PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 2, due September 8, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Free particle motion.* (**5 points**)
   For a free particle (moving in a constant potential $V(\vec{r}) = V_0$), show that the quantity $A(\vec{r}, \vec{p}, t) = \vec{r} - \vec{p}\,t/m$ is conserved. What does it represent physically ?

2. *Charged particle in a homogeneous magnetic field.* (**20 points**)
   The Hamiltonian for a particle with mass m and charge q in an electromagnetic field, as determined by the scalar and vector potentials $\phi(\vec{r}, t)$ and $\vec{A}(\vec{r}, t)$, reads

$$H(\vec{r}, \vec{p}, t) = \frac{1}{2\,m}\left[\vec{p} - \frac{q}{c}\,\vec{A}(\vec{r}, t)\right]^2 + q\,\phi(\vec{r}, t)\ .$$

   (a) Find the equations of motion for $\vec{r}(t)$, $\vec{p}(t)$, and $\vec{v}(t)$.
   (b) For a homogeneous, stationary magnetic field $\vec{B} = \text{const.}$, show that $\vec{A}(\vec{r}) = -\frac{1}{2}\,\vec{r} \times \vec{B}$ (in $d = 3$ dimensions).
   (c) Solve for the trajectory of a charged particle in a homogeneous magnetic field $\vec{E} = 0$, $\vec{B} = B\,\vec{e}_z$, and describe its motion.

3. *Compton effect.* (**10 points**)
   In Compton's famous experiment, Röntgen (X ray) photons are scattered off free electrons. Employ energy and momentum conservation (best combined in a relativistic four-vector) to derive the formula

$$\Delta\lambda = 2\lambda_c\,\sin^2(\theta/2)$$

   for the change of the photon wavelength as function of the scattering angle θ. Here, $\lambda_c = h/mc$ denotes the Compton wavelength.

4. *(*) Inverse Lorentz transformation.* (**5 points**)
   Consider a general Lorentz transformation $x^\mu \to x'^\mu = \Lambda^\mu{}_\nu\,x^\nu + a^\mu$, with $\Lambda^\mu{}_\rho\,g^{\rho\lambda}\,\Lambda^\nu{}_\lambda = g^{\mu\nu}$ and $a^\mu = \text{const.}$ Show that $\Lambda^{-1\mu}{}_\nu = \Lambda_\nu{}^\mu$ and find the inverse Lorentz transformation $x'^\mu \to x^\mu$.