

PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009
Homework assignment 1, due September 1, 2009 at 5.00 pm

The *Graduate Honor Code* applies to this assignment. Specifically:

- The use of sample solutions of any kind is prohibited.
- Please turn in your own legible and well-explained solutions.
- I support teamwork for solving the homework assignments, but each student must turn her/his solutions separately (no copying).

Should you encounter difficulties, please feel free to ask me for help: during office hours, the discussion meeting, or via email. I will grade your papers; homework will count 30 % towards your course grade. I encourage you to work on the more difficult extra-credit assignments (*) which are meant to help you improve your homework score. You will receive copies of my sample solutions (for your own use only !).

1. *Particle in a spherically symmetric potential.* (20 + 5 points)

A particle with mass m in $d = 3$ dimensions moves in a spherically symmetric potential $V(\vec{r}) = V(r)$, where $r = |\vec{r}|$.

(a) Show that the particle's kinetic energy can be written as

$$T(\vec{p}) = T(p_r, \vec{L}) = \frac{p_r^2}{2m} + \frac{\vec{L}^2}{2mr^2},$$

where $p_r = \vec{e}_r \cdot \vec{p}$ denotes its radial momentum component, and $\vec{L} = \vec{r} \times \vec{p}$ its angular momentum.

- (b) Express the particle's Lagrangian $L(r, \theta, \phi, \dot{r}, \dot{\theta}, \dot{\phi})$ and the angular momentum component L_z in terms of spherical coordinates.
- (c) Determine the corresponding canonically conjugate momenta and the Hamiltonian $H(r, \theta, \phi, p_r, p_\theta, p_\phi)$; compare with (a). List all cyclic coordinate(s) and the associated conserved quantities.
- (d) Through direct differentiation, show that $d\vec{L}/dt = 0$.
- (e) Find Hamilton's equations of motion for the radial components $\dot{r} = p_r/m$ and $\dot{p}_r = -\partial V_{\text{eff}}(r)/\partial r$; what is $V_{\text{eff}}(r)$ here ?
- (f) (*) *Extra-credit assignment:*

Let $V(r) = -GmM/r$ be the gravitational potential of a point mass M . Discuss the possible particle trajectories. For a circular orbit (radius r_c , period T_o), derive the virial theorem $E = -T = V/2$ and Kepler's third law of planetary motion $r_c^3/T_o^2 = GM/4\pi^2$.

2. *Poisson brackets.***(15 points)**

In Hamiltonian mechanics, the Poisson bracket between two observables $A(q_l, p_l, t)$ and $B(q_l, p_l, t)$ is defined via

$$\{A, B\} = \sum_l \left(\frac{\partial A}{\partial q_l} \frac{\partial B}{\partial p_l} - \frac{\partial B}{\partial q_l} \frac{\partial A}{\partial p_l} \right) .$$

- (a) Confirm the identity $\{A B, C\} = A \{B, C\} + \{A, C\} B$.
- (b) Consider a particle (mass m) in three dimensions. Compute the Poisson brackets of the components of the position, linear momentum, and angular momentum vectors $\{L_i, x_j\}$, $\{L_i, p_j\}$, $\{L_i, L_j\}$, and show that $\{L_i, A\} = 0$, where $A = \vec{r}^2, \vec{p}^2, \vec{L}^2$ is a scalar.
- (c) Let the particle be moving in a spherically symmetric potential $V(\vec{r}) = V(r)$ where $r = |\vec{r}|$. Show that all components of the angular momentum vector $\vec{L} = \vec{r} \times \vec{p}$ are conserved quantities.