PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 7, due October 13, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Angular momentum commutators.* (**20 points**)
   The angular momentum operator (in three dimensions) is $\vec{L} = \vec{r} \times \vec{p}$.
   (a) Determine the commutators $[L_i, x_j]$, $[L_i, p_j]$, and $[L_i, L_j]$. Confirm that a compact notation for the latter is $\vec{L} \times \vec{L} = i\hbar\, \vec{L}$.
   (b) Find $[L_i, \vec{r}^{\,2}]$, $[L_i, \vec{p}^{\,2}]$, and $\left[L_i, \vec{L}^{\,2}\right]$. What follows for $\left[\vec{L}, V(r)\right]$, where $r = |\vec{r}|$, and consequently for the energy eigenstates of spherically symmetric potentials ?
   (c) Are there common eigenstates of $L_x$ and $L_y$, or for $L_z$ and $\vec{L}^2$ ? Provide the uncertainty relation for $(\Delta L_x)_\psi\, (\Delta L_y)_\psi$ in an arbitrary state $\psi$. Under what condition does this product vanish ?

2. *Charged particle in a homogeneous electric field.* (**10 + 5 points**)
   A quantum particle with mass $m$ and charge $q$ is placed in a homogeneous electric field $\vec{E} = E_0\, \vec{e}_x$ (in $d = 3$ dimensions).
   (a) What is its potential energy? Which energy eigenvalue spectrum would you expect for this problem?
   (b) Find the Hamiltonian in the momentum representation and solve the stationary Schrödinger equation for $\phi_E(\vec{p})$. Normalize these energy eigenfunctions properly.
   (c) (*) From your result in (b), obtain the corresponding spatial wave functions $\varphi_E(\vec{r})$. *Hint*: you should arrive at *Airy functions*

$$\mathrm{Ai}(z) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{s^3}{3} + s\, z\right) ds \ .$$

3. *Schrödinger equation in the momentum representation.* (**5 points**)
   Derive the time-dependent Schrödinger equation in the momentum representation for a particle with mass $m$,

$$i\hbar\, \frac{\partial\, \phi(\vec{p}, t)}{\partial t} = \frac{\vec{p}^{\,2}}{2\, m}\, \phi(\vec{p}, t) + \frac{1}{(2\pi\hbar)^d} \int \widetilde{V}(\vec{p} - \vec{p}\,')\, \phi(\vec{p}\,', t)\, d^d p' \ ,$$

   where

$$\widetilde{V}(\vec{p}) = \int V(\vec{r})\, e^{-i\vec{p}\cdot\vec{r}/\hbar}\, d^d r \ .$$