PHYSICS 5455 — QUANTUM MECHANICS I — Fall 2009

Homework assignment 8, due October 20, 2009

The *Graduate Honor Code* applies to this assignment (see homework 1).

1. *Heisenberg picture.* **(10 + 10 points)**
   Find the time-dependent position and momentum operators in the Heisenberg picture for
   (a) a free particle in d dimensions, and
   (b) the one-dimensional harmonic oscillator, $V(x) = \frac{m}{2}\,\omega^2\,x^2$, via
   (i) direct explicit evaluation of $A_H(t) = e^{iHt/\hbar}\,A\,e^{-iHt/\hbar}$; *or*
   (ii) solving the equivalent Heisenberg equations of motion.
   (*) Extra credit: use *both* (i) and (ii).

2. *Parity operator.* **(10 points)**
   The parity operator is defined through $P\,f(\vec{r}) = f(-\vec{r})$.
   (a) Confirm that P is Hermitean, $P = P^\dagger$, and has eigenvalues ± 1.
   (b) Show that P anticommutes with momentum, $\{P, \vec{p}\} = 0$.
   (c) Demonstrate that $P_\pm = \frac{1}{2}(1 \pm P)$ represent projection operators onto even/odd eigenfunctions of P, $P P_\pm = \pm P_\pm$, and satisfy the properties $P_+ + P_- = 1$, $P_\pm P_\mp = 0$, $[P_+, P_-] = 0$, and $P_\pm^2 = P_\pm$.

3. *Galilean transformations.* **(10 points)**
   The physics in *inertial* reference frames is invariant under Galilean coordinate transformations $\vec{r} \to \vec{r}\,' = \vec{r} - \vec{v}\,t$, $t \to t' = t$, with $\vec{v} = \text{const.}$
   (a) How does the Hamiltonian for a single particle (mass m) change under this coordinate transformation ?
   (b) Confirm explicitly that the time-dependent Schrödinger equation remains intact under a Galilean transformation, provided the wave function transforms according to $\psi(\vec{r}, t) \to \psi'(\vec{r}\,', t')$, where

$$\psi'(\vec{r}\,', t') = e^{-im\left(\vec{v}\cdot\vec{r} - \vec{v}^{\,2}\,t/2\right)/\hbar}\,\psi(\vec{r}, t) = e^{-im\left(\vec{v}\cdot\vec{r}\,' + \vec{v}^{\,2}\,t'/2\right)/\hbar}\,\psi(\vec{r}\,' + \vec{v}\,t', t')\,.$$