Evaluating fitting models of the missing energy contribution of Ar and Ti nuclear shell orbitals using the E12-14-012 (e,e' p) scattering experiment at Jefferson Lab

Adam Dirican[1], Zachary Jerzyk[2], and Camillo Mariani[3]

[1]University of Maryland, College Park; [2]St. Norbert College; [3]Virginia Polytechnic Institute and State University

INTRODUCTION

The Deep Underground Neutrino Experiment (DUNE) seeks to probe CP-symmetry violation via the oscillation rates of the neutrino and antineutrino, detect supernovae neutrinos, and potentially invalidate several grand unification theories by making the first observation of proton decay. DUNE will be the world's largest argon-neutrion scattering experiment. However, little work has been done on electron-nucleus scattering for isospin nonsymmetric atoms, let alone neutrino-nucleus scattering for the argon-40 specific to DUNE. In Jefferson Lab (JLab) Hall A's experiment E12-14-012, the (e,e'p) scattering cross sections of argon-40 (N=22) and titanium-48 (Z=22) were measured against a detailed Monte Carlo (MC) simulation. These data were analyzed in order to determine if the choice of fit modeling would impose a systematic uncertainty on the argon-40 and titanium-48 spectroscopic factors.

JEFFERSON LAB EXPERIMENT

The (e,e'p) interaction involves an incident electron knocking a proton out of the nucleus of a target atom, yielding a proton and the electron as detectable products. An electron beam incident on gaseous argon-40 and solid titanium-48 targets produced these interactions at JLab, and spectrometers detected the final state electrons and protons.

Because argon-40 is isospin nonsymmetric, neutrino-proton scattering is different from neutrino-neutron scattering. Thus, titanium-48, having the same number of protons as argon-40 has neutrons, serves as an excellent proxy for neutrino-neutron interactions in argon-40.

Alongside the experimental data, an MC simulation was created to decompose the total nuclear spectral functions into their individual orbital contributions (Figure 1). The MC also allowed for a rigorous error analysis to be performed (See Figure 2 for schematic).

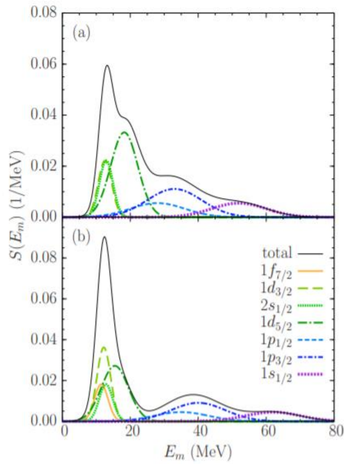


Fig. 1: Missing energy distribution of proton orbitals in (a) argon and (b) titanium. The innermost orbitals required an asymmetrical model since the distribution is increasingly asymmetric as orbitals are further from the nuclear surface.

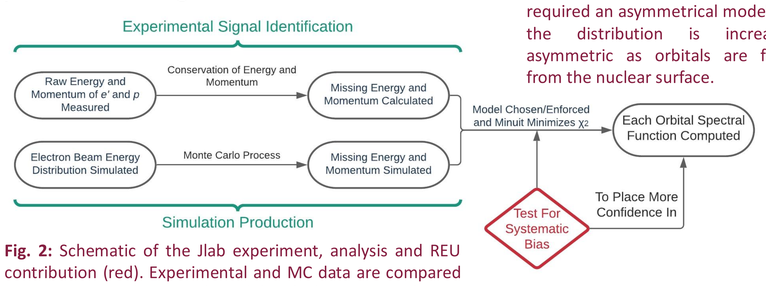


Fig. 2: Schematic of the Jlab experiment, analysis and REU contribution (red). Experimental and MC data are compared against various theoretical models which may introduce a systematic bias.

FITTING ANALYSIS

Each nuclear orbital could have its spectral function modeled with two conditions. The first was to enforce either a Gaussian (symmetric) or Maxwell-Boltzmann (nonsymmetric) distribution. The second was whether to enforce a cross-sectional dependence on the mean energy of the incoming electrons. Also, two other fitting options could be turned on or off: (1) to use missing momentum fit results, and (2), to implement an energy level penalty function using theory to constrain drift in the spectral functions, giving additional variations to test. See Figure 3 (below) for the complete list of models tested.

Given the numerous possible permutations (4^6 for argon; 4^7 for titanium), we identified nuclear theory constraints to reduce the number of models we would have to consider. Specifically, the innermost orbitals would not be reached by low energy incoming electrons, and so never had their full spectral distributions in the physical missing energy range. Thus, we modeled all of them with Maxwell-Boltzmann (nonsymmetric) distributions. Also, orbitals could be grouped for modeling according to their energies and whether they were expected to contribute similarly to the missing energy distribution.

