Determination of the Ti Spectral Function from JLab (e,e'p) Data

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Introduction

The Deep Underground Experiment (DUNE) will study neutrino oscillation to gain insight on the origin of matter, the relationship between the four forces, and black hole formation, among other fields. It will consist of a near detector, a far detector, and a neutrino beamline. In the detectors, the neutrinos and antineutrinos will interact with the protons and neutrons in the argon-40 nuclei, respectively. Since the energy of the incoming neutrinos will not be known precisely, it is essential to understand the nuclear structure of argon to reconstruct the beam energy and find the parameters of the oscillation probability. (e,e'p) scattering in titanium-48 was used to model the spectral function of argon's neutrons as (e,e'n) scattering is not efficient.

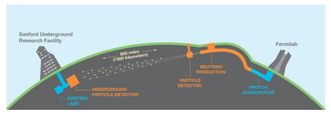

Figure 1: DUNE set-up

$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Equation 1: Neutrino oscillation probability

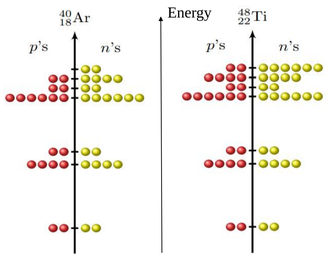

Figure 2: Proton and neutron shell occupancy of argon and titanium

Experimental Data

	E'_e (GeV)	θ_e (deg)	Q^2 (GeV^2/c^2)	$\|\mathbf{p}'\|$ (MeV/c)	$T_{p'}$ (MeV)	$\theta_{p'}$ (deg)	$\|\mathbf{q}\|$ (MeV/c)	p_m (MeV/c)
kin1	1.799	21.5	0.549	915	372	−50.0	865	50
kin2	1.716	20.0	0.460	1030	455	−44.0	846	184
kin3	1.799	17.5	0.370	915	372	−47.0	741	174
kin4	1.799	15.5	0.291	915	372	−44.5	685	230

Table 1: Kinematics used where E'_e is the outgoing electron's energy, θ_e the incoming electron angle, Q^2 the 4-momentum transfer squared, and $|\mathbf{p}'|$ the momentum, T'_p the kinetic energy, θ'_p the angle of the outgoing proton, and p_m the missing momentum

In the experiment E12-14-012 at Thomas Jefferson National Accelerator Facility (JLab) an electron beam with an energy of 2.2 GeV was aimed at gaseous argon and solid titanium, respectively. The resulting particles were detected using a proton and an electron spectrometer. The difference in momentum and energy in the interaction are the missing momentum p_m and the missing energy E_m. For titanium, 4 kinematics with varying incoming angles were used to sample different missing momentum.

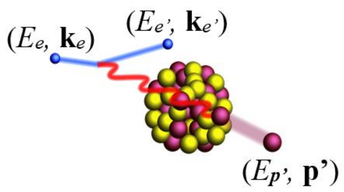

Figure 3: (e,e'p) scattering

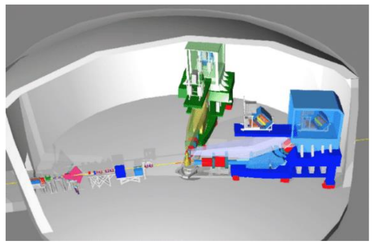

Figure 4: Experimental set-up

Data Analysis

We measured the cross section $\frac{d^6\sigma}{d\omega d\Omega_{e'} dT_{p'} d\Omega_{p'}} = \frac{Y(p_m, E_m)}{B \times lt \times p \times V_B \times C_{rad}}$ from the data and found the reduced cross section as a function of p_m and E_m, respectively. By dividing out the elementary cross section σ_{ep} – for which we used de Forest's σ_{CC1} – and the nuclear transparency $T(E_{p'})$ from $\frac{d^6\sigma_{IA}}{d\Omega_{e'} dE_{e'} d\Omega_{p'} dE_{p'}} \propto \sigma_{ep} P(p_m, E_m) T(E_{p'})$, we obtained the spectral functions, $P(p_m)$ and $P(E_m)$. The spectral function consists to 80% of the mean field and to 20% of the correlated part. We extracted the spectroscopic factors from the spectral functions and the peak energy positions and their distribution widths from $P(E_m)$. The spectroscopic factors were normalized to 80% of 22 for the total strength of the orbitals and to 20% of 22 for the correlated part.

Fitting Procedure

We fitted the previously obtained data against a Monte Carlo simulation using a ROOT code for each p_m and E_m. With TMinuit we minimized χ^2. The p_m fit had 9 fitting parameters, while the E_m fit had 25. For better comparison of the goodness of fit, we used the reduced χ^2 value, which is normalized by the number of degree of freedom. This means it is optimal at 1; if the value is larger than 1, the data does not match the fit well; and if it is smaller than 1, the data is either overfitted or the error is overestimated.

