

Dynamic scaling of a cluster growth process far from equilibrium

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Abstract. The dynamics of cluster growth in a two-lane driven diffusive system is analyzed by extensive Monte Carlo simulations. We observe good dynamical scaling for small values of a particle-particle exchange probability γ on relatively small lattices. As γ increases, the (effective) scaling exponents develop a dependence on γ , suggesting that the observed coarsening does not result in a genuine ordered phase but is instead part of a complex crossover.

1 Introduction

The study of idealized lattice models has proved to be invaluable for exploring the vast realm of systems far from equilibrium. Although simple to define, the behavior of these models is very rich and complex [1]. Since exact solutions are only rarely available, the initial insights into their properties are almost always gleaned from direct simulations. To develop the necessary confidence in the numerics, models for which both exact solutions and simulation data can be compared have received especially close scrutiny. It is therefore particularly startling if stark discrepancies between simulation data and analytical predictions are observed. For example, the exact steady-state solution for a simple exclusion process with two species on a ring shows that a “phase transition” suggested by simulation data and mean-field theory [2] is nothing but a finite-size effect [3,4]: Particle-particle correlations are controlled by a length scale which can become enormous (e.g., 10^{70} lattice sites) but nevertheless remains finite. Such extraordinary findings in a simple one-dimensional stochastic model reveal the unexpected, often counter-intuitive features of systems far from equilibrium.

In this work we study a closely related model, namely a system of two particle species [5,6], driven in opposite directions on two coupled lanes [7]. When two particles encounter one another, they may exchange positions with a small exchange rate γ . Monte Carlo simulations of static and dynamic properties suggest that the system undergoes a phase transition between a homogeneous, disordered phase and an ordered phase characterized by the presence of a single macroscopic particle cluster. Starting from an initially random configuration, this ordered phase is approached via a coarsening process in which clusters grow much faster than diffusively [8,9]. For this model, there is no exact solution which can provide rigorous insights into the nature

of the ordered “phase”. Yet again, it appears to be a finite-size effect, based on an analytic conjecture due to Kafri *et al* [10,11] and high-precision Monte Carlo data for steady-state cluster size distributions on rather large systems [13].

In this paper we show data from extensive computer simulations focusing on the dynamic scaling properties of the system, as it approaches the steady state. For small values of γ , we find a growth exponent of $2/3$ and good evidence of dynamical scaling. Yet, as γ increases, we are forced to adjust the scaling exponents in order to collapse the data on a universal curve. We believe that these findings provide further evidence for the destabilization of the ordered “phase” with increasing γ or system size.

2 The model and its properties

The model is defined as a stochastic driven diffusive lattice gas with random sequential dynamics that conserves the number of particles. Each site of a periodic $2 \times L$ lattice can be occupied by a ‘positive’ (+) or “negative” (−) particle, or remain empty (\emptyset). Biased in opposite directions, the two species of particles diffuse on the lattice. Their numbers are equal, i.e. the system is uncharged. The particles interact only through an excluded volume constraint. In one Monte Carlo step (MCS), the following steps are repeated $2L$ times: (i) a pair of two nearest-neighbor sites (a “bond”) is chosen at random; (ii) for bonds along the L direction, a $(+\emptyset)$ or a $(\emptyset-)$ pair is always exchanged while a $(+-)$ pair is exchanged with probability γ ; (iii) for transverse bonds, particle-hole pairs are always exchanged while particle-particle pairs are exchanged with probability γ . The chosen dynamical rules are such that the bias is from left to right (right to left) for the + (−) particles, and no backward jumps are allowed. In contrast, lane changes occur in an unbiased fashion.

Clusters of size s are defined as s particles connected by nearest-neighbor bonds regardless of their charge. To monitor the growth of such clusters quantitatively, we measure the residence distribution, $p(s, t)$, i.e., the probability that a randomly selected particle belongs to a cluster of size s at time t . It is naturally normalized, i.e. $\sum_{s=1}^L p(s, t) = N$, where N is the total number of particles in the system. The average cluster size is then defined by $l_c(t) = \sum_{s=1}^L s p(s, t)/N$.

In the stationary state, the residence distribution $p^*(s) = \lim_{t \rightarrow \infty} p(s, t)$ clearly distinguishes homogeneous from ordered configurations. When the system is populated only by small clusters, $p^*(s)$ is monotonically decreasing (uni-modal) in s . A bi-modal $p^*(s)$ signals the presence of a macroscopic cluster, coexisting with a low-density (“traveller”) region in the remainder of the system [13]. Our steady-state data show clearly that bi-modal distributions cross over to uni-modal ones, if either γ or L are increased [13]. In contrast, one-lane systems exhibit exponentially decaying (uni-modal) distri-

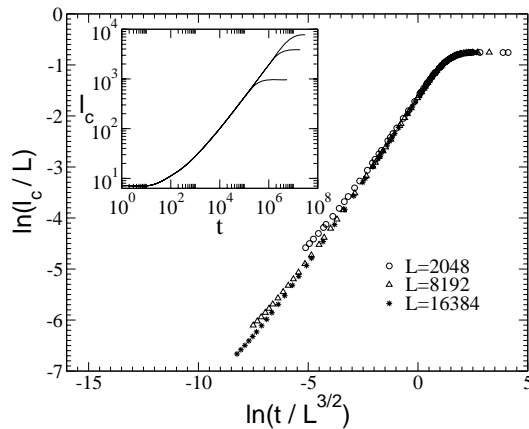


Fig. 1. Dynamic scaling for l_c/L^α vs. t/L^z for $\gamma = 0.1$ and $\rho = 0.5$. The inset shows the original data l_c vs. t .

butions over the whole parameter space, in agreement with an exact solution [15].