	Argon Model Index (File #)												Additional Variations				
Orbital	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1d3/2, 2s1/2	MD	MD	MI	MI	MD	MD	MI	MI	GD	GD	GI	GI	MD	MD	MD	MD	MD
1d5/2	MD	MD	MI	MI	GD	GD	GI	GI	GD	GD	GI	GI	MD	MD	MD	GI	GI
1p1/2, 1p3/2, 1s1/2	GI	GD	GD	GI	GI	GD	GD	GI	GI	GD	GD	GI	GI	GI	GI	GI	GD
Reduced Chi-Squared	1.106	6.586	6.153	1.321	1.106	6.553	5.070	1.350	1.101	6.827	4.880	1.130	0.661	1.008	0.540	1.356	4.894

Legend			
1st Character	G	Model with Gaussian
	M	Model with Maxwell Boltzmann
2nd Character	D	σ is dependent on the mean
	I	σ is independent of the mean

	Titanium Model Index (File #)												Additional Variations			pm fitting results not used	Energy level penalty function not used	Neither is used
Orbital	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15			
1f7/2, 1d3/2	GD	GD	GI	GI	MD	MD	MI	MI	MD	MD	MI	MI	GD	GD	GD			
2s1/2	GD	GD	GI	GI	GD	GD	GI	GI	MD	MD	MI	MI	GD	GD	GD			
1d5/2, 1p1/2, 1p3/2, 1s1/2	GI	GD	GD	GI	GI	GD	GD	GI	GI	GD	GD	GI	GI	GI	GI			
Reduced Chi-Squared	1.304	3.891	4.876	2.315	1.359	3.821	6.738	2.295	2.067	3.750	6.579	2.866	1.109	1.304	1.109			

Fig. 3: The final selection of tested spectral function models for each orbital in argon (top) and titanium (bottom). Rejected models are highlighted in red according to χ2 results, and the rightmost columns indicate additional models that were decided upon to provide a more complete data set of the models.

THE CODE

Using CERN's Minuit package in ROOT, minimization was performed on each orbital's cross-section spectral distribution against the detailed Monte Carlo simulation developed by JLab. Goodness of fit was calculated by χ2-minimization.

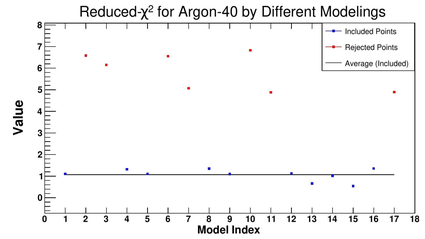
The code was developed for modularity, allowing it to function for both argon and titanium files, as well as in any newly developed files. Designed with a high degree of customization and suppression of output, various settings can be modified in the header of the file, and logic tables work to produce the fit and graphing as desired. These are shown in Figure 4 (right).

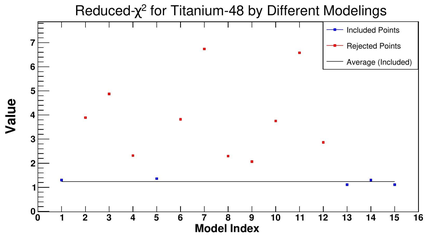
```
/*
Legend: First array value corresponds to orbital groups:
1 --> orbital parameters 0 and 1 (2s1/2 and 1d3/2)
2 --> orbital parameter 2 (1d5/2),
3 --> orbital parameter 3 (1p1/2 and 1p3/2)
4 --> orbital parameter 4 (1s1/2)

Second array value corresponds to two modeling choices:
        1) Gaussian (0) or Maxwell-Boltzmann (1) modeling
        2) Sigma dependece (0) or independence (1) on the mean field
both with associated booleans.

*/

int settings[4][2] = { {1,1} , {1,1} , {0,1} , {0,1}};
bool use_pm_fit_results = true;
bool use_energy_penalty_function = true;


bool CreateNewRunFile = false;
bool CreateGraphs = true;
bool SuppressSomeRuns = true;
vector <int> suppressedRuns = {2,3,6,7,10,11,12};
bool MakeLegend = true;

//Beginning of Adam and Zack's Contributions
#ifndef GLOBAL_VARIABLES
#define GLOBAL_VARIABLES
extern const string filesNameExtension = "runFile_v18-";
#endif
```

Fig. 4: Snapshot of the code header. Graphing and modeling options are contained as Booleans or 2D array entries.

RESULTS

After assessing the various models and eliminating those that produced non-physical parameters, we determined that the reduced-χ2 values of the remaining models did not require introducing a systematic error. If we had found large variations in the reduced-χ2 values, or in any of the calculated parameters, we would have implemented a more sophisticated δ-χ2 test to determine systematic biases. However, no such adjustments were necessary, and much greater confidence can be placed in the method for calculation of the spectroscopic factors in argon-40 and titanium-48. Our model analysis can be extended to other isospin nonsymmetric nuclei for neutrino-nucleus scattering in future experiments. See Figure 5 (top) for reduced-χ2 values of argon and Figure 6 (bottom) for titanium.





Figs. 5 (above) and 6 (below): Reduced-χ2 values show good agreement among several sets of models after non-physical models (in red) are removed. This indicates negligible systematic bias in the methodology.

CONCLUSIONS

Our results reveal that there is no need for an additional systematic error estimation for choice-of-fit in either argon-40 or titanium-48. This was especially well demonstrated for argon, which has more theoretical and computational development than titanium.

Future work with the JLab experiment focuses on achieving the same progress with the titanium minimization, refining our model of argon's neutron spectral functions. When the argon-40 neutrino-nucleus scattering is fully realized, far greater accuracy can be placed in Lar-TPCs like DUNE, and physics beyond the Standard Model can be fully investigated.

ACKNOWLEDGEMENTS

We acknowledge the Virginia Tech Center for Neutrino Physics for offering this REU program, Prof. Camillo Mariani for his attentive mentorship, and finally the National Science Foundation for their financial support in making this possible.

REFERENCES

- B. Abi, *et al.* (The DUNE Collaboration), Journal of Instrumentation 15, T08008 (2020).
- L. Gu, *et al.* (The Jefferson Lab Hall A Collaboration), Physical Review C103, 034604 (2021).
- J. Mougey, *et al.* Nuclear Physics A262, 461 (1976).
- A. Batz. Data Analysis of (e, e'p) Argon and Titanium Electron Scattering