Parameter	Value (MeV)	Uncertainty (MeV)
$E(1f_{7/2})$	11.32	0.10
$E(1d_{3/2})$	12.30	0.24
$E(2s_{1/2})$	12.77	0.25
$E(1d_{5/2})$	15.86	0.20
$E(1d_{5/2}) - E(1d_{3/2})$	3.57	0.31
$E(1p_{3/2}) - E(1p_{1/2})$	6.36	0.75

Table 2: Constraints on the E_m fit

$$\chi^2 = \sum_i \chi_i^2 = \sum_i \left(\frac{\sigma_i^{\text{red, obs}} - \sum_\alpha S_\alpha f_\alpha^{\text{pred}}(i)}{\sigma_{\sigma_i^{\text{red, obs}}}} \right)^2$$

Equation 2: χ^2 used for fit analysis

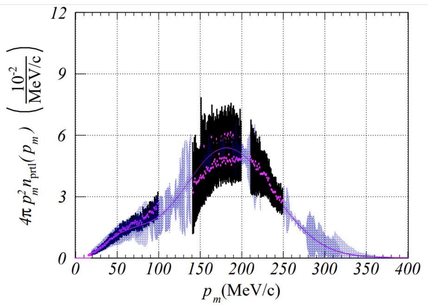

Figure 5: Partial momentum distribution for the missing energy range of 0–30 MeV

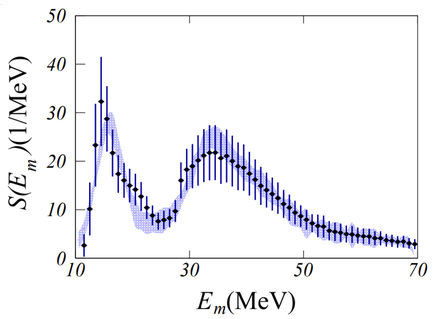

Figure 6: Missing energy distribution for kinematics 2

Missing Momentum

We obtained the spectroscopic factors with and without correlated part of the spectral function. The total spectroscopic strengths, which should be at 22 (as the total number of protons), match within uncertainty, so there is no large bias. The reduced $\chi 2$ value is slightly better without the correlated part at 0.57 but still either overfitted or the error is overestimated.

		w/ corr.	w/o corr.
α	N_α	S_α	
$1f_{7/2}$	2	0.83 ± 1.17	0.78 ± 1.35
$1d_{3/2}$	4	1.17 ± 0.22	1.34 ± 0.10
$2s_{1/2}$	2	2.02 ± 0.08	2.18 ± 0.08
$1d_{5/2}$	6	2.34 ± 1.34	2.34 ± 3.72
$1p_{1/2}$	2	2.46 ± 0.27	2.71 ± 1.19
$1p_{3/2}$	4	5.46 ± 1.69	5.46 ± 0.05
$1s_{1/2}$	2	2.17 ± 0.09	2.51 ± 0.08
corr.	0	1.20 ± 0.09	excluded
$\sum_\alpha S_\alpha$		22.16 ± 5.31	20.17 ± 4.14
d.o.f.		675	676
χ^2/d.o.f.		0.49	0.57

Table 3: Spectroscopic factors of p_m fit

Missing Energy

		all priors	w/o p_m	w/o corr.
α	N_α	S_α		
$1f_{7/2}$	2	1.53 ± 0.25	1.55 ± 0.28	1.24 ± 0.22
$1d_{3/2}$	4	2.79 ± 0.37	3.15 ± 0.54	3.21 ± 0.37
$2s_{1/2}$	2	2.00 ± 0.11	1.78 ± 0.46	2.03 ± 0.11
$1d_{5/2}$	6	2.25 ± 0.16	2.34 ± 0.19	3.57 ± 0.29
$1p_{1/2}$	2	2.00 ± 0.20	1.80 ± 0.27	2.09 ± 0.19
$1p_{3/2}$	4	2.90 ± 0.20	2.92 ± 0.20	4.07 ± 0.15
$1s_{1/2}$	2	2.14 ± 0.10	2.56 ± 0.30	2.14 ± 0.11
corr.	0	1.07 ± 0.07	0.96 ± 0.11	excluded
$\sum_\alpha S_\alpha$		20.72 ± 0.78	20.76 ± 1.05	19.10 ± 0.59
d.o.f		121	153	125
χ^2/d.o.f.		0.95	0.71	1.23

	E_α (MeV)			σ_α (MeV)	
α	w/ priors		w/o priors	w/ priors	w/o priors
$1f_{7/2}$	11.32 ± 0.10		11.31 ± 0.10	8.00 ± 5.57	8.00 ± 6.50
$1d_{3/2}$	12.30 ± 0.24		12.33 ± 0.24	7.00 ± 0.61	7.00 ± 3.84
$2s_{1/2}$	12.77 ± 0.25		12.76 ± 0.25	7.00 ± 3.76	7.00 ± 3.84
$1d_{5/2}$	15.86 ± 0.20		15.91 ± 0.22	2.17 ± 0.27	2.23 ± 0.29
$1p_{1/2}$	33.33 ± 0.60		33.15 ± 0.65	3.17 ± 0.45	3.03 ± 0.48
$1p_{3/2}$	39.69 ± 0.62		39.43 ± 0.68	5.52 ± 0.70	5.59 ± 0.70
$1s_{1/2}$	53.84 ± 1.86		52.00 ± 3.13	11.63 ± 1.90	13.63 ± 2.59
corr.	25.20 ± 0.02		25.00 ± 0.29	—	—

Table 4: Spectroscopic factors of E_m fit on the top, and the peak energy positions and their distribution widths on the bottom

We found the spectroscopic factors and peak energy positions and distribution widths with and without the p_m fit. Furthermore, we obtained the spectroscopic factors without the correlated spectral function. The total spectroscopic strength with and without the p_m fit match within uncertainty. There is a difference in the total spectroscopic strength when not using the correlated spectral function but it is small, indicating that there is no large bias. The peak energy positions and their distribution widths all match within uncertainty with and without the p_m fit. The reduced $\chi 2$ value is best when using all priors at 0.95.

Acknowledgements

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