NAME:

PHYSICS 4456 — INTRODUCTION TO QUANTUM MECHANICS II
Spring 2005 — Final exam — May 9, 2005

Please note: The Virginia Tech *honor code* applies to this final exam. This is a *closed book* exam. Please do *not* communicate with your fellow students ! You may ask me for assistance if you find the wording of any of the problem unclear. Please turn in your *entire* work at the end of this test.

Good luck and keep calm !!!

1. *Calcium atom.* (**15 points**)
   The neutral Calcium atom has $Z = 20$ electrons.
   (a) For given principal quantum number n, give the allowed values for the angular momentum quantum numbers ℓ and m.
   (b) What is the degeneracy of the nth hydrogen atom energy level ? Write down the electronic configurations of Ca and the Ca^+ ion.
   (c) Determine the ground state configurations of Ca and Ca^+, and provide the corresponding spectroscopic notations.
   (d) Which of the following excited states can be reached from the Ca atom ground state via electric dipole transitions: $4\,^3S_1$, $4\,^1P_1$, $4\,^1D_2$, $5\,^1S_0$? Use selection rules to explain your answers.

2. *Hydrogen atom: expectation values.* (**15 points**)
   Consider the hydrogen atom ground state wave function
   $\varphi_{100}(\vec{r}) = (\pi a_0^3)^{-1/2}\, e^{-r/a_0}$, with $r = |\vec{r}|$, $a_0 = \hbar^2/m_e e^2$, $E_1 = -e^2/2a_0$.
   (a) What is the most likely value of r to be found in a measurement ?
   (b) Compute the expectation values $\langle r \rangle_{100}$ and $\langle r^{-1} \rangle_{100}$.
   (c) Find the ground state expectation value of the kinetic energy.
   (d) Initially, let the atom be prepared in the superposition state $\psi(\vec{r}, t = 0) = \frac{1}{\sqrt{3}} \left[\sqrt{2}\,\varphi_{100}(\vec{r}) - \varphi_{310}(\vec{r})\right]$. What is $\psi(\vec{r}, t > 0)$? What are the probabilities of measuring $L_z = 0$ and $|\vec{L}| = 0$ in this state ? Calculate the atom's mean energy.

   Useful integrals : $$\int_0^\infty x^k\, e^{-ax}\, dx = \frac{k!}{a^{k+1}}\ .$$

This test sheet has **two** *pages.* **Please turn over !**

3. *Spin-orbit coupling.* **(12 points)**
Spin-orbit coupling in atoms is described by a Hamiltonian of the form $H_{\text{so}} = \epsilon_0 \vec{L} \cdot \vec{S}$. Here $\vec{L}$ is the orbital angular momentum, $\vec{S}$ the spin.
(a) For atoms with spin $s = 1/2$, what are the possible quantum numbers j and m_j for the total angular momentum $\vec{J} = \vec{L} + \vec{S}$?
(b) Which operators are simultaneously diagonalized by the basis states of the coupled representation ?
(c) Find the eigenvalues of H_{so} as function of the orbital angular quantum number ℓ.

4. *Two neutrons in infinite potential well.* **(14 points)**
Two neutrons (spin $s = 1/2$) are confined by an infinite potential well: $V(x) = 0$ for $0 < x < a$, ∞ otherwise.
(a) What are the single-particle wave functions and energy eigenvalues in this potential ?
(b) Assuming the neutrons are non-interacting, provide a complete description of their ground state (including spin and energy).
(c) Write down all possible excited states for this two-particle system.

5. *Spin perturbation theory.* **(12 points)**
A spin $s = 1/2$ particle is described by the Hamiltonian $H = H_0 + H'$, with $H_0 = A\,\vec{S}^2$, $H' = -b_x\,\sigma_x - b_z\,\sigma_z$.
(a) What are the eigenstates and eigenvalues of H_0 ?
(b) Use first-order perturbation theory with respect to H' to determine the energy eigenvalues and (normalized) eigenstates of H.
(c) Under which condition should perturbation theory be reliable ?
Pauli's spin matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

6. *Harmonic oscillator transition rate.* **(12 points)**
A particle with mass m and charge q is confined by a one-dimensional harmonic oscillator potential $V(x) = \frac{m}{2}\,\omega_0^2\,x^2$. At $t = 0$ it becomes exposed to a weak electric field $E_x(t) = E_0\,\exp(-t/\tau)$.
(a) Determine the time-dependent perturbation Hamiltonian $H'(t)$.
(b) Compute the transition probability, induced by the electric field, from the ground state (at $t = 0$) to the first excited state ($t = +\infty$).
(c) Discuss the limits $\omega_0\,\tau \to 0$ and $\omega_0\,\tau \to \infty$.
Recall $a = \sqrt{\frac{m\omega_0}{2\hbar}}\left(x + \frac{ip}{m\omega_0}\right)$.

Good luck !!! — Happy graduation / summer break !!!