PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 6, due October 6, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Inner product in position and momentum space.* (**5 points**)
Using the completeness relations for position and momentum eigenstates, show that the inner product in terms of position and momentum space wave functions reads explicitly

$$\langle\psi|\widetilde{\psi}\rangle = \int \psi^*(\vec{r})\,\widetilde{\psi}(\vec{r})\,d^dr = \frac{1}{(2\pi\hbar)^d}\int \phi^*(\vec{p})\,\widetilde{\phi}(\vec{p})\,d^dp \ ,$$

and reestablish that these wave functions are indeed related to each other through Fourier and inverse Fourier transforms.

$$\psi(\vec{r}) = \frac{1}{(2\pi\hbar)^d}\int \phi(\vec{p})\,e^{i\vec{p}\cdot\vec{r}/\hbar}\,d^dp \ , \quad \phi(\vec{p}) = \int \psi(\vec{r})\,e^{-i\vec{p}\cdot\vec{r}/\hbar}\,d^dr \ .$$

2. *Minimal uncertainty wave packet.* (**10 points**)
In one dimension, determine the normalized wave functions that minimize the uncertainty product $\Delta x\,\Delta p = \hbar/2$ in both position and momentum space.

3. *Superposition state and time evolution.* (**10 points**)
Consider a particle with mass m confined to an infinite potential well of width a. Let the system be initially prepared in the superposition state

$$\psi(x, t = 0) = C\ [\varphi_1(x) + 2\,\varphi_3(x) - 2\,\varphi_4(x)] \ .$$

(a) Determine the constant C and $\psi(x, t)$ for $t > 0$.
(b) Find the mean energy $\langle E(t)\rangle$ of this superposition state.

4. *Free particle superposition state.* **(15 points)**

Consider the wave function $\psi(\vec{r}, t=0) = C\left[2\cos(\vec{k}\cdot\vec{r}) + \sin(2\vec{k}\cdot\vec{r})\right]$, evolving under the free-particle Hamiltonian $H = \vec{p}^{\,2}/2\,m + V_0$.

(a) Write down the wave function $\psi(\vec{r}, t)$ for $t > 0$.

(b) If the energy is measured in the state $\psi(\vec{r}, t)$, which values can be found, and with what probabilities? Assuming a measurement at $t = t_0$ results in $E = \hbar^2\vec{k}^2/2m + V_0$, what is $\psi(\vec{r}, t)$ for $t > t_0$?

(c) If instead the linear momentum is measured in the state $\psi(\vec{r}, t)$, what are the possible results and their probabilities? Assuming a measurement at $t = t_0$ results in $\vec{p} = -\hbar\vec{k}$, what is $\psi(\vec{r}, t > t_0)$?

(d) If momentum is measured subsequent to the energy measurement in (b) with result $E = \hbar^2\vec{k}^2/2m + V_0$, what are now the possible results and associated measurement probabilities? If the energy is measured subsequent to the momentum measurement in (c) yielding $\vec{p} = -\hbar\vec{k}$, what are the possible results and probabilities ?