(-2, +2, 0)
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(20,1,1,1)	-	-	6	0	(0,0,0)
(20,1,1,1)	-	-	6	3	(0,0,0)
(1,20,1,2)	-	-	5	0	(+1,-1,0)
(1,20,1,2)	-	-	7	3	(-1,+1,0)

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- Symmetry Enhancement in the Infrared:
  For $N_c = 1$, Ψ transforms in the same way as Ω, $SU(N_3)$ and $SU(2)$ combine to form $SU(N_3 + 2)$
- For $N_1 = 2, N_2 = 2, N_3 = 2, N_c = 1$, the flavor symmetry is enhanced to E_6 at level 1 in the infrared

- Under the $(SU(2)^4)/\mathbb{Z}_2$ subgroup of E_6,
  $\mathbf{27} = (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1}) + (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2})$
  $+(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}) + \mathbf{3}(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$

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$+(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2}) + (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}) + \mathbf{3}(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$
- There are 54 states shared between the two phases and they form two copies of the decomposition above
  [Guo,Jia,Sharpe 1501.0098]

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