3 Monte Carlo simulations

Faced with the computational challenge to simulate a slow diffusion process on large lattices, we apply the “brute force” method. To improve the statistics of the simulations, we use a fast multi-spin coding algorithm for conserved dynamics (see, e.g., [12]). Our model is a three-state model so that we need at least two bits to store the state of a particular site on the lattice. Instead of encoding such a state by two consecutive bits in one machine word, we prefer to use one bit from two different words. In this way the single-bitwise algorithm for one Monte Carlo step becomes a little bit more efficient, but we suffer a (mild) speed penalty from working with two different words for the encoding. The multi-spin code that we use for a single bond exchange is more complicated than standard codes but, using hardware with 64-bit architecture, we benefit much more from simulating 64 lattices at the same time. The overall efficiency increases by about a factor of 35.

The inset of Fig. 1 shows l_c vs. t obtained from 4096 independent runs. Data are collected for lattices with $L = 2\text{K}$, 8K and 16K at particle density $\rho = 0.5$ and $\gamma = 0.1$. At $t = 0$ the initial configuration on the lattice is random. We test whether the data obey dynamic scaling of the form:

$$l_c(t) = L^\alpha F(t/L^z). \quad (1)$$

We obtain very good data collapse for large t ($t > 1000$ MCS), using the values $\alpha = 1.0$ and $z = 3/2$ for the dynamic exponent (see Fig. 1). This results

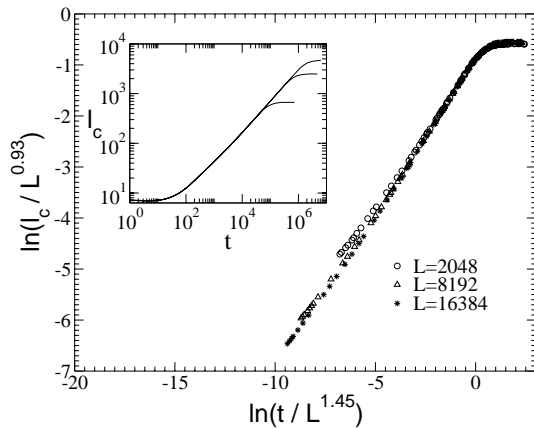


Fig. 2. Dynamic scaling for l_c/L^α vs. t/L^z for $\gamma = 0.3$ and $\rho = 0.5$. The inset shows the original data l_c vs. t .

in a cluster growth exponent $\beta = \alpha/z = 2/3$ with error bars much smaller than the symbols. For this small value of γ and for the L 's that we could simulate, the growth exponent β appears to coincide with the prediction from the Burgers equation [14]. Since the early-time behavior of $l_c(t)$ is essentially independent of L , as illustrated by the inset of Fig. 1, the scaling form of Eq. (1) cannot describe the full range of the data. The plateau in $l_c(t)$ at early times can be understood easily: starting from a random configuration, small clusters form almost immediately everywhere on the lattice, as oppositely charged particles block each other. Until particles make their way through these clusters and begin to escape from the opposite side, $l_c(t)$ cannot change significantly.

For this small value of $\gamma = 0.1$, the dynamic scaling exponents are consistent with the formation of a single macroscopic cluster whose size, in steady state, scales with the system size. However, a different picture emerges for larger values of γ . The data for $\gamma = 0.3$ are shown in Fig. 2. Here, a least squares minimization gives the best scaling collapse if $\alpha \approx 0.93(1)$ and $z \approx 1.45(1)$, whence $\beta \approx 0.64(1)$. It is tempting to conjecture that the apparent γ -dependence of the scaling exponents is evidence for the finite-size nature of the ordered “phase”. By increasing γ , we effectively destabilize the macroscopic cluster, which in turn produces the shift in the cluster growth exponents.

4 Conclusions

In this paper, we have discussed the dynamic scaling properties of coarsening particle clusters in a two-species driven diffusive system on two coupled

lanes. We find excellent data collapse with Burgers scaling exponents at small values of γ (0.1); in contrast, at larger γ (0.3), good data collapse can only be achieved with a different set of (effective) exponents. We believe that this is a *dynamic precursor* of dramatic differences in the steady states at these parameter values. While systems at small γ (for the L 's investigated) are characterized by a single macroscopic cluster in the long-time limit, systems at larger γ can already sustain several large clusters which, however, do no longer scale well with the system size. These findings serve as another indicator that the observed ordered states do not constitute a genuine thermodynamic phase.

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