PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 12, due November 17, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Dirac delta distribution potential.* (**15 + 15 points**)
   Consider the stationary Schrödinger equation for the one-dimensional Dirac delta-function potential $V(x) = \lambda\,\delta(x)$.
   (a) Obtain the continuity conditions for the energy eigenfunctions and their derivatives at $x = 0$.
   (b) Find the reflection and transmission coefficients for a plane wave scattering state ($E > 0$) incident from the left.
   (c) For $\lambda < 0$, find the normalized bound state wave function and its energy eigenvalue ($E < 0$).
   (d) (*) Determine the properly normalized scattering states ($E > 0$) as simultaneous eigenstates of the parity operator.
   (e) (*) Through explicit calculations, confirm the orthogonality and completeness relations for these eigenstates.
   Hint: treat the attractive and repulsive potential cases separately; you will need the Laplace transforms

$$\int_0^\infty e^{ikx}\,dx = \frac{i}{k} + \pi\,\delta(k)\ , \quad \int_{-\infty}^0 e^{ikx}\,dx = -\frac{i}{k} + \pi\,\delta(k)\ .$$

2. *One-dimensional δ potential: momentum representation.* (**10 points**)
   Find the energy eigenvalue $E < 0$ and normalized wave function $\varphi_E(p)$ for the bound state in the attractive potential $V(x) = -\lambda\,\delta(x)$, $\lambda > 0$.

3. *Delta distribution 'molecular' states.* (**15 points**)
   A quantum particle (mass m) is subject to the attractive potential $V(x) = -\lambda\,\delta(x-a) - \lambda\,\delta(x+a)$, with $\lambda > 0$.
   (a) Find transcendental equations for bound state(s) in this potential.
   (b) Sketch their graphical solutions. What is the condition for an odd parity bound state to exist ? Check the limit $a \to 0$; estimate the energy difference between the even / odd bound states as $a \to \infty$.