

## Homework assignment 14, due December 8, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Cylindrical quantum wire.* (15 points)

Consider a quantum wire along the  $x$  direction, with a confining potential  $V(\vec{r}) = -V_0$  for  $0 \leq y^2 + z^2 \leq a^2$ ,  $V(\vec{r}) = 0$  for  $y^2 + z^2 > a^2$ .

- Through a suitable separation of variables obtain the form of both the scattering and bound states in this potential. Write down the conditions that would determine the reflection and transmission coefficients and the binding energies (do *not* attempt a solution).
- In the limit  $V_0 \rightarrow \infty$ , obtain the bound state energies in the  $yz$  plane explicitly for the ground state and first five excited states.

2. *Spherical delta shell potential.* (15 points)

Consider a quantum particle in three dimensions, interacting with a spherical delta shell potential  $V(r) = -\lambda \delta(r - a)$ .

- Obtain its  $s$ -wave ( $\ell = 0$ ) scattering states and the reflection amplitude and coefficient for an incoming radial wave  $e^{-ikr}$ .
- Obtain the condition on  $\lambda$  for the existence of a bound state with  $\ell = 0$ . Determine its normalized wave function.
- Find the minimal potential strength required for the existence of a bound state for  $\ell > 0$ . *Hint:* consider the limit  $E \rightarrow 0$ .

3. *Hydrogen atom ground state in momentum space.* (10 points)

The ground state wave function of the hydrogen atom is ( $r = |\vec{r}|$ )

$$\varphi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} .$$

- Compute the corresponding wave function  $\phi(\vec{p})$  in momentum space, and check that it is correctly normalized.
- Evaluate  $\langle p^2 \rangle_{100}$  and find the mean kinetic and potential energies in the hydrogen atom ground state.