PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 11, due November 10, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Reflection-free potentials.* (**15 points**)
   A quantum particle moves in the potential (with $\mu > 0$)
   $$V(x) = -\frac{\mu(\mu+1)\,\hbar^2}{2\,m\,a^2\,[\cosh(x/a)]^2}\ .$$
   (a) Find the scattering states for this problem as follows: First, set $\varphi_k(x) = e^{ik\,x}\,\phi_k(y)$ with appropriately chosen k and $y = x/a$. Next, substitute $z = \tanh y$, and show that the resulting power series for $\phi_k(z)$ terminates (only) if μ is an integer.
   (b) For $\mu = 1, 2$, determine the scattering states ($E \geq 0$) explicitly, and find the allowed bound states ($E < 0$) and their energies.

2. *Double potential step.* (**15 points**)
   Consider scattering states in the one-dimensional double potential step: $V(x) = -\alpha\,V_0$ for $x < -a$, $V(x) = V_0$ for $|x| < a$, and $V(x) = 0$ for $x > a$, where a, α, and $V_0 > 0$.
   (a) Assuming an incident wave from the left, compute the reflection and transmission amplitudes for $E > V_0$. Check the limits $\alpha \to 0$ (rectangular barrier) and $a \to 0$ (potential step). Find the reflection and transmission coefficients, and confirm that $r + t = 1$.
   (b) Obtain the reflection and transmission amplitudes and coefficients in the situations $0 < E < V_0$, and $-\alpha\,V_0 < E < 0$.

3. *Tunneling through continuous potential barriers.* (**5 + 5 points**)
   Employ the (approximate) formula for tunneling through a continuous potential barrier to compute the transmission coefficient for ($V_0 > 0$)
   (a) "cold" field emission of electrons from a metal, $V(x) = -V_0$ for $x < 0$, $V(x) = -q\,E_0\,x$ for $x > 0$;
   (b) (*) alpha decay of radioactive nuclei, $V(r) = Z_1 Z_2\,e^2/r$ for $r > R$, $V(r) \approx -V_0$ for $r < R$, where R denotes the range of the nuclear force (radius of the nucleus), and for α decay $Z_1 = Z - 2$, $Z_2 = 2$.

4. (*) *Wave packet evolution near a resonance.* **(5 points)**
Consider a one-dimensional wave packet $\psi_i(x,t)$ incident on a potential well of width $2a$. Use the Breit–Wigner formula for the amplitude $T(E)$ near a resonance to compute the transmitted wave packet $\psi_t(x,t)$.
Hint: change the integration variable to energy, extend the integration limits to infinity, and apply contour integration in the complex E plane.